6

Chapter

Triangles

In the Chapter

In this chapter, you will be studying the following points:

- Two figures having the same shape but not necessarily the same size are called similar figures.
- All the congruent figures are similar but the converse is not true.
- Two polygons of the same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (i.e., proportion).
- If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.
- If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
- If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (AAA similarity criterion).
- If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (AA similarity criterion).
- If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (SSS similarity criterion).
- If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar (SAS similarity criterion).
- The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
- If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.
- In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Pythagoras Theorem).
- If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 6.1

Q.1. Fill in the blanks using the correct word given in brackets :

(i) All circles are .... (congruent, similar)

(ii) All squares are .... (similar, congruent)

(iii) All .... triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are .... and (b) their corresponding sides are .... (equal, proportional)

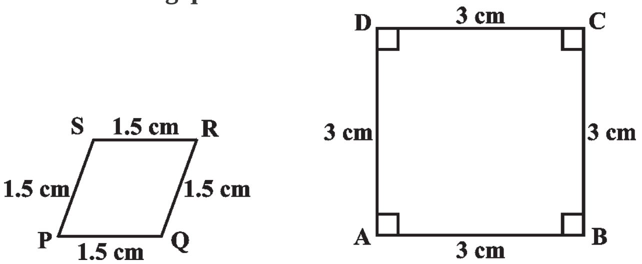
Ans. (i) similar, (ii) similar, (ii) equilateral, (iv) (a) equal, (b) proportional.

Q.2. Give two different examples of pair of (i) similar figures. (ii) non-similar figures.

Ans. (i) (a) Pair of equilateral triangle are similar figures. (b) Pair of squares are similar figure.

(ii) (a) A triangle and a quadrilateral form a pair of non-similar figures. (b) A square and a trapezium form a pair of non-similar figures.

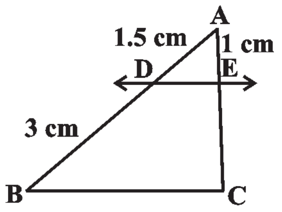
Q.3. State whether the following quadrilaterals are similar or not:



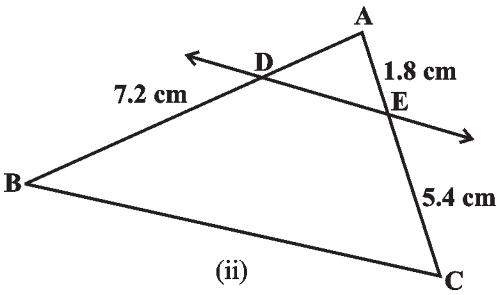
Ans. The two quadrilaterals, in figure are not similar because their corresponding angles are not equal. It is clear from the figure that. $\angle A$ is 90° but $\angle P$ is not 90°.

EXERCISE 6.2

Q.1. In Fig. , (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



(i)



(ii)

Ans. (i) in ΔABC, we have

$DE \parallel BC$

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$$

(By basic proportionality theorem)

$$\Rightarrow \quad \frac{1.5}{3} = \frac{1}{EC}$$

$$\Rightarrow \quad EC = \frac{3}{1.5} = 2 \text{ cm}$$

(ii) In ΔABC [fig. (ii)], we have

$DE \parallel BC$

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$$

(By basic proportionality theorem)

$$\Rightarrow \quad \frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$\Rightarrow \quad AD = \frac{7.2 \times 1.8}{5.4} = 2.4 \text{ cm.}$$

Q.2. E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF || QR :

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

Ans. (i) We have,

$$\frac{PE}{EQ} = \frac{3.9}{3} = \frac{1.3}{1} \quad ...(i)$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = \frac{1.5}{1} \quad ...(ii)$$

From (i) and (ii), we have

$$\frac{PE}{EQ} \neq \frac{PF}{FR}$$

Therefore, EF is not parallel to QR

[By using converse of Basic proportionality theorem]

(ii) We have

$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9} \quad ...(i)$$

$$\frac{PF}{FR} = \frac{8}{9} \quad ...(ii)$$

From (i) and (ii), we have

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Therefore, EF || QR

[Using converse of basic proportionality theorem]

(iii) We have,

$$\frac{PE}{EQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64} \quad ...(i)$$

$$\frac{PF}{FR} = \frac{0.36}{2.56} = \frac{36}{256} = \frac{9}{64} \quad ...(ii)$$

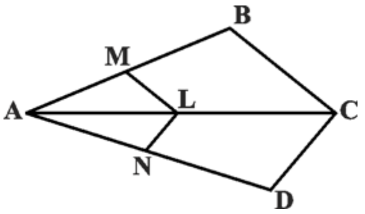
From (i) and (ii), we have

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Therefore, EF || QR

[Using converse of Basic proportionality theorem]

Q.3. In Fig. , if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Ans. In ΔABC, we have

LM || CB

$$\frac{AL}{AC} = \frac{AM}{AB} \quad ...(i)$$

[Using Basic proportionality theorem]

In ΔADC, we have

LN || CD

$$\Rightarrow \frac{AL}{AC} = \frac{AN}{AD} \quad ...(ii)$$

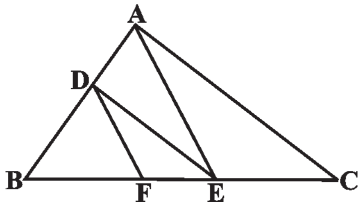
[Using Basic proportionality theorem]

From (i) and (ii), we have

$$\frac{AM}{AB} = \frac{AN}{AD} \quad \text{Proved.}$$

[Converse of Basic proportionality theorem]

Q.4. In Fig. 6.19, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



Ans. Given DE || AC and DF || AE

To Prove : $\frac{BF}{FE} = \frac{BE}{EC}$

Proof : In ΔABE, we have

DF || AE

Therefore, by using Basic proportionality theorem, we have

$$\frac{BF}{FE} = \frac{BD}{DA} \quad ...(i)$$

In ΔABC, we have

DE || AC

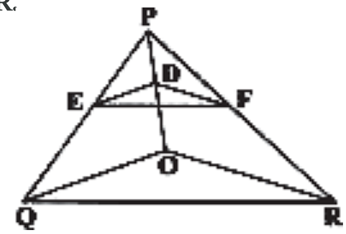
Therefore, by using Basic proportionality theorem, we have

$$\frac{BE}{EC} = \frac{BD}{DA} \quad ...(ii)$$

Comparing (i) and (ii), we get

$$\frac{BF}{FE} = \frac{BE}{EC} \quad \text{Hence proved.}$$

Q.5. In Fig., DE || OQ and DF || OR. Show that EF || QR.



Ans. In ΔPOQ, we have

DE || OQ

$$\Rightarrow \frac{PE}{EQ} = \frac{PD}{DO} \quad ...(i)$$

[Using basic proportionality theorem]

In ΔPOR, we have

DF || OR

$\Rightarrow \quad \frac{PD}{DO} = \frac{PF}{FR}$ ...(ii)

[Using basic proportionality theorem]

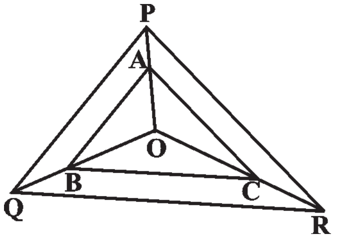
Comparing (i) and (ii), we get

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore \quad EF \parallel QR$

[Using converse of basic proportionality theorem]

Q.6. In Fig., A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



Ans. Given A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR.

To prove : BC || QR

Proof : In ΔOPQ , we have

AB || PQ

Therefore, by using basic proportionality theorem, we have

$\frac{OA}{AP} = \frac{OB}{BQ}$ ...(i)

In ΔOPR, we have

AC || PR

Therefore, by using basic proportionality theorem, we have

$\frac{OC}{CR} = \frac{OA}{AP}$ ...(ii)

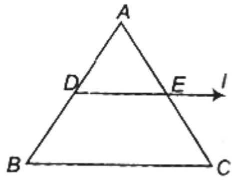
Comparing (i) and (ii), we get

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Therefore, by using converse of Basic proportionality theorem, we get

BC || QR.

Q.7. Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).



Ans. In ΔABC, D is the mid-point of AB

i.e., $\frac{AD}{DB} = 1$

As straight line l || BC

Line l is drawn through D and it meets AC at E.

By basic proportionality theorem.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\Rightarrow \quad \frac{AE}{EC} = 1$. [From Eq. (i)]

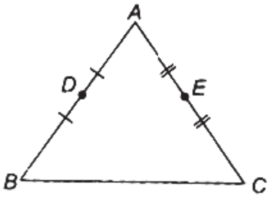
$\Rightarrow \quad AE = EC \Rightarrow \frac{AE}{EC} = 1.$

$\Rightarrow$ E is the mid-point of AC.

Hence proved.

Q.8. Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Ans. In ΔABC, D and E are mid-points of sides AB and AC, respectively.



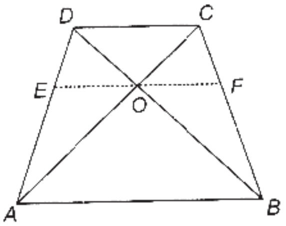
$\Rightarrow \quad \frac{AD}{DB} = 1$ and $\frac{AE}{EC} = 1.$ (see fig.)

