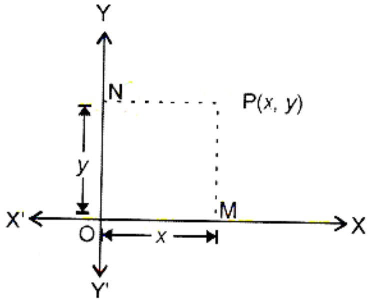
Chapter 7 Coordinate Geometry

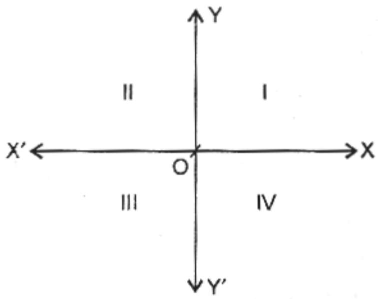
In the Chapter

In this chapter, you will be studying the following points:

- **Introduction :** It is a branch of mathematics which sets up a definite correspondence between the position of a point in a plane and a pair of algebraic numbers, called co-ordinates.
- **Co-ordinate Axes :** The adjoining figure shows two number line XOX' and YOY' intersecting each other at fixed point. Let O be the fixed point called origin and XOX' and YOY', the two perpendicular lines through O, called cartesian or rectangular co-ordinates axes.



- Axes of co-ordinates : In the figure OX and OY are called as x-axis and y-axis respectively and both together are know as axies of co-ordinates.
- **Abscissa :** The distance of the point P from Y-axis is called its abscissa. In the figure OM is the abscissa.
  The distance of the point P from x-axis is called its ordinate. On is the ordinate in the figure.
- **Quadrant :** The co-ordinate axis i.e., the horizontal axis XOX' divide the plane into four parts called the quadrants.
  (i) XOY is called the first quadrant.
  (ii) YOX' is called the second quadrant.
  (iii) X'OY' is called the third quadrant.
  (iv) Y'OX is called the fourth quadrant.



NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 7.1

Q.1. Find the distance between the following pair of points :

(i) (2, 3), (4, 1)

(ii) (– 5, 7), (– 1, 3)

(iii) (*a*, *b*), (– *a*, – *b*)

Ans. (i) Let A(2, 3) and B(4, 1) be the given points

Here, $x_1 = 2, y_1 = 3, x_2 = 4, y_2 = 1$

$\therefore$ $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4-2)^2 + (1-3)^2}$

$= \sqrt{(2)^2 + (-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$

(ii) Let A(–5, 7) and B(–1, 3) be the given points.

Here, $x_1 = -5, y_1 = 7$ and $x_2 = -1, y_2 = 3$.

$$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-1+5)^2 + (3-7)^2}$$

$$= \sqrt{(4)^2 + (-4)^2}$$

$$= \sqrt{16+16} = 4\sqrt{2}$$

(iii) Let A(a, b) and B(–a, –b) be the given points.
Here, $x_1 = a, y_1 = b$ and $x_2 = -a, y_2 = -b$

$$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-a-a)^2 + (-b-b)^2}$$

$$= \sqrt{(-2a)^2 + (-2b)^2}$$

$$= \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2}$$

Q.2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Ans. Let the given points be A(0, 0) and B(36, 15) then,

$$AB = \sqrt{(36-0)^2 + (15-0)^2}$$

$$= \sqrt{(36)^2 + (15)^2}$$

$$= \sqrt{1296+225} = \sqrt{1521}$$

$$= 39$$

Yes, we can find the distance between the two towns A and B discussed in section 7.2 and this distance = 39 km.

Q.3. Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.

Ans. Let the given points be A(1, 5), B(2,3) and C(–2, –11). Then

$$AB = \sqrt{(2-1)^2 + (3-5)^2}$$

$$\Rightarrow \quad AB = \sqrt{(1)^2 + (-2)^2}$$

$$\Rightarrow \quad AB = \sqrt{1+4} = \sqrt{5}$$

$$AC = (-2-1)^2 + (-11-5)^2$$

$$\Rightarrow \quad AC = \sqrt{(-3)^2 + (-16)^2}$$

$$\Rightarrow \quad AC = \sqrt{9+256}$$

$$\Rightarrow \quad AC = \sqrt{256}$$

$$\text{and,} \quad BC = \sqrt{(-2-2)^2 + (-11-3)^2}$$

$$\Rightarrow \quad BC = \sqrt{(-4)^2 + (-14)^2}$$

$$\Rightarrow \quad BC = \sqrt{16+196}$$

$$\Rightarrow \quad BC = \sqrt{212}$$

Here, we see that AB + BC ≠ AC, BC + AC ≠ AB and AB + AC ≠ BC.

Hence, the points A, B and C are not collinear.

Q.4. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

Ans. Let the given points be A(5,–2), B(6,4) and C(7,–2). Then

$$AB = \sqrt{(6-5)^2 + \{4-(-2)\}^2}$$

$$= \sqrt{(1)^2 + (6)^2} = \sqrt{1+36}$$

$$= \sqrt{37}$$

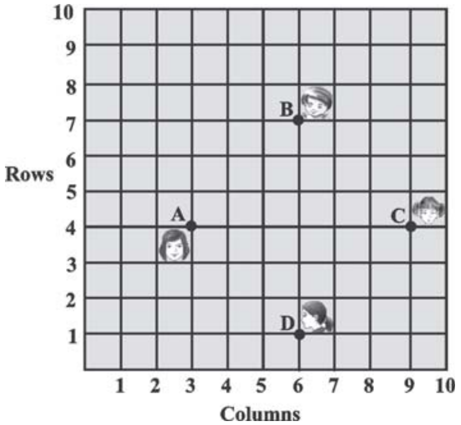
$$BC = \sqrt{(7-6)^2 + (-2-4)^2}$$

$$= \sqrt{(1)^2 + (-6)^2}$$

$$= \sqrt{1+36} = 37$$

Since, AB = BC

Q.5. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 7.8. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.



Ans. Let A(3,4), B (6,7), C(9,4) and D(6,1) be the given points. Then

Now, $AB = \sqrt{(6-3)^2+(7-4)^2}$

$= \sqrt{(3)^2+(3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(9-6)^2+(4-7)^2}$

$= \sqrt{(3)^2+(-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(6-9)^2+(1-4)^2}$

$= \sqrt{(-3)^2+(-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$DA = \sqrt{(3-6)^2+(4-1)^2}$

$= \sqrt{(-3)^2+(3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$AC = \sqrt{(9-3)^2+(4-4)^2} = 6$

$BD = \sqrt{(6-6)^2+(1-7)^2} = 6$

We see that,

$AB = BC = CD = DA$

and $AC = BD$

Therefore, ABCD is a square.

Here, Champa is correct.

