Chapter 3

Pair of Linear Equations in Two Variables

In the Chapter

In this chapter, you will be studying the following points:

- Two linear equations in the same two variables are called a pair of linear equations in two variables. The most general form of a pair of linear equations is
  $a_1x + b_1y + c_1 = 0$
  $a_2x + b_2y + c_2 = 0$
  where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers, such that
  $a_1^2 + b_1^2 \neq 0,\ a_2^2 + b_2^2 \neq 0.$
- A pair of linear equations in two variables can be represented, and solved, by the:
  (i) graphical method
  (ii) algebraic method
- Graphical Method :
  The graph of a pair of linear equations in two variables is represented by two lines.
  (i) If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent**.
  (ii) If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is **dependent (consistent)**.
  (iii) If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent**.
- Algebraic Methods : We have discussed the following methods for finding the solution(s) of a pair of linear equations :
  (i) Substitution Method
  (ii) Elimination Method
  (iii) Cross-multiplication Method
- If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situations can arise :

  (i) $\frac{a_1}{b_2} \neq \frac{b_1}{b_2}$: In this case, the pair of linear equations is consistent.

  (ii) $\frac{a_1}{b_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$: In this case, the pair of linear equations is inconsistent.

  (iii) $\frac{a_1}{b_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$: In this case, the pair of linear equations is dependent and consistent.
- There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 3.1

Q.1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Ans. Let present age of Aftab be x years and present age of his daughter by y years.

Case I. Seven years ago

$$\text{Age of Aftab} = (x-7) \text{ years}$$
$$\text{Age of his daughter} = (y-7) \text{ years}$$

According to question :

$$(x-7) = (y-7)$$
$$\Rightarrow \quad x-7 = 7y-49$$
$$\Rightarrow \quad x-7y = -42$$

Case II.

Three years later,

$$\text{Age of Aftab} = (x+3) \text{ years}$$
$$\text{Age of his daughter} = (y+3) \text{ years}$$

According to question :

$$x+3 = 3(y+3)$$
$$\Rightarrow \quad x+3 = 3y-9$$
$$\Rightarrow \quad x-3y = 6$$

So, algebraic expression be

$$x-7y = -42$$
$$x-3y = 6$$

Graphical representation

For eq. (i), we have

$$x-7y = -42$$
$$\Rightarrow \quad x = 7y-42$$

Thus, we have following table :

x	0	42
y	6	12

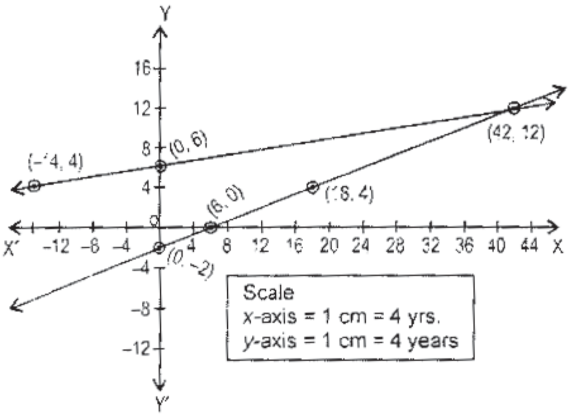
From eqn. (*ii*), we have

$$x-3y = 6$$
$$\Rightarrow \quad x = 3y+6$$

Thus, we have following table

x	0	6	42	18
y	–2	0	12	4

When we plot the graph of equation. We find that both the lines intersect at the point (42, 12). Therefore, $x = 42, y = 12$ is the solution of the given system of equations.



Q.2. The coach of a cricket of a cricket team buys 3 bats and 6 balls for Rs. 3900. Later, she buys another bat and 3 more balls of the same kind for Rs. 1300. Represent this situation algebraically and graphically.

Ans. Let the cost of 1 bat be Rs x and cost of 1 ball be Rs. y

Case I.

$$\text{Cost of 3 bats} = 3x$$
$$\text{Cost of 3 balls} = 3y$$

According to question,

$$3x+6y = 3900$$

Case II. Cost 1 bat $= x$

$$\text{Cost of 3 more balls} = 3y$$

According to question,

$$x+3y = 1300$$

So, algebraically representation be

$$3x+6y = 3900$$
$$x+3y = 1300$$

Graphical representation

We have,
$$3x+6y = 3900$$
$$\Rightarrow \quad 3(x+2y) = 3900$$
$$\Rightarrow \quad x+2y = 1300$$
$$\Rightarrow \quad x = 1300-2y$$

Thus, we have following table :