$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow DE \parallel BC$

(By converse of basic proportionality theorem)

Q.9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Ans. We draw, EOF || AB (also || CD)



In ΔACD, $OE || CD$

$\Rightarrow \frac{AE}{ED} = \frac{AO}{OC}$

(Basic proportionality theorem) ...(i)

In ΔABD, $OE || BA$

$\Rightarrow \frac{DE}{EA} = \frac{DO}{OB}$

(Basic proportionality theorem)

$\Rightarrow \frac{AE}{ED} = \frac{OB}{OD}$ ...(ii)

From Eqs. (i) and (ii), we get

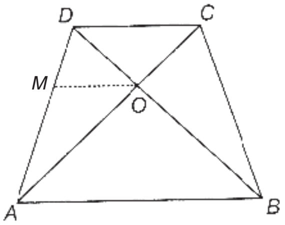
$$\frac{AO}{OC} = \frac{OB}{OD}$$

i.e., $\frac{AO}{BO} = \frac{CO}{DO}$

Hence Proved.

Q.10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Ans. Given a quadrilateral ABCD in which AC and BD are diagonals, which intersect each other at O.



To Prove : ABCD is a trapezium such that AB || DC.

Const. : Draw a line OM || AB.

Proof : In ΔADB, we have

OM || AB

Therefore, by using Basic proportionality theorem, we have

$$\frac{DM}{MA} = \frac{DO}{OB}$$

$\Rightarrow \frac{AM}{DM} = \frac{OB}{OD}$ ...(i)

[Taking reciprocals of both sides]

It is given that,

$$\frac{AO}{BO} = \frac{CO}{DO}$$

$\Rightarrow \frac{AO}{OC} = \frac{OB}{OD}$ ...(ii)

Comparing (i) and (ii), we get

$$\frac{AM}{DM} = \frac{OA}{OC}$$

Therefore, by using converse of basic proportionality theorem, we have

OM || DC

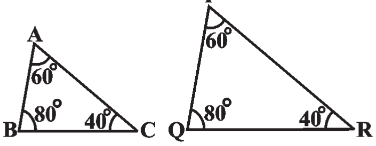
But OM || AB (by construction)

$\Rightarrow$ AB || DC

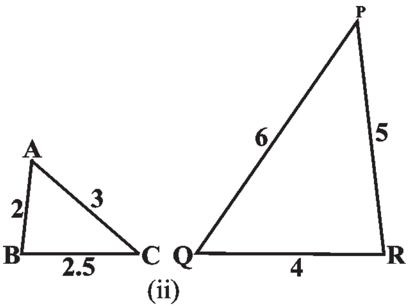
Hence, ABCD is a trapezium.

EXERCISE 6.3

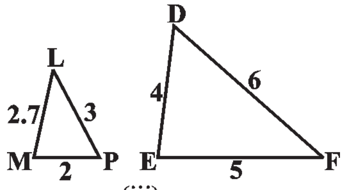
Q.1. State which pair of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pair of similar triangles in the symbolic form :



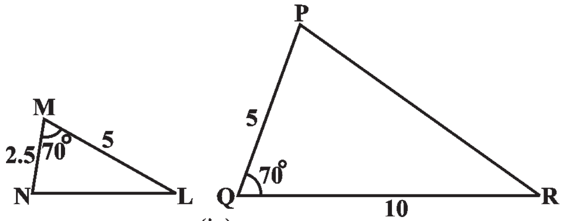
(i)



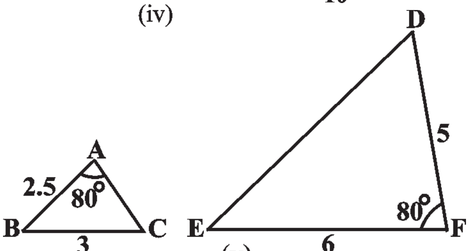
(ii)



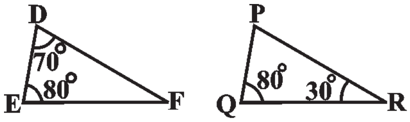
(iii)



(iv)



(v)



(vi)

Ans. (i) In ΔABC and ΔPQR, we have

$$\angle A = \angle P$$
$$\angle B = \angle Q$$
and $$\angle C = \angle R$$
$\therefore$ $$\Delta ABC \sim \Delta PQR$$

[Using AAA similarity condition]

(ii) In ΔABC and ΔQRP, we have

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

$\therefore$ $$\Delta ABC \sim \Delta QRP$$

[Using SSS similarity condition]

(iii) No. In ΔLMP and ΔDEF

$$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}, \frac{LP}{DF} = \frac{3}{6} = \frac{1}{2} \text{ and } \frac{LM}{EF} = \frac{27}{5} \neq \frac{1}{2}$$

i.e. $$\frac{MP}{DE} = \frac{LP}{DF} \neq \frac{LM}{EF}$$

Here, all correspinding sides are not equal in proportional.

Thus, the two triangles are not similar.

(iv) In ΔMNL and ΔQPR, we have

$$\frac{ML}{QR} = \frac{MN}{QP} = \frac{1}{2}$$

and $$\angle NML = \angle PQR$$
$$\Delta MNL \sim \Delta QPR$$

[Using SAS similarity condition]

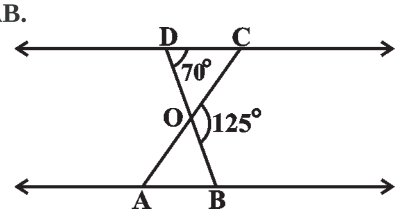
(v) No.

(vi) In ΔDEF and ΔPQR, we have

$$\angle D = \angle P$$
$$\angle E = \angle Q$$
and $$\angle F = \angle R$$
$\therefore$ $$\Delta DEF \sim \Delta PQR$$

[Using AAA similarity conditon]

Q.2. In Fig., $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Ans. $\angle DOC + 125° = 180°$

(DOC is a straight line)

$\Rightarrow$ $\angle DOC = 180° - 125° = 55°$

$\angle DCO + \angle COD + \angle DCO = 180°$

(Sum of three angles of ΔODC)

$\Rightarrow$ $\angle DCO + 70° + 55° = 180°$

Þ $\angle DCO + 125° = 180° – 125° = 55°$

Now, we are given that, $\Delta ODC \sim \Delta OBA$

Þ $\angle OCD = \angle OAB$

Þ $\angle OAB = \angle OCD = \angle DCO = 55°$

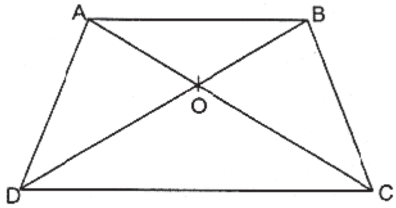
i.e., $\angle OAB = 55°$

Hence, we have $\angle DOC = 55°$, $\angle DCO = 55°$ and $\angle OAB = 55°$.

Q.3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Ans. Given : AC and BD are diagonals of a trapezium ABCD with AB | | DC, which intersect each other at the point O.

To prove : $\frac{OA}{OC} = \frac{OB}{OD}$



Proof : In ΔAOB and ΔCOD, we have

$\angle OAB = \angle OCD$ [Alternate angles]

and $\angle OBA = \angle ODC$ [Alternatre angles]

Therefore, by using AA similarity condition.

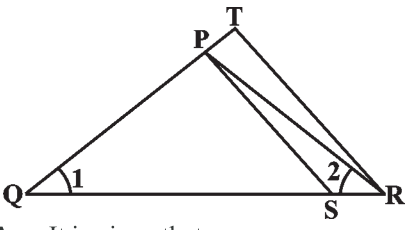
$\Delta AOB \sim \Delta COD$

$\Rightarrow \quad \frac{OA}{OC} = \frac{OB}{OD}$

[Proportional sides of similar triangles]

Q.4. In Fig., $\frac{QR}{QS} = \frac{QT}{PR}$ = and $\angle 1 = \angle 2$.

Show that ΔPQS ~ ΔTQR.



Ans. It is given that,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

$\Rightarrow \quad \frac{QS}{QR} = \frac{PR}{QT}$ ...(i)

[Taking reciprocals of both sides]

and $\angle 1 = \angle 2$ [given]

$\Rightarrow$ PR = PQ ...(ii)

Now, in ΔPQS and ΔTQR, we have

$\frac{QS}{QR} = \frac{PQ}{QT}$ [PR = PQ]

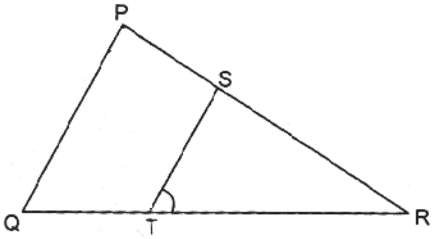
and $\angle Q = \angle Q$ [common]

Therefore, by using SAS similarity condition

$\Delta PQS \sim \Delta TQR$.

Q.5. S and T are points on sides PR and QR of Δ PQR such that ∠P = ∠RTS. Show that Δ RPQ ~ Δ RTS.

Ans. Given : S and T are points on sides PR and QR and ΔPQR such that ∠P = ∠RTS



To prove : ΔRPQ ~ ΔRTS

Proof : In ΔRPQ and ΔRTS, we have

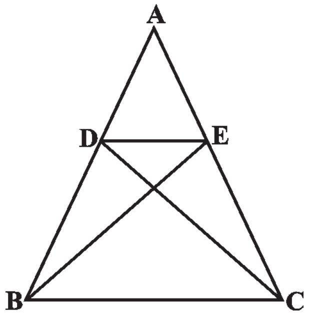
$\angle RPQ = \angle RTS$ [Given]

and $\angle QRP = \angle SRT$ [common]

Therefore by using AA similarity condition

$\Delta RPQ \sim \Delta RTS$

Q.6. In Fig., if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



Ans. Given : ΔABE ≅ ΔACD

To prove : ΔADE ~ ΔABC

Proof : We have

$\Delta ABE \cong \Delta ACD$

$\therefore$ AB = AC [by CPCT]

and AD = AE [by CPCT]

$\therefore \quad \frac{AB}{AD} = \frac{AC}{AE}$

and $\angle DAE = \angle BAC$ [common]

Therefore, by using SAS similarity condition

$\Delta ADE \sim \Delta ABC$.