Q.6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (– 1, – 2), (1, 0), (– 1, 2), (– 3, 0)

(ii) (–3, 5), (3, 1), (0, 3), (–1, – 4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Ans. (i) Let the given points be A(–1, –2), B(1,0), C(–1,2) and D(–3,0)

Then, $AB = \sqrt{\{1-(-1)\}^2+\{0-(-2)\}^2}$

$\Rightarrow AB = \sqrt{(1+1)^2+(-2)^2}$

$\Rightarrow AB = \sqrt{(2)^2+(-2)^2}$

$\Rightarrow AB = \sqrt{4+4} = \sqrt{8}$

$BC = \sqrt{(-1-1)^2+(2-0)^2}$

$\Rightarrow BC = \sqrt{4+4} = \sqrt{8}$

$CD = \sqrt{\{-3-(-1)\}^2+(0-2)^2}$

$\Rightarrow CD = \sqrt{(-3+1)^2+(-2)^2}$

$\Rightarrow CD = \sqrt{(-2)^2+(-2)^2}$

$\Rightarrow CD = \sqrt{4+4} = \sqrt{8}$

and $DA = \sqrt{\{-1-(-3)\}^2+(-2-0)^2}$

$\Rightarrow DA = \sqrt{(-1+3)^2+(-2)^2}$

$\Rightarrow DA = \sqrt{(2)^2+(-2)^2}$

$\Rightarrow DA = \sqrt{4+4} = \sqrt{8}$

Thus, we have

$AB = BC = CD = DA$

$\Rightarrow$ All sides are equal.

Also, $AC = \sqrt{\{-1-(-1)\}^2+\{2-(-2)\}^2}$

$\Rightarrow AC = \sqrt{(-1+1)^2+(2+2)^2}$

$\Rightarrow AC = \sqrt{0+(4)^2}$

$\Rightarrow AC = \sqrt{16} = 4$

and $BD = \sqrt{(-3-1)^2+0}$

$\Rightarrow = \sqrt{(-4)^2} = \sqrt{16} = 4$

Since, all four sides of the quadrilateral are equal and diagonals are also equal, so the given points form a square.

(ii) Let the given points be A(–3, 5), B(3, 1), C(0, 3) and D(–1,–4)

Then, $AB = \sqrt{\{3-(-3)\}^2+(1-5)^2}$

$\Rightarrow AB = \sqrt{(3+3)^2+(1-5)^2}$

$\Rightarrow AB = \sqrt{(6)^2+(-4)^2}$

$\Rightarrow$ AB = $\sqrt{36+16} = \sqrt{52}$

BC = $\sqrt{(0-3)^2+(3-1)^2}$

$\Rightarrow$ BC = $\sqrt{(-3)^2+(2)^2}$

$\Rightarrow$ BC = $\sqrt{9+4} = \sqrt{13}$

AC = $\sqrt{(-3)^2+4} = \sqrt{13}$

$\therefore$ AB = BC + AC

$\therefore$ A, B and C are collinear

So, quadrilateral ABCD is not formed.

(iii) Let the given points be A(4,5), B(7,6), C(4,3) and D(1,2)

Then AB = $\sqrt{(7-4)^2+(6-5)^2}$

$\Rightarrow$ AB = $\sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}$

BC = $\sqrt{(4-7)^2+(3-6)^2}$

$\Rightarrow$ BC = $\sqrt{(-3)^2+(-3)^2}$

$\Rightarrow$ BC = $\sqrt{9+9} + \sqrt{18}$

CD = $\sqrt{(1-4)^2+(2-3)^2}$

$\Rightarrow$ CD = $\sqrt{(-3)^2+(-1)^2}$

$\Rightarrow$ CD = $\sqrt{9+1} = \sqrt{10}$

and DA = $\sqrt{(4-1)^2+(5-2)^2}$

$\Rightarrow$ DA = $\sqrt{(3)^2+(3)^2}$

$\Rightarrow$ DA = $\sqrt{9+9} = \sqrt{18}$

Here, we have

AB = CD = $\sqrt{10}$

And, BC = DA = $\sqrt{18}$

$\Rightarrow$ Opposite sides of quadrilateral are equal.

Also, AC = $\sqrt{(4-4)^2+(3-5)^2}$

$\Rightarrow$ AC = $\sqrt{0+(-2)^2}$

$\Rightarrow$ AC = $\sqrt{0+4} = \sqrt{4} = 2$

and BD = $\sqrt{(1-7)^2+(2-6)^2}$

$\Rightarrow$ BD = $\sqrt{(-6)^2+(-4)^2}$

$\Rightarrow$ BD = $\sqrt{36+16} = \sqrt{52}$

Here, sides AB = DC and BC and AD. Therefore, quadrilateral formed by given points is a parallelogram.

Q.7. Find the point on the *x*-axis which is equidistant from (2, –5) and (–2, 9).

Ans. Let P(x, 0) be a point on axis and A(2, –5) a nd B (–2, 9) are two given points from which P is equidistant.

$\therefore$ PA = PB

$\Rightarrow$ $PA^2 = PB^2$

$\Rightarrow$ $(2-x)^2+(-5-0)^2 = (-2-x)^2+(9-0)^2$

$\Rightarrow$ $4+x^2-4x+25 = 4+x^2+4x+81$

$\Rightarrow$ $8x = -56$

$\Rightarrow$ $x = -7$

$\therefore$ Thus, required point on the x-axis is (–7, 0).

Q.8. Find the values of *y* for which the distance between the points P(2, – 3) and Q(10, *y*) is 10 units.

Ans. Here PQ = 10 units, (given)

$\Rightarrow$ $PQ^2 = 100$

$\Rightarrow$ $(10-2)^2+(y+3)^2 = 100$

$\Rightarrow$ $(8)^2+y^2+8+6y = 100$

$\Rightarrow$ $64+y^2+6y = 91$

$\Rightarrow$ $y^2+6y-27 = 0$

$\Rightarrow$ $y^2+9y-3y-27 = 0$

$\Rightarrow$ $y(y+9)-3(y+9) = 0$

$\Rightarrow$ $(y+9)(y-3) = 0$

$\Rightarrow$ $y = -9$ or 3.

Q.9. If Q(0, 1) is equidistant from P(5, –3) and R(*x*, 6), find the values of *x*. Also find the distances QR and PR.

Ans. Here, PQ = QR (given)

$\therefore$ $PQ^2 = QR^2$

$\Rightarrow$ $(5-0)^2+(-3-1)^2 = (x-0)^2+(6-1)^2$

$\Rightarrow$ $(5)^2+(-4)^2 = (x)^2+(5)^2$

$\Rightarrow$ $25+16 = x^2+25$

$\Rightarrow$ $x^2 = 16$

$\therefore$ $x = +4$.

Therefore, co-ordinates of R are R (+4, 6)

Now, QR = $\sqrt{(0\pm4)^2+(1-6)^2} = \sqrt{41}$

PR = $\sqrt{(\pm4-5)^2+\{6-(-3)\}^2}$

$= \sqrt{(4-5)^2+81}$ or $\sqrt{(-4-5)^2+81}$

$= \sqrt{82}$ or $9\sqrt{2}$

Q.10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (– 3, 4).

Ans. Let P (x, y), A (3, 6) and B (–3, 4) are the given points.