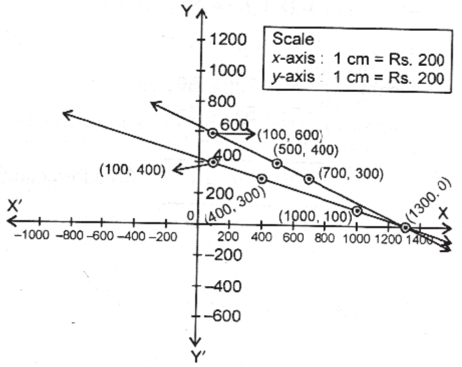
x	1300	500	700	100
y	–2	0	12	4

We have,
$$x+3y = 1300$$
$$\Rightarrow \quad x = 1300-3y$$

Thus, we have following table :

x	1300	100	400	1000
y	0	400	300	100

When we plot the graph of equations, we find that both the lines intersect at the point (1300, 0). Therefore, $x = 1300, y = 0$ is the solution of the given system of equations.



Q.3. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs. 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs. 300. Represent the situation algebraically and geometrically.

Ans. Let the cost of 1 kg of apples be Rs. x and of 1 kg of grapes be Rs. y. So, algebraic representation

$$2x + y = 160$$

$$4x + 2y = 300$$

$$\Rightarrow \quad 2x + y = 150$$

Graphical representation, we have

$$2x + y = 160$$

$$\Rightarrow \quad y = 160 - 2x$$

x	50	40
y	60	80

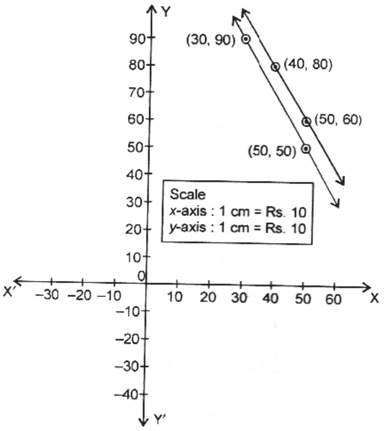
Graphical representation, we have

$$2x + y = 150$$

$$\Rightarrow \quad y = 150 - 2x$$

x	50	30
y	50	80

When we plot the graph of the equations we find that two lines do not intersect i.e. they are parallel.



EXERCISE 3.2

Q.1. Form a pair of linear equations in the following problems and find their solutions graphically.

(*i*) 10 students of Class X took part in Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(*ii*) 5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of pencil and that of one pen.

Ans. (*i*) Let the number of boys be x and number of girls be y.

Case I. $x + y = 10$ ....(*i*)

Case II. $y = x + 4$

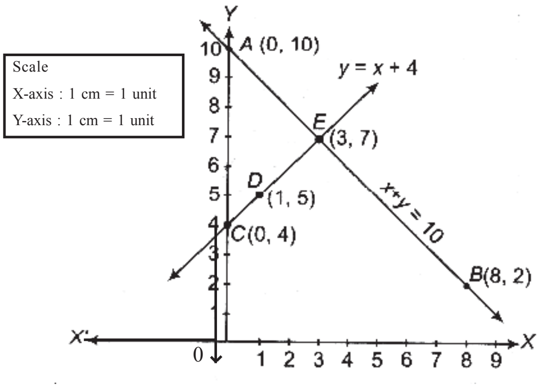
$x - y = -4$ ...(*ii*)

We have, $x + y = 10$

$$x = 10 - y$$

Thus, we have following table :

x	6	3	5
y	4	7	5



We have, $x - y = -4$

$\Rightarrow$ $x = y - 4$

Thus, we have following table :

x	0	3	1
y	4	7	5

When we plot the graph of the given equation, we find that both lines intersect at the point (3, 7). So x = 3, y = 7 is the required solution of the pair of linear equation.

Hence, the number of boys be 3 and the number of girls be 7, who took part in quiz.

(*ii*) Let cost of 1 pencil be Rs. x and cost of 1 pen be Rs. y.

Case I. Cost of pencils = 5x

Cost of 7 pens = 7y

According to question,

$$5x + 7y = 50$$

Case II. Cost of pencils = 7x

Cost of 5 pens = 5y

According to question,

$$7x + 5y = 46$$

Thus, we have

$$\begin{aligned} 5x + 7y &= 50 \\ 7x + 5y &= 46 \\ \hline 12x + 12y &= 96 \\ \hline \Rightarrow \quad 12(x+y) &= 0 \\ \Rightarrow \quad x + y &= 8 \end{aligned}$$

Again,

$$\begin{aligned} 5x + 7y &= 50 \\ 7x + 5y &= 46 \\ - \quad - \quad &\ - \\ \hline -2x + 2y &= 4 \\ \hline \Rightarrow \quad -2(x-y) &= 4 \\ \Rightarrow \quad x - y &= -2 \end{aligned}$$