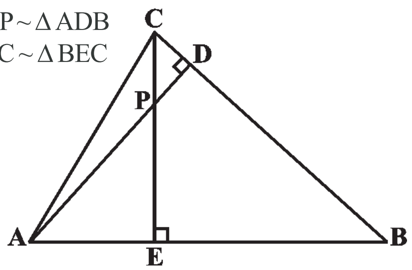
Q.7. In Fig., altitudes AD and CE of Δ ABC intersect each other at the point P. Show that:

(i) Δ AEP ~ Δ CDP

(ii) Δ ABD ~ Δ CBE

(iii) Δ AEP ~ Δ ADB

(iv) Δ PDC ~ Δ BEC



Ans. AD and CE are altitudes, which intersect each other at P.

(i) In ΔAEP and ΔCDP,

$\angle AEP = \angle CDP = 90°$ [given]

and $\angle APE = \angle CPD$ [vertically opposite angles]

Therefore, by using AA similarity condition

$\Delta AEP \sim \Delta CDP$

(ii) In ΔABD and ΔCBE

$\angle ADB = \angle CEB = 90°$ [given]

and $\angle B = \angle B$ [common]

Therefore, by using AA similarity condition

$\Delta ABD = \Delta CBE$

(iii) In ΔAEP and ΔADB

$\angle AEP = \angle ADB = 90°$ [given]

and $\angle PAE = \angle DAB$ [common]

Therefore, by using AA similarity condition

$\Delta AEP \sim \Delta ADB$

(iv) in ΔPDC and ΔBEC

$\angle PDC = \angle CEB = 90°$ [given]

$\angle PCD = \angle ECB$ [common]

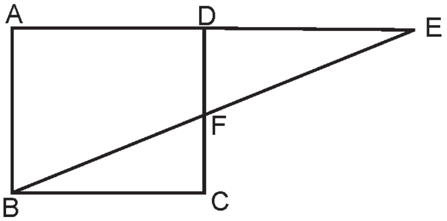
Therefore, by using AA similarity condition

$\Delta PDC \sim \Delta BEC$

Q.8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB.

Ans. Given : E is a point on side CD produced of parallelogram ABCD. BE intersects CD at F.

To prove : ΔABE ~ ΔCFB.



Proof : In ΔABE and ΔCFG, we have

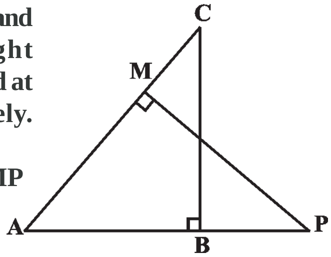
$\angle AEB = \angle CBF$ [Altrnate angles]

and $\angle A = \angle C$ [Opposite angles of a parallelogram]

Therefore, by using AA similarity condition.

$\Delta ABE \sim \Delta CFB$

Q.9. In Fig., ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:



(i) ΔABC ~ Δ AMP

(ii) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$

Ans. Given : ΔABC and ΔAMP right angled at B and M respectively.

(i) In ΔABC and ΔAMP,

$\angle ABC = \angle AMP = 90°$ (Given)

and $\angle A = \angle A$ (common)

Therefore, by using AA similarity condition

$\Delta ABC \sim \Delta AMP$

(ii) $\Delta ABC \sim \Delta AMP$

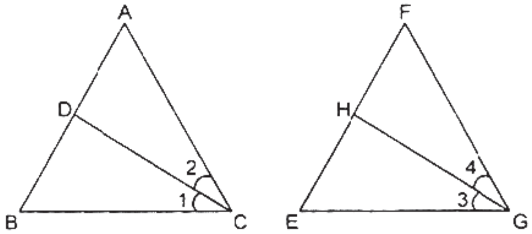
$\Rightarrow \dfrac{AB}{AM} = \dfrac{BC}{MP} = \dfrac{AC}{AP}$

$\Rightarrow \dfrac{AC}{AP} = \dfrac{BC}{MP}$ Hence Proved.

Q.10. CD and GH are respectively the bisectors of ∠ ACB and ∠ EGF such that D and H lie on sides AB and FE of Δ ABC and ∠ EFG respectively. If Δ ABC ~ Δ FEG, show that:

(i) $\dfrac{CD}{GH} = \dfrac{AC}{FG}$ (ii) Δ DCB ~ Δ HGE

(iii) Δ DCA ~ Δ HGF



Ans. (i) We have,

$\Delta ABC \sim \Delta FEG$ ...(i)

$\Rightarrow \angle A = \angle F$

and $\angle C = \angle G$

$\Rightarrow \dfrac{1}{2}\angle C = \dfrac{1}{2}\angle G$

$\Rightarrow \angle 1 = \angle 3$ and $\angle 2 = \angle 4$ ...(ii)

[∴ CD and GH are bisectors of ∠C and ∠G respectively]

Thus, in Δs ACD and FGH, we have

$\angle A = \angle F$ [from (i)]

$\angle 2 = \angle 4$ [from (ii)]

Therefore, by AA-criterion of similarity, we have

ΔACD ~ ΔFGH or ΔDCA ~ ∠HGF

(ii) We have,

$\Delta ACD \sim \Delta FGH$

$\Rightarrow \quad \frac{AC}{FG} = \frac{CD}{GH}$

(iii) in Δs DCB and HGE, we have

$\angle 1 = \angle 3$ [From (ii)]

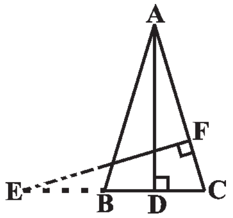
$\angle B = \angle E$

[ΔABC ~ ΔFEG]

Thusm by AA-criterion of similarity, we have

ΔDCB ~ ΔHGF.

Q.11. In Fig., E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that Δ ABD ~ Δ ECF.



Ans. Given : ΔABC is an isosceles Δ with AB= AC, AD ⊥ BC and EF ⊥ AC.

Now, in ΔABD and ΔECF, we have

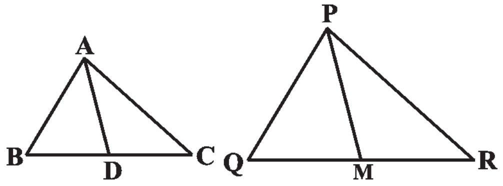
$\angle ABD = \angle ECF$ [ÐB = ÐC]

and $\angle ADB = \angle EFC = 90°$

Therefore, by using A.A. similarity condition

ΔABD ~ ΔECF

Q.12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see Fig.). Show that Δ ABC ~ Δ PQR.



Ans. Given: ΔABC and ΔPQR in which AD and PM are medians drawn on sides BC and QR respectively. It is also given that:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

To prove : ΔABC ~ ΔPQR

Proof : We have

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

Thereforem by using SSS similarity condition

ΔABD ~ ΔPQM

$\Rightarrow \quad \angle B = \angle Q$

Now, in ΔABC and ΔPQR, we have

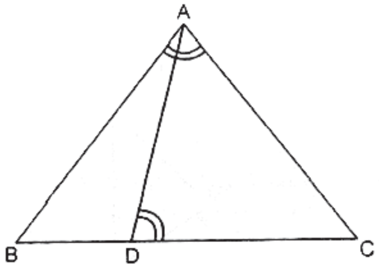
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

and $\angle B = \angle Q$

Therefore, by using SAS similarity condition

ΔABC ~ ΔPQR.

Q.13. D is a point on the side BC of a triangle ABC such that ∠ ADC = ∠ BAC. Show that CA^2 = CB.CD.



Ans. Given : ΔABC in which D is a point in BC such that ∠ADC = ∠BAC

To prove : $\frac{CA}{CD} = \frac{CB}{CA}$

Proof : In ΔDAC and ΔABC,

$\angle ADC = \angle BAC$ [given]

and $\angle C = \angle C$ [common]

Thereforem by using AA similarity condition

ΔDAC ~ ΔABC

$$\Rightarrow \quad \frac{DA}{AB} = \frac{AC}{BC} = \frac{DC}{AC}$$

$$\Rightarrow \quad \frac{AC}{BC} = \frac{DC}{AC}$$

$\Rightarrow \quad AC^2 = CB \times CD$ Hence Proved.

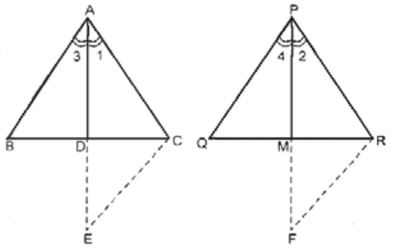
Q.14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that Δ ABC ~ Δ PQR.

Ans. Given : Triangle ABC and ΔPQR in which AD and PM are medians drawn on sides BC and QR respectively. It is given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

To Prove : $\Delta ABC \sim \Delta PQR$

Const : Produced AD to E such that AD = DE and PM to F such that PM = MF.