$\therefore$ According to the question, PA = PB

$$\Rightarrow \quad (PA)^2 = (PB)^2$$

$$\Rightarrow \quad (x-3)^2+(y-6)^2 = (x+3)^2+(y-4)^2$$

$$\Rightarrow x^2-6x+9+y^2-12y+36 = x^2+9+6x+y^2+16-8$$

$$\Rightarrow \quad 12x+4y-20 = 0$$

$$\Rightarrow \quad 3x+y-5 = 0$$

EXERCISE 7.2

Q.1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.

Ans. Let the coordinates of the point is (x, y)

Here, $x_1 = -1, y_1 = 7, x_2 = 4, y_2 {-3}, m_1 = 2, m_2 = 3.$

$$\therefore \quad x = \frac{m_1x_2+m_2x_1}{m_1+m_2} = \frac{2\times4+3\times(-1)}{2+3}$$

$$= \frac{8-3}{5}$$

$$= \frac{5}{5} = 1$$

$$\text{and} \quad y = \frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{2\times(-3)+3\times(7)}{2+3}$$

$$= \frac{-6+21}{5}$$

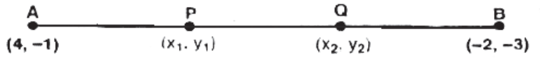
$$= \frac{15}{5} = 3.$$

Hence, required coordinates are (1, 3).

Q.2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

Ans. Let (x_1, y_1) and $Q(x_2, y_2)$ be the points of trisection of the line segment AB, where A (4, –1) and B (–2, 3)

So, AP = PQ= QB



$$\therefore \quad AQ : QB = 2 : 1$$

$$\therefore \quad x_2 = \frac{2\times(-2)+1\times(4)}{2+1}$$

$$= \frac{-4+3}{3} = \frac{0}{3} = 0$$

$$y_2 = \frac{2\times(-3)+1\times(-1)}{2+1}$$

$$= \frac{-6-1}{3} = \frac{-7}{3}$$

The coordinates of Q are $\left(0, \frac{-7}{3}\right)$, which is also, the mid point of PB.

$$\therefore \quad \frac{x_1+(-2)}{2} = 0 \quad \text{and} \quad \frac{y_1+(-3)}{2} = \frac{-7}{3}$$

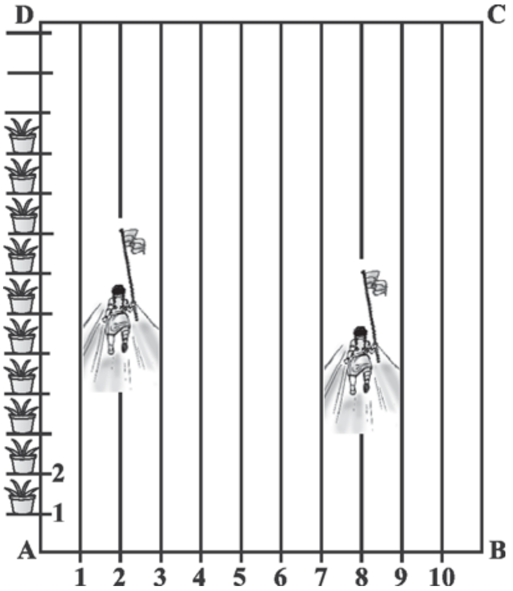
$$\Rightarrow x_1 - 2 = 0, \qquad \Rightarrow y_1 - 3 = \frac{-14}{3}$$

$$\Rightarrow x_1 = 2, \qquad \Rightarrow y_1 = \frac{-5}{3}$$

$$p\,(x_1, y_1) = P\left(2, -\frac{5}{3}\right)$$

$\therefore$ The required coordinate of the two points of trisection are $\left(2, \frac{-5}{3}\right)$ and $\left(0, \frac{-7}{3}\right)$.

Q.3. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. 7.12. Niharika runs $\frac{1}{4}$ th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$ th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Ans. From figure, the position of green flag posted by Nikarika is given by

$$P\left(2, \frac{1}{4}\times 100\right) i.e., P(2, 25)$$

and that of red flag posted by Preet is given by

$$Q\left(8, \frac{1}{5}\times 100\right) i.e., Q(8, 20)$$

Now, $\quad PQ = \sqrt{(8-2)^2 + (20-25)^2}$

$$= \sqrt{(6)^2 + (-5)^2}$$

$$= \sqrt{36+25}$$

$$= \sqrt{61}$$

$\therefore$ The required distance between both the flag = $\sqrt{61}$ m.

Again, let R be the position of the blue flag posted by Rashmi in the halfway of the line segment PQ.

$$\therefore \quad R = \left(\frac{2+8}{2}, \frac{25-20}{2}\right)$$

$$\Rightarrow \quad R = (5, 22.5)$$

Hence, the blue flag is on the 5th line at a distance of 22.5 m above AB.

Q.4. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

Ans. Let, the ratio of k : 1.

$$\therefore \quad -1 = \frac{k\times(6) + 1\times(-3)}{k+1},$$

Using $\quad x = \dfrac{m_1x_2 + m_2x_1}{m_1+m_2}$

$\Rightarrow \quad -(k+1) = 6k - 3$

$\Rightarrow \quad 7k = 2$

$\Rightarrow \quad k = \dfrac{2}{7}$

$\therefore \quad k : 1 = 2 : 7$

Hence, the required ratio is 2 : 7.

Q.5. Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the *x*-axis. Also find the coordinates of the point of division.

Ans. Let the required ratio of k : 1. So, the coordinates of the point M of division A(1, –5) and B(–4, 5) are $\left(\dfrac{kx_2 + 1.x_1}{k+1}, \dfrac{ky_2 + 1.y_1}{k+1}\right)$

i.e., $\left(\dfrac{-4k+1}{k+1}, \dfrac{5k-5}{k+1}\right)$

But according to question, line segment joining A(1,–5) and B (–4, 5) is divided by the x-axis. So y-coordinates must be zero.

\ $\quad \dfrac{5k-5}{k+1} = 0$

Þ $\quad 5k - 5 = 0$

Þ $\quad 5k = 5$

Þ $\quad k = 1$

So, the required ratio is 1 : 1 and the point of division M is $\left(\dfrac{-4(1)+1}{1+1}, \dfrac{5(1)-5}{1+1}\right)$ *i.e.,* $\left(\dfrac{-4+1}{2}, 0\right)$

i.e., $\left(-\dfrac{3}{2}, 0\right)$

Q.6. If (1, 2), (4, *y*), (*x*, 6) and (3, 5) are the vertices of a parallelogram taken in order, find *x* and *y*.

Ans. Let P(1, 2), Q (4, y), R (x, 6) and S (3, 5) are the given vertices of parallelogram.

$\therefore$ Mid point of PR = Mid point of QS

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

$$\Rightarrow \left(\frac{1+x}{2}, 4\right) = \left(\frac{7}{2}, \frac{y+5}{2}\right)$$

$$\therefore \frac{1+x}{2} = \frac{7}{2} \text{ and } 4 = \frac{y+5}{2}$$

$\Rightarrow$ $x = 6$ and $y = 3$.

Q.7. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).

Ans. Let the coordinates of point A be (x, y)

We know that centre is mid point of the diameter

$$\therefore 2 = \frac{x+1}{2} \quad \text{and} \quad -3 = \frac{y+4}{2}$$

$x = 3$ and $y = -10$.

Hence, the required coordinates of point A are (3, – 10),

Q.8. If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB.

Ans. Let the coordinates of P be (x, y)

Here, $$\frac{AP}{AB} = \frac{3}{7}$$

or $$\frac{AP}{AB - AP} = \frac{3}{7-3}$$

or $$\frac{AP}{PB} = \frac{3}{4}$$

Let P (x, y) divides the join A (–2, –2) and B (2, –4) in the ratio 3 : 4.

Using the formula,

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

we get, $$x = \frac{3.(2) + 4.(-2)}{3+4}$$

$$= \frac{6-8}{7} = \frac{-2}{7}$$

$$y = \frac{3.(-4) + 4.(-2)}{3+4}$$

$$= \frac{-12-8}{7} = \frac{-20}{7}$$

Hence, the coordinate of P are $\left(-\frac{2}{7}, \frac{-20}{7}\right)$.

Q.9. Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

Ans. Let points P, Q and R divide the line segement AB in four equal parts, such that

AP : PB = 1 : 3, AQ : QB = 1 : 1, AR : RB = 3 :1

Find the coordinate P (x_1, y_1), we take ratio 1 : 3.

$$\therefore x_1 = \frac{1.(2) + 3(-2)}{1+3} \qquad y_1 = \frac{1 \times 8 + 3 \times 2}{1+3}$$

$$= \frac{2-6}{4} \qquad = \frac{8+6}{4}$$

$$= -\frac{4}{4} \qquad = \frac{14}{4}$$

$$x_1 = -1, \qquad y_1 = \frac{7}{2}$$

Now to find the point $Q(x_2, y_2)$, we take ratio 1 : 1.

$$\therefore x_2 = \frac{-2+2}{2} \qquad y_2 = \frac{2+8}{2}$$

$$x_2 = \frac{0}{2} \qquad y_2 = \frac{10}{5}$$

$$x_2 = 0 \qquad y_2 = 5$$

Hence $Q(x_2, y_2) = Q(0, 5)$

Again, to find the point $R(x_3, y_3)$, we take ratio 3 : 1.

$$\therefore x_3 = \frac{3 \times (2) + 1 \times (-2)}{3+1}$$

$$x_3 = \frac{6-2}{4}$$

$$x_3 = \frac{4}{4}$$

$$x_3 = 1$$

$$\therefore y_3 = \frac{3 \times (8) + 1 \times (2)}{3+1}$$

$y_3 = \frac{24+2}{4}$

$y_3 = \frac{26}{4}$

$y_3 = \frac{13}{2}$

$\therefore \quad R(x_3, y_3) = R\left(1, \frac{13}{2}\right)$

Hence, the coordinates of the points, which divide the line segement AB in four equal parts are $\left(-1, \frac{7}{2}\right)$, (0, 5) and $\left(1, \frac{13}{2}\right)$.

Q.10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order.

[Hint : Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)]

Ans. Let A(3, 0), B(4, 5), C(–1, 4) and D(–2, –1) be the vertices of the rhombus ABCD.

$\therefore$ Diagonal, $AC = \sqrt{(-1-3)^2 + (4-0)^2}$

$\left[\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$= \sqrt{(-4)^2 + 4^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

Diagonal, $BD = \sqrt{(-2-4)^2 + (-1-5)^2}$

$= \sqrt{(-6)^2 + (-6)^2}$

$= \sqrt{36+36}$

$= \sqrt{72} = 6\sqrt{2}$

$\therefore$ Area of the rhombus ABCD $= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= 2 \times 6 \times \sqrt{2} \times \sqrt{2} = 12 \times 2$

= 24 sq units.

EXERCISE 7.3

Q.1. Find the area of the triangle whose vertices are :

(i) (2, 3), (–1, 0), (2, – 4)

(ii) (–5, –1), (3, –5), (5, 2)

Ans. (i) Let the given points be A(2,3), B(–1,0) and C(2,–4)

Here, we have

$x_1 = 2, y_1 = 3$

$x_2 = -1, y_2 = 0$

and $x_3 = 2, y_3 = -4$

Now, Area of ΔABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}\{2[0-(-4)] + (-1)[-4-3] + 2(3-0]\}$

$= \frac{1}{2}(8+7+6)$

$= \frac{1}{2} \times 21 = \frac{21}{2}$ sq. units.

(ii) Let the given points be A(–5, –1), B(3, –5) and C(5, 2).

Here, we have

$x_1 = -5, y_1 = -1$

$x_2 = 3, y_2 = -5$

and $x_3 = 5, y_3 = 2$

Now, Area of ΔABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[(-5)\{-5-2\} + (3)\{2-(-1)\} + (5)\{(-1)-(-5)\}]$

$= \frac{1}{2}[35 + 9 + 20] = 32$ square units.

Q.2. In each of the following find the value of '*k*', for which the points are collinear.

(i) (7, –2), (5, 1), (3, *k*)

(ii) (8, 1), (*k*, – 4), (2, –5)

Ans. (i) Let the given points be A(7, –2), B(5,1) and C(3, K).

Here, we have,

$x_1 = 7, y_1 = -2$

$x_2 = 5, y_2 = 1$

and $x_3 = 3, y_3 = K$

Now, Area of ΔABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[7(1-k) + 5\{k - (-2)\} + 3(-2-1)]$

$= \frac{1}{2}[7 - 7k + 5k + 10 - 9]$

$= \frac{1}{2}[8 - 2k] = 4 - k.$

If the points are collinear, then area of the triangle = 0

$\Rightarrow \quad 4 - k = 0$

$k = 4$

(ii) Let the given points be A (8,1), B(K,–4) and C(2,–5)

Here, we have

$x_1 = 8, y_1 = 1$

$x_2 = K, y_2 = -4$

and $x_3 = 2, y_3 = -5$

Now, Area of ΔABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[8\{-4 - (-5)\} + k(-5 - 1) + 2\{1 - (-4)\}]$

$= \frac{1}{2}[8 - 6k + 10]$

$= \frac{1}{2}[18 - 6k] = 9 - 3k.$

If the points are collinear, then area of the triangle = 0

$\Rightarrow \quad 9 - 3k = 0$

$\Rightarrow \quad 3k = 9$

$\Rightarrow \quad k = \frac{9}{3} = 3 \Rightarrow k = 3.$

Q.3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

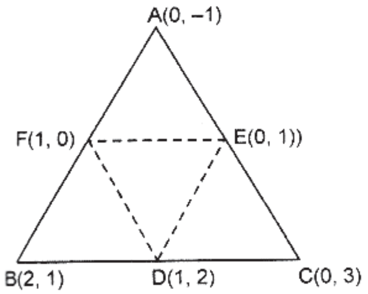
Ans. Let A(0,–1), B(2,1) and C(0,3) be the vertices of ΔABC. Let D, E F be the mid-points of sides BC, CA and AB respectively. Then the coordinates of D, E and F are (1, 2), (0, 1) and (1, 0) respectively.