Proof : In ΔABD and ΔCDE

$AD = DE$ [by construction]

$\angle ADB = \angle CDE$ [vertically opposite angles]

and $BD = DC$ [AD is a median]

Therefore, by using SAS congruency condition

$\Delta ABD \cong \Delta CED$

$\Rightarrow AB = CE$ [by CPCT]

Similarly, we can prove

$\Delta PQM \cong \Delta RMF$

$\Rightarrow PQ = RF$ [by CPCT]

It is given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{CE}{RF} = \frac{AC}{PR} = \frac{2AD}{2PM} \quad \begin{bmatrix} AB = CE \\ PQ = RF \end{bmatrix}$$

$$\Rightarrow \frac{CE}{RF} = \frac{AC}{PR} = \frac{AE}{PR}$$

Therefore, by using SSS congruency condition

$\Delta ACE \cong \Delta PRF$

$\Rightarrow \angle 1 = \angle 2$ ...(i)

Similarly, $\angle 3 = \angle 4$ ...(ii)

Adding (i) and (ii), we get

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle A = \angle P$

Now, in ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{AC}{PR}$$

and $\angle A = \angle P$

Therefore, by using SAS similarity condition

$\Delta ABC \sim \Delta PQR$ Hence proved.

Q.15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Ans. In fig. (i), AB is a pole and behind it a sun is risen which casts a shadow of length BC = 4 cm and makes a angle θ to the horizontal and in fig. (ii). PM is a height of the tower and behind a sun risen which casts a shadow of length, NM = 28 cm.

In ΔACB and ΔPNM

$\angle C = \angle N = \theta$

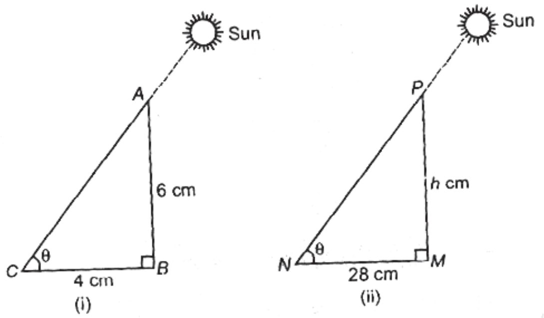
and $ABC = PMN = 90°$

$\therefore \Delta ABC \sim \Delta PMN$

(AAA similarity criterion)

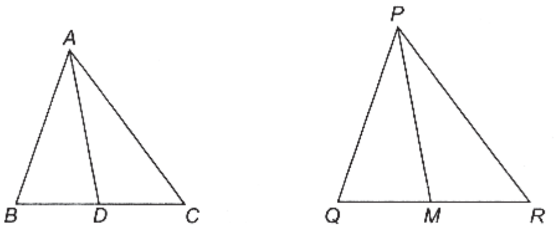
$$\Rightarrow \frac{AB}{PM} = \frac{BC}{MN} \Rightarrow \frac{AB}{BC} = \frac{PM}{MN}$$

$$\Rightarrow \frac{6}{4} = \frac{h}{28} \Rightarrow h = \frac{6 \times 28}{4} = 42m$$



Q.16. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Ans. Given : AD and PM are medians of triangle ABC and ΔPQR. It is given that $\Delta ABC \sim \Delta PQR$.



To prove : $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof : It is given that

$\Delta ABC \sim \Delta PQR$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR}$

[Proportional sides of two similar triangles]

$\Rightarrow \quad \frac{AB}{PQ} = \frac{2BD}{2QM}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM}$

[AD and PM are medians]

Now, in ΔABD and ΔPQM, we have

$\frac{AB}{PQ} = \frac{BD}{QM}$

and $\quad \angle B = \angle Q$

[ΔABC ~ ΔPQR]

Therefore, by using AA similarity condition

ΔABD ~ ΔPQM

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM}$

[Proportional sides of two similar triangles]

EXERCISE 6.4

Q.1. Let Δ ABC ~ Δ DEF and their areas be, respectively, 64 cm² and 121 cm². If EF = 15.4 cm, find BC.

Ans. It is given that

ΔABC ~ ΔDEF

We know that the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$

$\Rightarrow \quad \frac{64}{121} = \frac{BC^2}{(15.2)^2}$

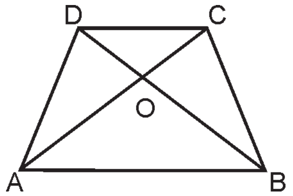
$\Rightarrow \quad \left(\frac{8}{11}\right)^2 = \left(\frac{BC}{15.4}\right)^2$

$\Rightarrow \quad \frac{8}{11} = \frac{BC}{15.4}$

$\Rightarrow \quad BC = \frac{15.4 \times 8}{11} = 11.2 \text{ cm}$

Hence, BC = 11.2 cm.

Q.2. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.



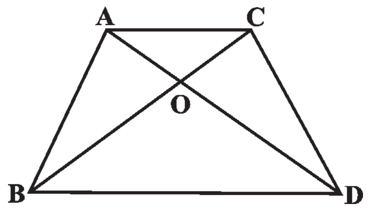
Ans. $\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2}$

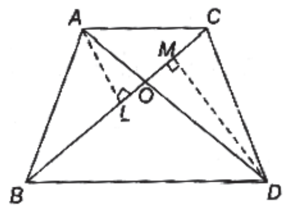
(using property of area of similar triangles)

$= \frac{(2CD)^2}{CD^2} \qquad (AB = 2CD)$

$= \frac{4 \times CD^2}{CD^2} = \frac{4}{1}$

Q.3. In Fig., ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(\Delta ABC)}{ar(\Delta COD)} = \frac{AO}{DO}$.





Ans. Draw AL ⊥ BC and DM ⊥ BC (see fig.)

In ΔOLA and Δ OMB

$\angle ALO = \angle DMO = 90°$

and $\quad \angle AOL = \angle DOM$

(vertically opposite angle)

$\therefore \quad \Delta OLA \sim \Delta OMD$

(AAA similarity criterion)

$\Rightarrow \quad \frac{AL}{DM} = \frac{AO}{DO}$ ...(i)

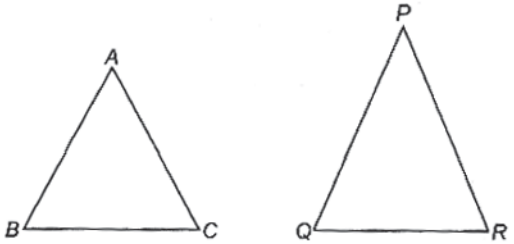
Now, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times (BC) \times (AL)}{\frac{1}{2} \times (BC) \times (DM)} = \frac{AL}{DM} = \frac{AO}{DO}$

(By Eq. (i)]

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Q.4. If the areas of two similar triangles are equal, prove that they are congruent.

Ans. Let ΔABC ~ ΔPQR and ar (ΔABC) = ar (ΔPQR)



i.e., $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = 1$

$\Rightarrow \quad \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{PR^2} = 1$

(Using propoerty of area of similar triangles)

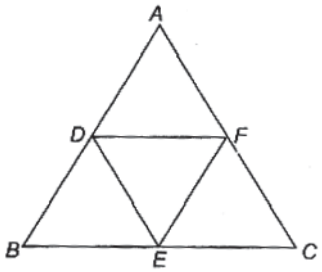
⇒ AB = PQ, BC = QR and CA = PR

(SSS proportionality criterion)

⇒ ΔABC ≅ ΔPQR.

Q.5. D, E and F are respectively the mid-points of sides AB, BC and CA of Δ ABC. Find the ratio of the areas of Δ DEF and Δ ABC.

Ans. Draw a ΔABC taking mid-points D, E and F on AB, BC and AC and join them.



Here, DF = $\frac{1}{2}$ BC, DE = $\frac{1}{2}$ CA

and EF = $\frac{1}{2}$ AB ...(i)

(D, E and F are mid-points of sides AB, BC and CA, respectively)

$\Rightarrow \quad \frac{DF}{BC} = \frac{DE}{CA} = \frac{EF}{AB} = \frac{1}{2}$

(SSS proportionality criterion)

⇒ ΔDEF ~ ΔCAB

$\Rightarrow \quad \frac{ar(\Delta DEF)}{ar(\Delta CAB)} = \frac{DE^2}{CA^2}$

$= \frac{\left(\frac{1}{2}CA\right)^2}{CA^2} = \frac{1}{4}$ [From Eq. (i)]

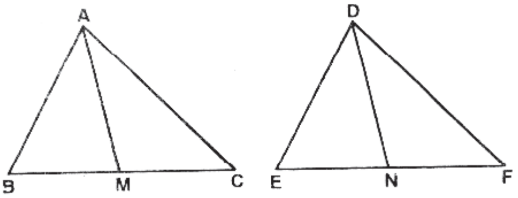
(Using propoerty of area of similar triangle)

$\Rightarrow \quad \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$ [ar(ΔCAB = ar (ΔABC)]

Hence, the required ratio is 1 : 4.

Q.6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Ans. Let ΔABC and ΔDEF are two similar triangles and AM and DN be their respective medians.