Now, Area of ΔABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$=$ Area of ΔABC $= \frac{1}{2}[0(1-3) + 2(3-(-1) + 0(0-1)]$

$=$ Area of ΔABC $= \frac{1}{2}[0+8+0] = 4$ sq. units.



Area of ΔABC $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

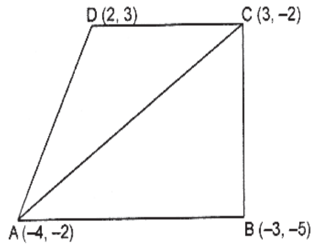
$\Rightarrow$ Area of ΔABC $= \frac{1}{2}[1(1-0) + 0(0-2) + 1(2-1)]$

$\Rightarrow$ Area of ΔABC $= \frac{1}{2}[1+1] = 1$ sq. units.

$\therefore$ Area of ΔABC = Area of ΔABC = 1 : 4.

Q.4. Find the area of the quadrilateral whose vertices, taken in order, are (– 4, – 2), (– 3, – 5), (3, – 2) and (2, 3).

Ans. Let the vertices of quadrilateral are A(– 4, – 2), B(– 3, – 5), C(3, – 2) and D(2, 3).



Area of ΔABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Here, we have

$x_1 = -4, \quad y_1 = -2$
$x_2 = -3, \quad y_2 = -5$
$x_3 = 3 \quad y_3 = -2$

Now, area of ΔABC

$= \frac{1}{2}[-4(-5+2)+(-3)(-2+2)+3(-2+3)]$

$= \frac{1}{2}[(-4\times-3)+(-3\times0)+(3\times3)]$

$= \frac{1}{2}[12+0+9]$

$= \frac{1}{2}\times 21$

= 10.5 sq. units.

We know that, area of ΔACD

$= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

Here, we have

$x_1 = -4, \quad y_1 = -2$
$x_2 = 3, \quad y_2 = -2$
$x_3 = 2 \quad y_3 = 3$

Now, area of ΔABC

$= \frac{1}{2}[(-4)(-2-3)+3(3+2)+2(-2+2)]$

$= \frac{1}{2}[(-4\times-5)+(3-5)+(2\times0)]$

$= \frac{1}{2}[20+15+0] = \frac{1}{2}\times35 = 17.5$ sq. units

Hence, the area of quadrilateral

ABCD = ar (ΔABC) + ar (ΔACD)
= (10.5 + 17.5)
= 28 sq. units.

Q.5. You have studied in Class IX, (Chapter 9, Example 3), that a median of a triangle divides it into two triangles of equal areas. Verify this result for Δ ABC whose vertices are A(4, – 6), B(3, –2) and C(5, 2).

Ans. ∴ Area of ΔABC

Here, we have

$x_1 = 4, \quad y_1 = -6$
$x_2 = 3, \quad y_2 = -2$
and $x_3 = 5 \quad y_3 = 2$

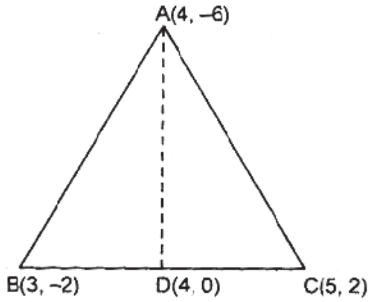
$= \frac{1}{2}[(4\times-2)+(3\times2)+(5\times-6) - (3\times-6+5\times-2+4\times2)]$

⇒ Area of ΔABC

$= \frac{1}{2}[(-8+6-30)-(-18-10+8)]$

⇒ Area of ΔABC

$= \frac{1}{2}[-32+20] = 6$ sq. units.



⇒ Area of ΔABC

$= \frac{1}{2}[\{(4\times(-2)+3\times0+4\times(-6)\} - \{3\times(-6)+4\times(-2)+4\times0\}]$

⇒ Area of ΔABC

$= \frac{1}{2}[(-8+0-24)-(-18-8+0)]$

⇒ Area of ΔABC

$= \frac{1}{2}[(-32+26)] = 3$ sq. units.

$\therefore \quad \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta ABD} = \frac{6}{3} = \frac{2}{1}$

⇒ Area of ΔABC = 2 (Area of ΔABD)

EXERCISE 7.4 (Optional)

Q.1. Determine the ratio in which the line 2x + y – 4 = 0 divides the line segment joining the points A(2, – 2) and B(3, 7).

Ans. Let the line $2x + y - 4 = 0$ divide the line segment joining the points A(2, –2) and B(3, 7) in the radio λ : 1. Let the point of intersection be P.

Then,

Co-ordinates of p are given by

$$P\left(\frac{(\lambda)(3)+(1)(2)}{\lambda+1}\right),\left(\frac{(\lambda)(7)+(1)(-2)}{\lambda+1}\right)$$

$$\Rightarrow \quad P\left(\frac{3\lambda+2}{\lambda+1},\frac{7\lambda-2}{\lambda+1}\right)$$

P lies on the line $2x+y-4=0$

$$\therefore \quad 2\left(\frac{3\lambda+2}{\lambda+1}\right)+\left(\frac{7\lambda-2}{\lambda+1}\right)-4 = 0$$

$\Rightarrow \quad 2(3\lambda+2)+(7\lambda–2)–4(\lambda+1) = 0$

$\Rightarrow \quad 6x+y+7\lambda-2-4\lambda-4 = 0$

$\Rightarrow \quad 9\lambda-2 = 0$

$$\Rightarrow \quad \lambda = \frac{2}{9}$$

Hence, the required ratios is 2 : 9.

Q.2. Find a relation between *x* and *y* if the points (*x*, *y*), (1, 2) and (7, 0) are collinear.

Ans. If the given points are collinear, then the area of the triangle with these points as vertices will be zero.

$$\therefore \quad \frac{1}{2}[x(2–0)+1(0–y)+7(y–2)] = 0$$

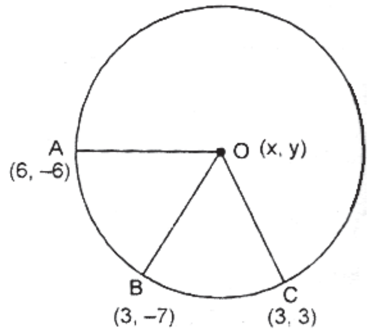
$$\Rightarrow \quad \frac{1}{2}[2x-y+7y-14) = 0$$

$\Rightarrow \quad 2x+6y-14 = 0$

$\Rightarrow \quad x+3y-7 = 0$

Q.3. Find the centre of a circle passing through the points (6, – 6), (3, – 7) and (3, 3).

Let the given point be A(6, –6), B (3, –7) and C(3, 3)



Let the centre of the circle be O(x, y)

Then, OA = OB = OC [radii of circle]

$\Rightarrow \quad (OA)^2 = (OB)^2 = (OC)^2$ ...(i)

From (i) we have

$OA^2 = OB^2$

$\Rightarrow \quad (x–6)^2+(y+6)^2 = (x-3)^2+(y+7)^2$

$(x–6)^2+(y+6)^2 = (x-3)^2+(y+7)^2$

$\Rightarrow \quad x^2-12x+36+y^2+12y+36 = x^2-6x+9+y^2+14y+49$

$\Rightarrow \quad 6x+2y = 14$

$\Rightarrow \quad 4x+y = 7$

Again we have,

$(OB)^2 = (OC)^2$

$\Rightarrow \quad (y+7)^2 = (y-3)^2$

$\Rightarrow \quad y^2+49+14y = y^2+9-6y$

$\Rightarrow \quad 20y = 9-49$

$\Rightarrow \quad 20y = 40$

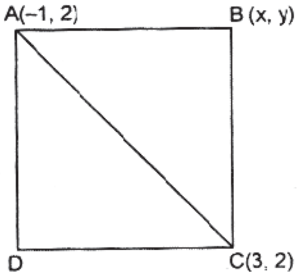
$\Rightarrow \quad y = –2$ ...(ii)

Putting the value of (iii) in (ii) we get, $x = 3$.

Hence, centre of a circle = O(3, –2)

Q.4. The two opposite vertices of a square are (–1, 2) and (3, 2). Find the coordinates of the other two vertices.

Ans. Let A(–1, 2) and C(3,2) be the two opposite vertices of a square ABCD. Let B(x, y) be the unknown vertex.



Then, AB = BC

$\Rightarrow \quad AB^2 = BC^2$

$\Rightarrow \quad (x+1)^2+(y-2)^2 = (x-3)^2+(y-2)^2$

$\Rightarrow \quad x^2+2x+1+y^2-4y+2 = x^2-6x+9+y^2-4y+4$

$\Rightarrow \quad 8x = 8$

$\Rightarrow \quad x = 1$

Also, $AB^2+BC^2 = AC^2$

[$\angle B = 90°$ and therefore using Pythagoras theorem]

$\Rightarrow \quad (x+1)^2+(y-2)^2+(x-3)^2+(y-2)^2$

$$= (3+1)^2 + (2-2)^2$$

$\Rightarrow \quad x^2 + 2x + 1 + y^2 - 4y + 4 + x^2 - 6x + 9 + y^2 - 4y + 4 = 16$

$\Rightarrow \quad 2x^2 + 2y^2 - 4x - 8y + 2 = 0$

$\Rightarrow \quad x^2 + y^2 - 2x - 4y + 1 = 0$

(Dividing throughout by 2]

Putting x = 1, we get

$\Rightarrow \quad 1 + y^2 - 2 - 6y + 1 = 0$

$\Rightarrow \quad y(y-2) = 0$

$\Rightarrow \quad y = 0, 4$

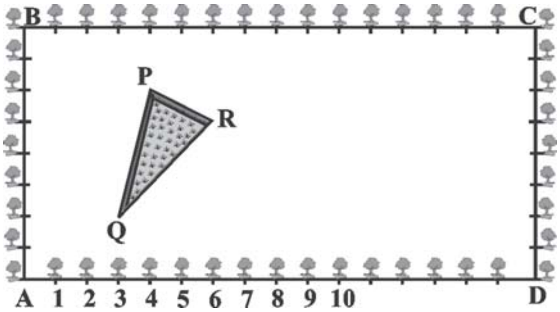
Hence, the other vertices are (1, 0) and (1, 4).

Q.5. The Class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar are planted on the boundary at a distance of 1m from each other. There is a triangular grassy lawn in the plot as shown in the Fig. 7.14. The students are to sow seeds of flowering plants on the remaining area of the plot.

(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin?

Also calculate the areas of the triangles in these cases. What do you observe?



Ans. (i) Taking A as origin, AD and AB as coordiante axis.

The coordinate of P are (3, 6)

The coordinate sof Q are (3, 2)

The coordinates of R are (6, 5)

Area of ΔPQR

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[4(2-5) + 3(5-6) + 6(6-2)]$$

$$= \frac{1}{2}[4(-3) + 3(-1) + 6(4)]$$

$$= \frac{1}{2}[-12 - 3 + 24] = \frac{9}{2} \text{ sqauare units.}$$

(ii) Taking C a origin, CB and CD as coordiante axes.

The coordinates of P are (12, 2)

The coordinates of Q are (13, 6)

The coordinates of R are (10, 3)

Area of ΔPQR

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[12(6-3) + 13(3-2) + 10(2-6)]$$

$$= \frac{1}{2}[12 \times 3 + 13 \times 1 + 10(-4)$$

$$= \frac{1}{2}[36 + 13 = 40]$$

$$= \frac{1}{2}(9) = \frac{9}{2} \text{ square units.}$$

Thus we observe : Areas are the same in both the cases.

Q.6. The vertices of a Δ ABC are A(4, 6), B(1, 5) and C(7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the Δ ADE and compare it with the area of Δ ABC. (Recall Theorem 6.2 and Theorem 6.6).

Ans. We have

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{AB}{AD} = \frac{AC}{AE} = 4$$

$$\Rightarrow \quad \frac{AD + DB}{AD} = \frac{AE + EC}{AE} = 4$$

[Taking reciprocals of both sides]

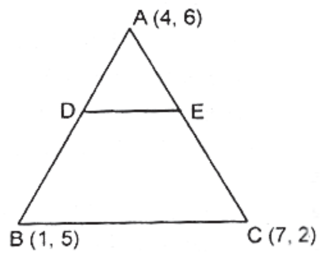
$$\Rightarrow \quad 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE} = 4$$

$$\Rightarrow \quad \frac{DB}{AD} = \frac{EC}{AE} = 3$$

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$$

⇒ AD : DB= AE : EC = 1 : 3

⇒ D and E divide AB and AC respectiely in the ratio 1 : 3.



So, the coordinates of D and E are

$$D\left(\frac{1+12}{1+3},\frac{5+18}{1+3}\right)=E\left(\frac{13}{4},\frac{23}{4}\right)$$

and $E\left(\frac{7+12}{1+3},\frac{2+18}{1+3}\right)=\left(\frac{19}{4},5\right)$ respectively.