ΔABC ~ ΔDEF

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{\frac{1}{2}BC}{\frac{1}{2}EF}$

$= \frac{BM}{EN}$

(M and N are mid points of BC and EF)

$\frac{AB}{DE} = \frac{BM}{EN}$

$\angle B = \angle E$

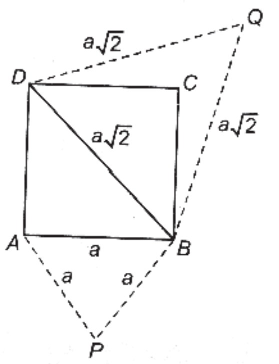
$\Delta ABM \sim \Delta DEN$ (SAS similarity)

$\Rightarrow \quad \frac{AB}{DE} = \frac{AM}{DN}$ ...(i)

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AM^2}{DN^2}$$

Q.7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Ans. ΔABM and ΔBDM are described on side AB and diagonal BD resp.



$$ar(\Delta ABM) = \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{\sqrt{3}}{4}a^2,$$

where a is side of square

$$ar(\Delta BDN) = \frac{\sqrt{3}}{4} \times (a\sqrt{2})^2$$

$[BD^2 = AB^2 + AD^2 = a^2 + a^2 = 2a^2 \; BD = \sqrt{2}a]$

$$= \frac{\sqrt{3}}{4} \times 2a^2$$

$$\therefore \quad \frac{ar(\Delta ABM)}{ar(DBDN)} = \frac{\frac{\sqrt{3}}{4} \times a^2}{\frac{\sqrt{3}}{4} \times 2a^2} = \frac{1}{2}$$

$ar(\Delta ABM) = \frac{1}{2}$ ar. (ΔBDM). Hence Proved

Tick the correct answer and justify :

Q.8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is

(A) 2 : 1 **(B) 1 : 2**

(C) 4 : 1 **(D) 1 : 4**

Ans. (c) 4 : 1

Justification : $\Delta ABC \sim \Delta BDE$ (AA similarity)

(each angle of equilateral trianlge is 60°)

$$\frac{ar(\Delta ABC)}{ar(DBDE)} = \frac{AB^2}{BD^2}$$

$$= \frac{AB^2}{\left(\frac{1}{2}BC\right)^2} = \frac{4AB^2}{AB^2}$$

$$= \frac{4}{1}$$

$ar(\Delta ABC) : ar(\Delta BDE) = 4 : 1$

Q. 9. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3 **(B) 4 : 9**

(C) 81 : 16 **(D) 16 : 81**

Ans. (d) 16 : 81

Justification : Ratio of areas of two similar trianlges is equal to the ratio of the squares of their corresponding sides.

$\therefore$ Ratio of areas of two triangles $= (4)^2 : (9)^2$

$= 16 : 81.$

EXERCISE 6.5

Q.1. Sides of some triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Ans. (i) $24^2 + 7^2 = 576 + 49 = 625 = 25^2$

Yes, these are the sides of right angled triangle.

Length of hypotenuse = 25 cm.

(ii) $3^2 + 6^2 = 9 + 36 = 45 \neq 8^2$.

No, these are not the sides of right angled triangle.

(iii) $50^2 + 80^2 = 2500 + 6400 = 8900 \neq 100^2$

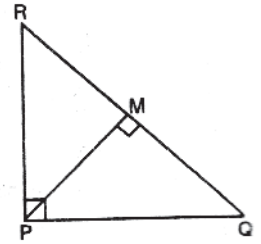
No, these are not the sides of right angled triangle.

(iv) $12^2 + 5^2 = 144 + 25 = 169 = 13^2$

Yes, these are the sides of right angled triangle.

Length of hypotenuse = 13 cm.

Q.2. PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that PM^2 = QM . MR.



Ans. If a perpendicular is drawn from the vertex of a right angled triangle to the hypotenuse then triangle on both sides of the perpendicular are similar to the whole triangle and to each other.

∴ $\Delta PMR \sim \Delta QMP$

⇒ $\frac{PM}{QM} = \frac{MR}{PM}$

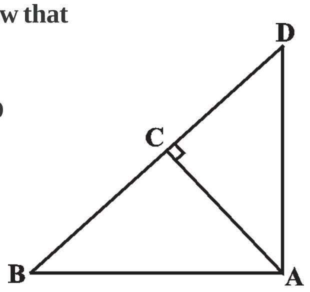
⇒ $PM^2 = QM.MR.$

Q.3. In Fig. 6.53, ABD is a triangle right angled at A and AC ⊥ BD. Show that

(i) AB^2 = BC . BD

(ii) AC^2 = BC . DC

(iii) AD^2 = BD . CD



Ans. (i) In ΔABD and ΔABC,

$\angle BAD = \angle BCA = 90°$ (given)

$\angle B = \angle B$ (common)

$\Delta ABD \sim \Delta CBA$ (AA similarity)

⇒ $\frac{AB}{BC} = \frac{BD}{AB}$

⇒ $AB^2 = BC.BD.$ Proved.

(ii) In ΔABD and ΔACD,

$\angle BAD = \angle ACD = 90°$ (given)

$\angle D = \angle D$ (common)

$\Delta ABD \sim \Delta CAD$ (AA similarity) ...(ii)

⇒ $\frac{AD}{CD} = \frac{BD}{AD}$

⇒ $AD^2 = BD.CD.$ Proved.

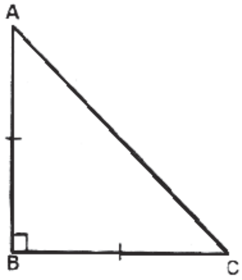
(iii) From (i) and (ii)

$\Delta CAD \sim \Delta CBA = 90°$

⇒ $\frac{AC}{BC} = \frac{CD}{AC}$

⇒ $AC^2 = BC.CD.$ Proved.

Q.4. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.



Ans. ABC is an isosceles triangle.

where $\angle C = 90°$

and $AC = BC$ ...(i)

Using Pythagoras theorem,

$AB^2 = AC^2 + BC^2$

⇒ $AB^2 = AC^2 + AC^2$ [From (i)]

⇒ $AB^2 = 24C^2$ Proved.

Q.5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Ans. Given, ABC is an isosceles triangle

where $AC = BC$

and $AB^2 = 24C^2$

⇒ $AB^2 = AC^2 + AC^2$

⇒ $AB^2 = AC^2 + BC^2$ (AC = BC)

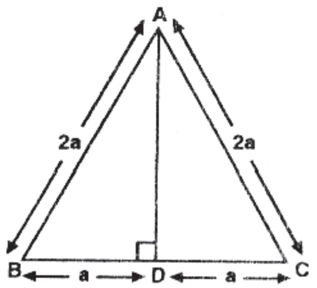
⇒ $\angle C = 90°$

∴ ΔABC is right angled triangle.

Q.6. ABC is an equilateral triangle of side 2*a*. Find each of its altitudes.

Ans. ABC is equilateral triangle with

AB = BC = CA = 2*a*



AD is it altitude.

∴ $AB^2 = AD^2 + BD^2$

⇒ $(2a)^2 = AD^2 + \left(\frac{1}{2}BC\right)^2$

$\Rightarrow \quad 4a^2 = AD^2 + a^2$

$\Rightarrow \quad AD^2 = 4a^2 - a^2$

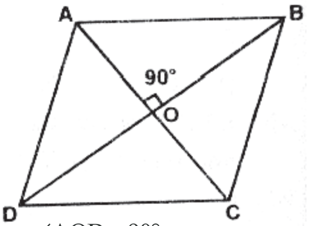
$= 3a^2$

$\therefore \quad AD = \sqrt{3}a$

Hence, length of each of its altitude = $\sqrt{3}a$.

Q.7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Ans. We know that diagonals of the rhombus bisect each other at right angles.



$\angle AOB = 90°$

$\Rightarrow \quad AB^2 = AO^2 + OB^2 \quad ...(i)$

Similarly, $BC^2 = BO^2 + OC^2 \quad ...(ii)$

$CD^2 = OC^2 + OD^2 \quad ...(iii)$

$DA^2 = OD^2 + OA^2 \quad ...(iv)$

On Adding (i), (ii), (iii) and (iv), we get

$AB^2 + BC^2 + CD^2 + DA^2$

$= 2(OA^2 + OB^2 + OC^2 + OD^2]$

$$= 2\left[\left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 + \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2\right]$$

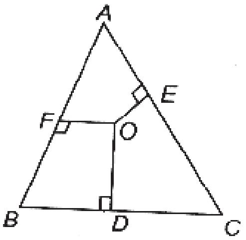
$$= 2 \times \frac{1}{2}\left[AC^2 + BD^2\right]$$

$\Rightarrow \quad AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$. Proved.

Q.8. In Fig., O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$,

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.



Ans. (i) Join OA, OB and OC.

In ΔOAF, $OA^2 = AF^2 + OF^2$

In ΔOBD, $OB^2 = OD^2 + BD^2$

In ΔOCE, $OC^2 = CE^2 + OE^2$

(i) L.H.S.

$= OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

$= (AF^2 + OF^2) + (OD^2 + BD^2) + (CE^2 + OE^2) - OD^2 - OE^2 - OF^2$

$= AF^2 + BD^2 + CD^2 = $ R.H.S. Proved.

(ii) Using (i)

$AF^2 + BD^2 + CE^2$

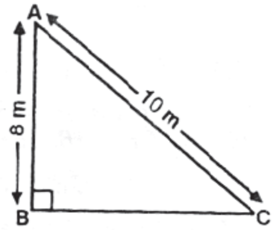
$= OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

$= (OA^2 - OE^2) + (OC^2 - OD^2) + (OB - OF^2)$

$= AE^2 + CD^2 + BF^2$ Proved.

Q.9. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Ans. Here, AC is a ladder of length 10m, A be the window and AB be the wall.



$\therefore \quad AC^2 = AB^2 + BC^2$

$\Rightarrow \quad (10)^2 = (8)^2 + BC^2$

$\Rightarrow \quad BC^2 = 100 - 64 = 36$ m

$\Rightarrow \quad BC = \sqrt{36} = 6$ m

Hence, foot of the ladder is at 6m from the base of wall.

Q.10. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Ans. Here, AB is a pole of height 18 m and AC is a wire of length 24 m.

Now, by using Pathagoras theorem,

$AC^2 = AB^2 + BC^2$

$(24)^2 = (18)^2 + BC^2$

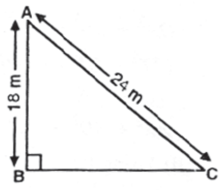
$5 + 6 = 324 + BC^2$

$BC^2 = 576 - 324$

$BC^2 = 252$

$BC = \sqrt{252}$

$= 6\sqrt{7}$.