Now,

∴ Area of ΔADE

$$=\frac{1}{2}\left|\left(4\times\frac{23}{4}+\frac{13}{4}\times5+\frac{19}{4}\times6\right)-\left(\frac{13}{4}\times6+\frac{19}{4}\times\frac{23}{4}+4\times5\right)\right|$$

⇒ Area of ΔABC

$$=\frac{1}{2}\left|\left(\frac{92}{4}+\frac{65}{4}+\frac{114}{4}\right)-\left(\frac{78}{4}+\frac{437}{16}+29\right)\right|$$

⇒ Area of ΔABC

$$=\frac{1}{2}\left|\left(\frac{271}{4}-\frac{1069}{16}\right)\right|=\frac{1}{2}\times\frac{15}{16}=\frac{15}{32} \text{ sq. units}$$

and

∴ Area of ΔABC

$$=\frac{1}{2}[(4\times5+1\times2+7\times6)-(1\times6+7\times5+4\times2)]$$

⇒ Area of ΔABC

$$=\frac{1}{2}[(20+2+42)-(6+35+8)]$$

⇒ Area of ΔABC

$$=\frac{1}{2}[64-49]=\frac{15}{2} \text{ sq. units.}$$

$$\therefore \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC}=\frac{15/32}{15/2}=\frac{1}{16}$$

Hence, Area of ΔABC = 1 : 16.

Q.7. Let A (4, 2), B(6, 5) and C(1, 4) be the vertices of Δ ABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1

(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.

(iv) What do you observe?

[Note : The point which is common to all the three medians is called the *centroid* and this point divides each median in the ratio 2 : 1.]

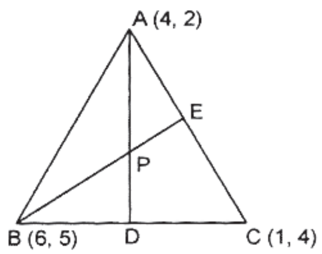
(v) If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Ans. (i) The median from A meets BC at D.

∴ D is the mid-point of BC. Then coordinates of D are

⇒ $D\left(\frac{6+1}{2},\frac{5+4}{2}\right)$ [Using mid-point formula]

⇒ $D\left(\frac{7}{2},\frac{9}{2}\right)$



(ii) Coordinates of P are

$$=P\left(\frac{(2)\left(\frac{7}{2}\right)+(1)(4)}{2+1},\frac{(2)\left(\frac{9}{2}\right)+(1)(2)}{2+1}\right)$$

$$=P\left(\frac{11}{3},\frac{11}{3}\right)$$

(iii) Now coordinates of E are

$$E\left(\frac{4+1}{2},\frac{2+4}{2}\right) \Rightarrow E\left(\frac{5}{2},3\right)$$

Co-ordinates of Q are

$$Q\left(\frac{(2)\left(\frac{5}{2}\right)+(1)(6)}{2+1},\frac{(2)(3)+(1)(5)}{2+1}\right)$$

$$\Rightarrow \quad Q\left(\frac{11}{3},\frac{11}{3}\right)$$

And co-ordinates of F are

$$F\left(\frac{4+6}{2},\frac{2+5}{2}\right) \Rightarrow F\left(5,\frac{7}{2}\right)$$

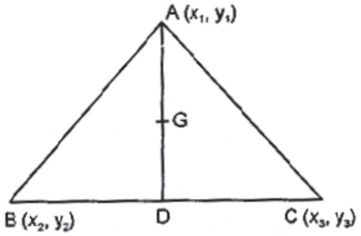
Co-ordinates of R are

$$R\left(\frac{(2)(5)+(1)(1)}{2-1},\frac{(2)\left(\frac{7}{2}\right)+(1)(4)}{2+1}\right)$$

$$\Rightarrow \quad R\left(\frac{11}{3},\frac{11}{3}\right)$$

(iv) We observe that P, Q, R are the same point.

(v) We know that the centroid of a triangle divides each median of the triangle from the vertex in the ratio 2 : 1. Let D be the mid-point of BC. Then,



Co-ordinates of D is

$$= D\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2}\right)$$

Let G(x, y) be the centroid of DABC. Then G will divide AB internally in the ratio 2 : 1.

$$\therefore \quad x = \frac{2\left(\frac{x_2+x_3}{2}\right)+1(x_1)}{2+1}$$

$$= \frac{x_1+x_2+x_3}{3}$$

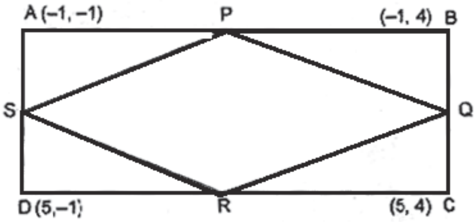
and $$y = \frac{2\left(\frac{y_2+y_3}{2}\right)+1(y_1)}{2+1}$$

$$= \frac{y_1+y_2+y_3}{3}$$

Hence, the co-ordinates of the centroid are given by $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$.

Q.8. ABCD is a rectangle formed by the points A(–1, –1), B(– 1, 4), C(5, 4) and D(5, – 1). P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.

Ans. We have, following given points



A (–1, –1)
B (–1, 4)
C (5, 4)
and D (5, –1)

Therefore, co-ordinates of P are

$$P\left(\frac{-1-1}{2}.\frac{-1+2}{2}\right) \Rightarrow P\left(-1,\frac{3}{2}\right)$$

Co-ordinates of Q are

$$= Q\left(\frac{-1+5}{2},\frac{9+4}{2}\right) \Rightarrow \quad Q(2,4)$$

Co-ordinates of R are

$$= R\left(\frac{5+5}{2},\frac{-1+4}{2}\right) \Rightarrow \quad P\left(5,\frac{3}{2}\right)$$

And, co-ordinates of S are

$$= S\left(\frac{-1+5}{2}.\frac{-1-1}{2}\right) \Rightarrow S(2,-1)$$

Now,

$\therefore \quad PQ = \sqrt{(2+1)^2 + \left(4 - \frac{3}{2}\right)^2}$

$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$

$QR = \sqrt{(5-1)^2 + \left(\frac{3}{2} - 4\right)^2}$

$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$

$RS = \sqrt{(2-5)^2 + \left(-1 - \frac{3}{2}\right)^2}$

$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$

$SP = \sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2}$

$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$

$PR = \sqrt{(5+1)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2} = 6$

$QS = \sqrt{(2-2)^2 + (4+1)^2} = 5$

We see that

PR = QR = RS = SP and PR ≠ QS.

Therefore PQRS is a rhombus.

Additional Questions

Q.1.Points P(–4, 2) lies on the line segment joining the points A(–4, 6) and B(–4, 6)

Ans. We intend to prove that P, A and B are collinear

= – 4 (6 + 6) + (–4) (–6, –2) + (–4) (2– 6)

= – 4 (12) –4(–8) –4 (–4)

=–48 + 32 + 16 = 0

Thus points are collinear.

Hence, P lies on AB and the statement is true.

Q.2. Points A(3,1), B (12, –2) and C(0, 2) cannot be the vertices of a triangle.

Ans.

$AB = \sqrt{(12-3)^3 + (-2-1)^2} = \sqrt{81+9} = \sqrt{90}$

$BC = \sqrt{(12)^2 + (-2-2)^2} = \sqrt{144+16} = \sqrt{160}$

$CA = \sqrt{(0-3)^2 + (2-1)^2} = \sqrt{9+1} = \sqrt{10}$

Clearly AB + CA < BC

Thus, A, B and C do not form a triangle.

Hence, the statement is false.

Q.3.A circle has its centre at the origin and a point P(5, 0) lies on it. The point Q(6, 8) lies outside the circle.

Ans. Radius = $\sqrt{(5)^2 + 9^2} = 5$

$\therefore \quad OQ = \sqrt{(6)^2 + (8)^2} = 10$

Now OQ > 5.

∴ Q lies outside the circle.

Hence the statement is true.

Q.4. Points A(4,3), B(6,4), C(5, –6) and D(–3, 5) are the vertices of a parallelogram.

Ans. $AB = \sqrt{(4-6)^2 + (3-4)^2} = \sqrt{4+1} = \sqrt{5}$

$CD = \sqrt{(5+3)^2 + (-6-5)^2} = \sqrt{64+21} = \sqrt{185}$

$\therefore$ AB ≠ CD

Thus ABCD is not a parallelogram.

Hence the statement is false.

Q.5. Find the points of *x*-axis which are at a distance of $2\sqrt{5}$ from the point (7, –4). How many such points are there ?

Ans. Let the point be P(*x*, 0)

Also let A (7, –4)

Now $\quad PA = 2\sqrt{5}$

$\Rightarrow \quad \sqrt{(x+7)^2 + (0+4)^2} = 2\sqrt{5}$

$\Rightarrow \quad (x-7)^2 + 16 = 20$

$\Rightarrow \quad (x-7)^2 = 4 \Rightarrow x - 7 = +2 \Rightarrow x = 7 + 2$

$\Rightarrow \quad x = 9, 5$

Thus, there are two required points (9, 0) and 5, 0)

Q.6. Find the value of *m* if the points (5, 1), (–2, –3) and (8, 2m) are collinear.

Ans. For collinearity :

$x_1(y_2-y_1)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow$ $5(-3-2m)+(-2)(2m-1)+8(1+3)=0$

$\Rightarrow$ $-15-10m-4m+2+32=0$

$\Rightarrow$ $-14m+19=0 \Rightarrow m=\frac{19}{14}$.

Q.7. Find the area of the triangle whose vertices are (–8, 4), (–6,6) and (–3, 9)

Ans. Area of Δ

$= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$= \frac{1}{2}[-8(6-9)-6(9-4)+(-3)(4-6)]$

$= \frac{1}{2}[24-30+6] = \frac{1}{2}\times 0 = 0$

Hence, Area of $\Delta = 0$.

Q.8. Find the value of *k* if the points A(*k* + 1), 2*k*), B(3*k*, 2*k* + 3) and C(5*k* – 1, 5*k*) are collinear.

Ans. For collinearity

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow$ $(k+1)(2k+3-5k)+3k(5k-2k)+(5k-1)(2k-2k-3)=0$

$\Rightarrow$ $(k+1)(-k+1)+k(3k)-(5k-1)=0$

$\Rightarrow$ $-(k^2+1+3k^2-5k+1=0$

$\Rightarrow$ $-k^2+1+3k^2-5k+1=0$

$\Rightarrow$ $2k^2-5k+2=0$

$\Rightarrow$ $2k^2-4k-k+2=0$

$\Rightarrow$ $2k(k-2)-1(k-2)=0$

$\Rightarrow$ $(k-2)(2k-1)=0 \Rightarrow k=2, \frac{1}{2}$

Q.9. Find a point which is equidistant from the point (A –5, 4) and B(–1, 6)? How many such points are there ?

Ans. Let the required points be P(*x*, *y*)

$\therefore$ PA = PB

$\sqrt{(x+5)^2+(y-4)^2} = \sqrt{(x+1)^2+(y-6)^2}$

$\Rightarrow$ $x^2+25+10x+y^2+16-8 = x^2+2x+1+y^2+36-12y$

$\Rightarrow$ $10x-8y+41 = 2x-12y+37$

$\Rightarrow$ $8x+4y = -4$

$\Rightarrow$ $2x+y = -1$

$\Rightarrow$ $y = -1-2x$

One such point is when $x=0, y=-1$

$\therefore$ Required one point is (0, –1) and there are such infinite number of points which are solutions of $2x+y+1=0$.

Q.10. Raj starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office, what is the extra distance travelled by Raj in reaching his office ? (Assume that distance covered are in straight lines). If the house is stiatuted at (2, 4), Bank at (5, 8), school at 13, 14) and office at (13, 26) and co-ordinates are in km

Ans. Let positions of house is A (2, 4)

Position of Bank is B(5, 8)

Position of school is C (13, 14)

Position of office is D(13, 26).

Distance

$AB = \sqrt{(5-2)^2+(8-4)^2} = \sqrt{9+16} = 5$

$BC = \sqrt{(13-5)^2+(14-8)^2} = \sqrt{64+36} = 10$

$CD = \sqrt{(13-13)^2+(26-14)^2} = 12$

$AD = \sqrt{(13-2)^2+(26-4)^2}$

$= \sqrt{(121-484)} = \sqrt{509} = 24.6$ km

Extra distance travelled

$= (5+10+12)-24.6$

$= 27-24.6 = 2.4$ km

Multiple Choice Questions

Q.1. If (–2, –1), (a, 0), (4, b) and 1,2) are the vertices of a parallelogram, then the values of a and b are :

(a) (1, 3) (b) (1, –1)

(c) –1, 1) (d) (3, –1)

Ans. (a)

Q.2. If the distance between the points (2, –2) and (–1, *x*) is 5, one of the values of *x* is :

(a) –2 (b) 2

(c) –1 (d) 1

Ans. (b)

Q.3. The midpoint of the line-segment joining the point A(–2, 8) and B(–6, –4) is :

(a) (–4, –6) (b) (2, 6)

(c) (–4, 2) (d) (4, 2)

Ans. (c)

Q.4. The point P which divides the line segment joining the points A(2, –5) and B(5, 2) in the ratio 2 : 3 lines in the quadrant.

(a) I (b) II

(c) III (d) IV

Ans. (d)

Q.5. The distance of the point P(2, 3) from the x-axis is :

(a) 2 (b) 3

(c) 1 (d) 5

Ans. (b)

Q.6. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is :

(a) 5 (b) 12

(c) 11 (d) $7 + \sqrt{5}$

Q.7. The area of a triangle with vertices A(3, 0), B(7, 0) and C(8, 4) is :

(a) 14 (b) 28

(c) 8 (d) 5

Ans. (c)

Q.8. The points (–4, 0), (4, 0), (0, 3) are the vertices of a :

(a) right triangle

(b) isosceles triangle

(c) equilateral triangle

(d) scalene triangle

Ans. (b)

Q.9. The area of a triangle with vertices (a, $b + c$), (b, $c + a$) and (c, $a + b$) is :

(a) $(a + b + c)^2$ (b) 0

(c) $a + b + c$ (d) abc

Ans. (b)

Q.10. If points A(1, 2), O(0, 0) and C(a, b) are collinear, then :

(a) $a = b$ (b) $a = 2b$

(c) $2a = b$ (d) $a = -6$

Ans. (c)