The required distance is $6\sqrt{7}$ m.

Q.11. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the

same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes travel after $1\frac{1}{2}$ hours?

Ans. The distance travelled by first plane in north direction

$$= 1000 \times \frac{3}{2} = 1500 \text{ km}$$

$\therefore$ AB $= 1500$ km

The distance travelled by second aeroplane in west direction

$$= 1200 \times \frac{3}{2} = 1800 \text{ km}$$

$\therefore$ BC $= 1800$ km

$\therefore$ Using Pythagoras theorem,

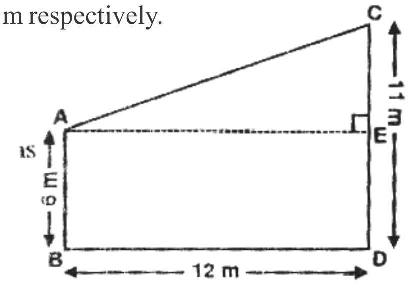
$$AC^2 = (1500)^2 + (1800)^2$$
$$= 2250000 + 3240000$$
$$= 5490000$$

$\Rightarrow$ AC $= 300\sqrt{61}$

Hence, the required distance $= 300\sqrt{61}$ km.

Q.12. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Ans. Here AB and CD be the poles of height 6 m and 11 m respectively.



Draw AE $\perp$ CD

$\therefore$ CE $= 11 - 6 = 5$ m

and AE $=$ BD $= 12$ m

Now in ΔAEC, using Pythagoras theorem

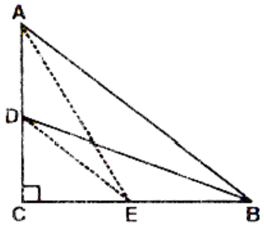
$$AC^2 = CE^2 + AE^2$$
$$= (5)^2 + (12)^2$$
$$= 25 + 144 = 169$$

$\therefore$ AC $= \sqrt{169} = 13$ m.

Q.13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Ans. In right angled triangles ACE and DCB

$$AE^2 = AC^2 + CE^2 \quad ....(i)$$
$$BD^2 = BC^2 + CD^2 \quad ...(ii)$$



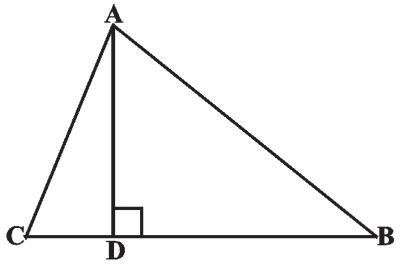
Adding (i) and (ii), we get

$$AE^2 + BD^2 = AC^2 + CE^2 + BC^2 + CD^2$$
$$= (AC^2 + BC^2) + (CE^2 + CD^2)$$
$$= AB^2 + DE^2$$

[in ΔABC $AB^2 = AC^2 + BC^2$ and in ΔDEC $DE^2 = DC^2 + CE^2$]

Hence proved.

Q.14. The perpendicular from A on side BC of a Δ ABC intersects BC at D such that DB = 3 CD (see Fig.). Prove that $2AB^2 = 2AC^2 + BC^2$.



Ans. Here, DB $= 3$CD

BC $=$ DB $+$ CD

$= 3$CD $+$ CD $= 4$CD

$\therefore$ CD $= \frac{1}{4}$ BC

and DB $= 3$CD $= \frac{3}{4}$ BC

In ΔABD,

$$AB^2 = AD^2 + BD^2$$
$$= AC^2 - CD^2 + BD^2$$
$$= AC^2 - \left(\frac{1}{4}BC\right)^2 + \left(\frac{3}{4}BC\right)^2$$

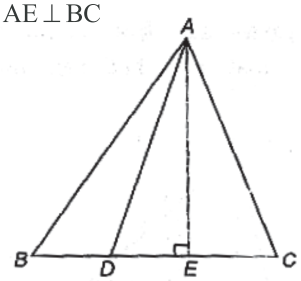
$\Rightarrow$ $16AB^2 = 16AC^2 - BC^2 + 9BC^2$

$\Rightarrow$ $16AB^2 = 16AC^2 + 8BC^2$

$\Rightarrow$ $2AB^2 = 2AC^2 + BC^2$ Proved.

Q.15. In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC. Prove that $9AD^2 = 7AB^2$.

Ans. Draw $AE \perp BC$



In ΔADE,

$AD^2 = AE^2 + DE^2$

$= AB^2 - BE^2 + DE^2$

$= AB^2 - BE^2 + (BE - BD)^2$

$= AB^2 - BE^2 + BE^2 + BD^2 - 2BE.BD$

$= AB^2 + BD^2 - 2BE.BD$

$$= AB^2 + \left(\frac{BC}{3}\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right),$$

$$BD = \frac{BC}{3} \text{ and } BE = \frac{BC}{2}$$

$$= AB^2 + \frac{BC^2}{9} - \frac{BC^2}{3} \qquad (AB = BC)$$

$9AD^2 = 9AB^2 + AB^2 - 3AB^2$

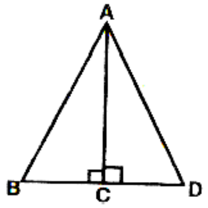
$9\,AD^2 = 7AB^2$. Proved.

Q.16. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Ans. ABC is an equilateral triangle and AD is one of its altitude.

Now, in ΔABD,

$AB^2 = AD^2 + BD^2$



$$= AD^2 + \left(\frac{BC}{2}\right)^2 \qquad \left(BD = \frac{1}{2}BC\right)$$

$$\Rightarrow \quad AB^2 = AD^2 + \frac{BC^2}{4}$$

$$\Rightarrow \quad AB^2 = AD^2 + \frac{AB^2}{4} \qquad (AB = BC)$$

$$\Rightarrow \quad AB^2 - \frac{AB^2}{4} = AD^2$$

$$\Rightarrow \quad \frac{3AB^2}{4} = AD^2$$

$\therefore \quad 3AB^2 = 4AD^2$

Q.17. Tick the correct answer and justify : In Δ ABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. The angle B is :

(A) 120° **(B) 60°**

(C) 90° **(D) 45°**

Ans. (c) 90°

Justification :

$AB^2 = (6\sqrt{3})^2 = 108, AC^2 = (12)^2 = 144$

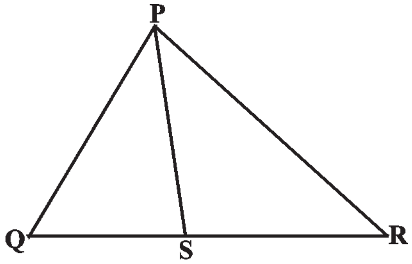
and $BC^2 = (6)^2 = 36$

$\therefore \quad AB^2 + BC^2 = 108 + 36 = 144 = AC^2$

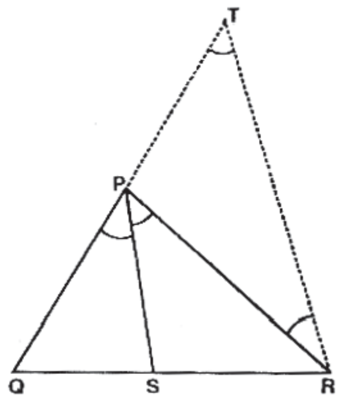
$\therefore \quad \angle B = 90°$, (Converse of Pythagoras theorem).

EXERCISE 6.6 (Optional)

Q.1. In Fig., PS is the bisector of $\angle$ QPR of Δ PQR. Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$.



Ans. We have $\angle QPS = \angle RPS$



as PS is internal bisector of $\angle P$. (given)

We have to prove that, $\frac{QS}{SR} = \frac{PQ}{PR}$

Construction : Draw a line through R parallel to PS which intersect QP produced to T.

Proof : $\angle SPR = \angle PRT$ ...(i) (Alt. $\angle S$)

and $\angle QPS = \angle PTR$ ...(ii) (Corresponding ÐS)

But $\angle QPS = \angle RPS$ ...(iii)

We have, $\angle PRT = \angle PTR$ [Using (i) (ii) and (iii)]

$\Rightarrow$ PT = PR

[sides opp. to equal angles are equal]

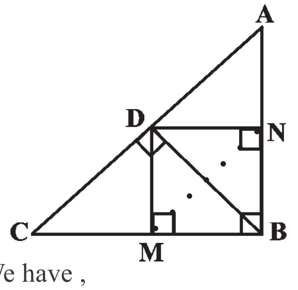
In DQRT, RT || PS

$\Rightarrow \frac{QS}{SR} = \frac{QP}{PT}$ (Basic proportionality theorem)

$\Rightarrow \frac{QS}{SR} = \frac{QP}{PR}$

Q.2. In Fig., D is a point on hypotenuse AC of Δ ABC, such that BD ⊥ AC, DM ⊥ BC and DN ⊥ AB. Prove that :

(i) $DM^2 = DN . MC$ (ii) $DN^2 = DM . AN$



Ans. We have ,

AB ⊥ BC and DM ⊥ BC.

$\Rightarrow$ AB || DM

Similarly, we have

CB ⊥ AB and DN ⊥ AB

$\Rightarrow$ CB || DN

Hence, quadrilateral BMDN is a rectangle

$\therefore$ BM = ND

(i) In DBMD, we have

$\angle 1 + \angle BMD + \angle 2 = 180°$

$\Rightarrow \angle 1 + 90° + \angle 2 = 180°$

$\Rightarrow \angle 1 + \angle 2 = 90°$

Similarly, in ΔDMC, we have

$\angle 3 + \angle 4 = 90°$

Since BD ⊥ AC. Therefore

$\angle 2 + \angle 4 = 90°$

Now, $\angle 1 + \angle 2 = 90°$ and $\angle 2 + \angle 3 = 90°$

$\Rightarrow \angle 1 + \angle 2 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 3$

Also, $\angle 3 + \angle 4 = 90°$ and $\angle 2 + \angle 3 = 90°$

$\Rightarrow \angle 3 + \angle 4 = \angle 2 + \angle 3 \Rightarrow \angle 2 = \angle 4$

Thus, in Δs BMD and DMC, we have

$\angle 1 = \angle 3$ and $\angle 2 = \angle 4$

Therefore, by using AA similar condition

ΔBMD ~ ΔDMC

$\Rightarrow \frac{BM}{DM} = \frac{MD}{MC}$

$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC}$ [BM = ND]

$\Rightarrow DM^2 = DN \times MC$

(ii) Similarly, as we have proved in part (i), we can prove

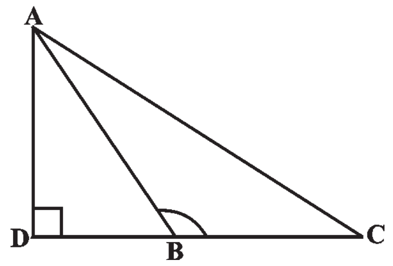
ΔBND ~ ΔDNA

$\Rightarrow \frac{BN}{DN} = \frac{ND}{NA}$

$\Rightarrow \frac{DM}{DN} = \frac{DN}{AN}$

$\Rightarrow DN^2 = DM \times AN$ Proved.

Q3. In Fig., ABC is a triangle in which ∠ ABC > 90° and AD ⊥ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2 BC . BD$.



Ans. $\angle ABC > 90°$

and $\angle ADC = 90°$ (given)

In ΔADC, $AC^2 = AD^2 + DC^2$ ...(i)

(Using pythagoras theorem)

But in ΔADB

$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = AB^2 - BD^2$ ...(ii)

On using equation (i) in (ii) we get

$AC^2 = AB^2 - BD^2 + DC^2$

$= AB^2 - BD^2 + (BD + BC)^2$

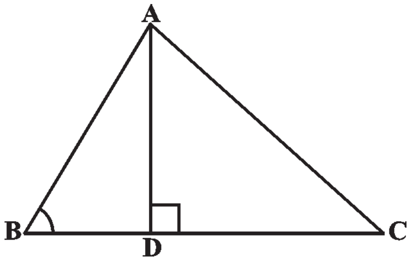
$= AB^2 - BD^2 + BD^2 + BC^2 + 2BC. BD.$

$\Rightarrow AC^2 = AB^2 + BC^2 + 2BC. BD.$

Hence Proved.

Q.4. In Fig., ABC is a triangle in which $\angle ABC < 90°$ and $AD \perp BC$. Prove that

$AC^2 = AB^2 + BC^2 - 2\,BC \cdot BD$.



Ans. $\angle ABC < 90°$

and $\angle ADC = 90°$ (given)

In ΔADC

$AC^2 = AD^2 + CD^2$ ...(i)

(Using Pythagoras theorem)

In ΔABD,

$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = AB^2 - BD^2$ ...(ii)

On using equation (ii) in equation (i), we get

$AC^2 = AB^2 - BD^2 + CD^2$

$= AB^2 - BD^2 + (BC - BD)^2$

$= AB^2 - BD^2 + BC^2 + BD^2 - 2\,BC.\,BD$

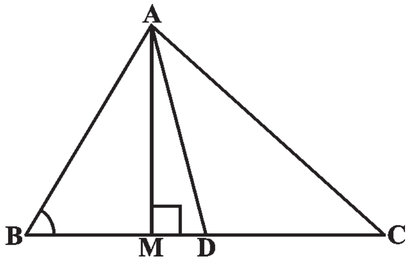
$\Rightarrow AC^2 = AB^2 + BC^2 - 2BC \cdot BD$. Hence proved.

Q.5. In Fig., AD is a median of a triangle ABC and $AM \perp BC$. Prove that :

(i) $AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2\,AD^2 + \frac{1}{2}\,BC^2$



Ans. (i) In ΔAMC, $\angle M = 90°$

$AC^2 = AM^2 + MC^2$

(Using Pythagoras theorem)

or $AC^2 = AM^2 + (DM + DC)^2$

$= AM^2 + DM^2 + DC^2 + 2DC.\,DM$

$= AD^2 + \left(\frac{BC}{2}\right)^2 + 2.\left(\frac{BC}{2}\right).\,DM,$

$= AD^2 + BC.\,DM + \left(\frac{BC}{2}\right)^2$

(ii) In ΔAMB,

$AB^2 = AM^2 + BM^2$

$AB^2 = AM^2 + (BD - DM)^2$

$= AM^2 + BD^2 + DM^2 - 2BD.\,DM$

$= (AM^2 + DM^2) + BD^2 - 2BD.\,DM$

[In ΔAMD $AD^2 = AM^2 + MD^2$ and D is mid point of BC $\therefore BD = \frac{BC}{2}$]

$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) BM$

$= AD^2 - BC.\,DM + \left(\frac{BC}{2}\right)^2$

(iii) From part (i) and (iii),we have

$AC^2 + AB^2 = AD^2 + BC.\,DM + \left(\frac{BC}{2}\right)^2$

$+ AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

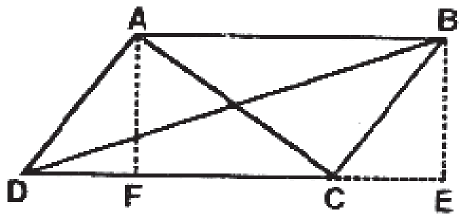
$= 2AD^2 + \frac{1}{2}BC^2$ Hence Proved.

Q.6. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Ans. In Fig. ABCD is a parallelogram

To prove that $AC^2 + BD^2$

$= AB^2 + BC^2 + CD^2 + DA^2$



Construction : Draw $AF \perp DC$ and $BE \perp DC$ produced

Proof : $\Delta ADF \cong \Delta BCE \Rightarrow DF = CE$

In ΔDBC, $\angle C > 90°$, hence

$\therefore \quad BD^2 = DC^2 + BC^2 + 2\, DC.\, BE$ ...(i)

In ΔADC, $\angle D < 90°$

$\therefore \quad AC^2 = AD^2 + DC^2 - 2DC.AF$

$= AD^2 + AB^2 - 2DC.BE,$ ...(ii)

[DF = CE as proved above and AB = DC sides of parallelogram]

On adding equation (i) and equation (ii), we get

$BD^2 + AC^2 = BC^2 + CD^2 + AD^2 + AB^2$

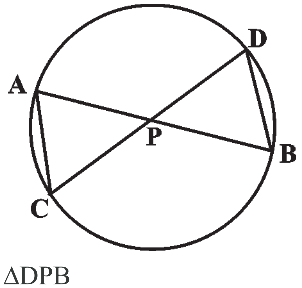
$= AB^2 + BC^2 + CD^2 + AD^2$

Q.7. In Fig., two chords AB and CD intersect each other at the point P. Prove that :

(i) Δ APC ~ Δ DPB

(ii) AP . PB = CP . DP

Ans.



(i) In ΔAPC and ΔDPB

$\angle CAP = \angle BDP,$
(angles in the same segment)

$\angle PCA = \angle PBD$
(angles in the same segment)

$\therefore \quad \Delta APC \sim \Delta DPB$ (AA similarity)

Hence proved.

(ii) From (i), $\Delta APC \sim \Delta DPB$

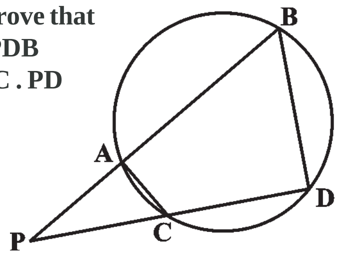
$$\Rightarrow \quad \frac{AP}{DP} = \frac{PC}{PB}$$

$\therefore \quad AP.\, PB = CP.\, DP$ Hence proved.

Q.8. In Fig., two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) Δ PAC ~ Δ PDB

(ii) PA . PB = PC . PD



Ans. (i) In ΔPAC and ΔPDB,

$\angle PAC = \angle PDB$
(exterior angles of cyclic quadrilateral)

$\angle P = \angle P$ (common)

$\therefore \quad \Delta PAC \sim \Delta PDB$ (AA similarity)

Hence proved.

(ii) From (i)

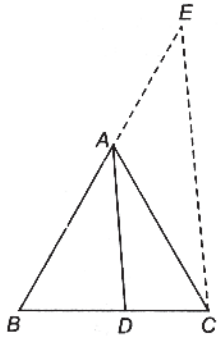
$\Delta PAC \sim \Delta PDB$

$$\Rightarrow \quad \frac{PA}{PD} = \frac{PC}{PB}$$

$\Rightarrow \quad PA.\, PB = PC.\, PD.$ Hence proved.

Q.9. In Fig. , D is a point on side BC of Δ ABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of ∠ BAC.

Ans. Given : ABC is a triangle and D is a point or BC such that



$$\frac{BD}{CD} = \frac{AB}{AC}$$

To prove : AD is the bisector of $\angle BAC$.

Construction :

Produce BA to E such that

AE = AC.

Join CE.

Proof : In ΔAEC,

$AE = AC$ (by construction)

$\Rightarrow \quad \angle AEC = \angle ACE,$ ...(i)

[angles opp. to equal sides of a triangle are equal]

Given, $$\frac{BD}{CD} = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{BD}{CD} = \frac{AB}{AE} \quad (AE = AC)$$

$\therefore \quad AD \,||\, EC,$ (By converse of basic proportionality theorem)

Since CA is a transversal, we have

$\angle BAD = \angle AEC,$ ...(ii)
(Corresponding angles)

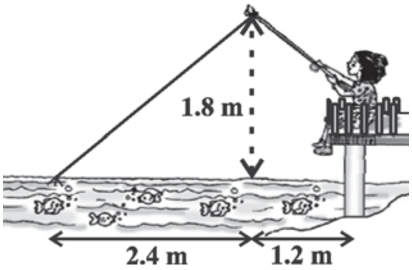
and $\angle DAC = \angle ACE$ ...(iii)
(Alternate angles)

From (i), (ii) and (iii), we have

$\angle BAD = \angle DAC$

Hence, AD bisects $\angle BAC$. Hence proved.

Q.10. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much



string does she have out (see Fig.)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?

Ans. The string does have after tearing

$$= \sqrt{(1.8)^2 + (2.4)^2}$$

$$= \sqrt{3.24 + 5.76}$$

$$= \sqrt{9.00} \qquad = 3 \text{ m}$$

The length of string she has cut = 3m

Rate of pulling out of string = 5 cm/ sec

time = 12 sec

length of string pulled = (12× 5) cm = 60 cm = 0.60m

remaining string = (3 – 0.6) m = 2.4 m

Now, in ΔABP

$$PA^2 = AB^2 + BP^2$$

$\Rightarrow$ $(2.4)^2 = (1.8)^2 + BP^2$

$\Rightarrow$ $5.76 = 3.24 + BP^2$

$\Rightarrow$ $BP^2 = 5.76 - 3.24 = 2.52$

$\Rightarrow$ $BP = \sqrt{2.52} = 1.59$ (approx)

Hence, required horizontal distance

$= (1.59 + 1.2)$ m

$= 2.79$ m.

Additional Questions

Q.1. Is the triangle with sides 25 cm, 5 cm and 24 cm a right triangle? Give reasons for your answer.

Ans. Since, $24^2 + 5^2 = 576 + 25 = 601 \neq 625 = 25^2$

$\therefore$ The given Δ is not a right triangle.

Q.2. It is given ΔDEF ~ ΔRPQ. Is it true to say that ∠D = ∠R and ∠E = ∠P? why?

Ans. ΔDEF ~ ΔRQP

$\therefore$ $\angle D = \angle R$

$\angle F = \angle Q$

$\Rightarrow$ $\angle F \neq \angle P$

Q.3. Is the following statement true ? Why?

"Two quadrilaterals are similar, if their corresponding angles are equal".

Ans. No, corresponding sides must also be proportional.

Q.4. Two dies and the perimeter of one triangle are respectively three times the corresponding sides and the perimeterof the other triangle. Are the two triangles similar? Why ?

Ans. Yes, as the corresponding two sides and the perimeters are equal, their third sides will also be equal.

Q.5. If in two right triangles, one fo the acute angles of one triangle is equal to an acute angles of theotehr triangle, can you say that the two triangles will be similar ? why ?

Ans. Yes, by AAA criterion, the two Δs will be similar.

Q.6. If ΔABC ~ ΔDEF, AB = 4 cm, DE = 6 cm, EF = 9 cm and FD = 12 cm,find the perimeter of ΔABC.

Ans. ΔABC ~ DDEF

AB = 4 cm

DE = 6 cm

EF = 9 cm

FD = 12 cm

Now, $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

$\Rightarrow$ $\dfrac{4}{6} = \dfrac{BC}{9} = \dfrac{AC}{12}$

$\Rightarrow$ $BC = \dfrac{4}{6} \times 9 = 6$ cm

$AC = \dfrac{4}{6} \times 12 = 8$ cm

$\therefore$ Perimeter of ΔABC,

$= AB + BC + AC$

$= 4$ cm $+ 6$ cm $+ 8$ cm

$= 18$ cm.

Q.7. Corresponding sides of two similar triangles are in the ratio of 2 : 3. If the area of the smaller triangle is 48 cm², find the area of the larger triangle.

Ans. Since, ratios of areas of two similar Δs are equal to the ratio of the squares of their corresponding sides.

Let the area of the larger triangle be x cm^2.

$\therefore \quad \frac{48}{x} = \frac{2^2}{3^2}$

$\Rightarrow \quad \frac{48}{x} = \frac{4}{9}$

$\Rightarrow \quad x = \frac{9}{4} \times 48$

$= 108$

$\therefore$ Area of thelarger Δ is 108 cm^2.

Q.8. Area of two similar triangles are 36 cm^2 and 100 cm^2. If the lengthof a side of the larger triangle is 20 cm, find the length of the corresponding side of the smaller triangle.

Ans. Let the length of the smaller triangle be x cm. Since, ratios of area of two similar triangles is equal to the ratio of squares of their correspondind sides.

$\therefore \quad \frac{36}{100} = \frac{x^2}{20^2}$

$\Rightarrow \quad x^2 = \frac{36}{100} \times 400$

$= 144$

$\Rightarrow \quad x = 12$ cm

$\therefore$ Length of the corresponding side of the smaller triangle is 12 cm.

Q.9. It is given that ΔABC ~ ΔEDF such that AB = 5 cm, AC = 7 cm, DF = 15cm and DE = 12 cm. Find the lengths of the remaining sides of the triangle.

Ans. Since, ΔABC ~ ΔEDF (Given)

$\therefore \quad \frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$ (CPST)

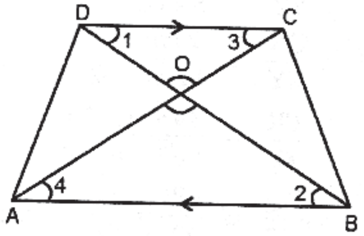
$\Rightarrow \quad \frac{5}{12} = \frac{BC}{15} = \frac{7}{EF}$

$\Rightarrow \quad BC = \frac{25}{4}, EF = \frac{84}{5}$

$\therefore$ Remaining sides of the triangles are $\frac{25}{4}$ cm and $\frac{84}{5}$ cm, i.e, 625 cm and 16.8 cm.

Q.10. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the area of triangles AOB and COD.

Ans. Given : ABCD is a trapezium in which AB || DC and AB = 2CD.



To find : $\frac{ar(\Delta AOB)}{ar(\Delta COD)}$

Proof : $\angle 1 = \angle 2$ (alternate $\angle$s)

$\angle 3 = \angle 4$ (alternate $\angle$s)

$\angle DOC = \angle AOB$ (V.O.A)

$\therefore$ ΔAOB ~ ΔDOC (By AAA similarity)

We know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

i.e., $\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2} = \frac{(2CD)^2}{CD^2} = \frac{4CD^2}{CD^2}$

$= \frac{4}{1}$ or 4 : 1.

Multiple Choice Questions

Q.1. The perimeters of two similar triangle ABC and PQR are 60 cm and 36 cm respectively. If PQ = 9 cm, then AB equals :

(a) 6 cm (b) 3.5 cm
(c) 15 cm (d) 20 cm

Ans. (c)

Q.2. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of trianlges ABC and BDE is :

(a) 2 :1 (b) 1 : 2
(c) 4 :1 (d) 1 : 4

Ans. (c)

Q.3. ΔABC ~ ΔDEF. If AB = 4 cm, BC = 3.5 cm, CA = 2.5 cm and DF = 7.5cm, then the perimeter of ΔDEF is :

(a) 10 (b) 14 cm

(c) 30 cm (d) 25 cm

Ans. (c)

Q.4. If ΔABC ~ ΔPQR, perimeter of ΔABC = 20 cm, perimeter of ΔPQR = 40 cm and PR = 8 cm, then the length of AC is :

(a) 8 cm (b) 6 cm

(c) 4 cm (d) 5 cm

Ans. (c)

Q.5. If ΔABC ~ ΔEDF and ΔABC is not similar to ΔDEF, then which of the following is not true ?

(a) AC. EF = AC. FD

(b) AB. EF = AC. DE

(c) BC. DE = AB. EF

(d) BC. DE = AB. FD

Ans. (c)

Q.6. It is given that ΔABC ~ ΔDEF, ∠A = 30°, ∠C = 50°, AB = 5 cm, AC = 8cm and DF = 7.5 cm. Then the following is true :

(a) DE = 12 cm, ∠F = 50°

(b) DE = 12 cm, ∠F = 100°

(c) EF = 12 cm, ∠D = 100°

(d) EF = 12 cm, ∠D = 30°

Ans. (b)

Q.7. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when :

(a) ∠B = ∠E (b) ∠A = ∠D

(c) ∠B = ∠D (d) ∠A = ∠F

Ans. (c)

Q.8. If in two triangles ABCand PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

(a) ΔPQR ~ ΔCAB

(b) ΔPQR ~ ΔABC

(c) ΔCBA ~ ΔPQR

(d) ΔBCA ~ ΔPQR

Ans. (a)

Q.9. In triangles ABC and DEF, ∠B = ∠E, ∠F = ∠C and AB = 3 DE. Then, the two triangles are

(a) congruent but not similar

(b) similar but not congruent

(c) neither congruent nor similar

(d) congruent as well as similar

Ans. (b)

Q.10. The lengths of the diagonals of a rhombus are 16 cm and 12 cm. Then, the length of the sides of the rhombus is :

(a) 9 cm (b) 10 cm

(c) 8 cm (d) 20 cm

Ans. (b)