We have, $x + y = 8$

$\Rightarrow$ $x = 8 - y$

Thus, we have following table :

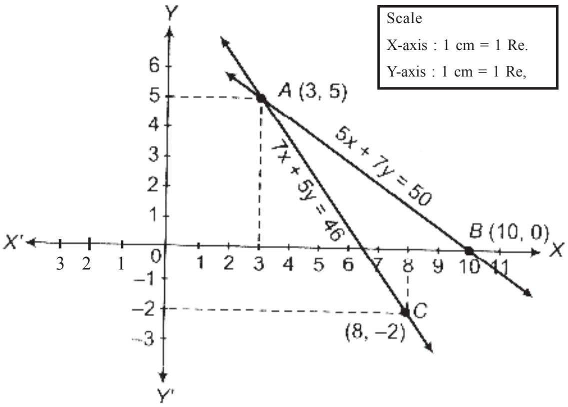
x	4	3	6
y	4	5	2

We have, $x - y = -2$

$x = y - 2$

Thus, we have following table :

x	0	3	2
y	2	5	4



When we plot the graph of the given equation, we find that both the lines intersect at the point (3, 5). So $x = 3$, $y = 5$ is the required solution of the pair of linear eqation

Hence, the cost of 1 pencil be Rs. 3, cost of 1 pen be Rs. 5.

Q.2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ and without drawing them, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide.

(*i*) $5x - 4y + 8 = 0, 7x + 6y - 9 = 0$

(*ii*) $9x + 3y + 12 = 0, 18x + 6y + 24 = 0$

(*iii*) $6x - 3y + 10 = 0, 2x - y + 9 = 0$

Ans. Comparing the given equations with standard form of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

(*i*) $a_1 = 5, b_1 = -4, c_1 = 8$

$a_2 = 7, b_2 = 6, c_2 = -9$

$$\therefore \quad \frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Thus, the lines representing the pair of linear equations are intersecting.

(*ii*) $a_1 = 9, b_1 = 3, c_1 = 12$

$a_2 = 18, b_2 = 6, c_2 = 24$

$$\therefore \frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Thus, the lines representing the pair of linear equations are intersecting.

(*iii*) $a_1 = 6, b_1 = -3, c_1 = 10$
$a_2 = 2, b_2 = -1, c_2 = 9$

$$\therefore \quad \frac{a_1}{a_2} = \frac{6}{2} = 3, \frac{b_1}{b_2} = \frac{-3}{-1} = 3, \frac{c_1}{c_2} = \frac{10}{9}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, the lines representing the pair of linear equations are parallel.

Q.3. On comparing the rations $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent or inconsistent.

(*i*) $3x + 2y = 5; 2x - 3y = 7$
(*ii*) $2x - 3y = 8; 4x - 6y = 9$
(*iii*) $\frac{3}{2}x + \frac{5}{3}y = 7; 9x - 10y = 14$
(*iv*) $5x - 3y = 11; -10x + 6y = -22$
(*v*) $\frac{4}{3}x + 2y = 8; 2x + 3y = 12$

Ans. (*i*) $3x + 2y = 5; 2x - 3y = 7$

Here, $a_1 = 3, b_1 = 2, c_1 = 5$
$a_2 = 2, b_2 = -3, c_2 = 7$

$$\therefore \quad \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-2}{3}, \frac{c_1}{c_2} = \frac{5}{7}$$

Clearly, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given lines are intersecting. So, the given pair of linear equations has exactly one solution and therefore it is consistent.

(*ii*) $2x - 3y = 8; 4x - 6y = 9$

Here, $a_1 = 2, b_1 = -3, c_1 = 8$
$a_2 = 4, b_2 = -6, c_2 = 9$

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \text{ and } \frac{c_1}{c_2} = \frac{8}{9}$$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given lines are parallel. So, the given pair of linear equations has no solution and therefore it is inconsistent.

(*iii*) $\frac{3}{2}x + \frac{5}{3}y = 7; 9x - 10y = 14$

Here, $a_1 = \frac{3}{2}, b_1 = \frac{5}{3}, c_1 = 7$
$a_2 = 9, b_2 = -10, c_2 = 14$

$$\frac{a_1}{a_2} = \frac{3/2}{9} = \frac{1}{6}$$

$$\frac{b_1}{b_2} = \frac{5/3}{-10} = \frac{1}{6}$$

and $\frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$

Clearly, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given lines are intersecting. So the given pair of linear equations has exactly one solution and therefore it is consistent.

(*iv*) $5x - 3y = 11; -10x + 6y = -22$

Here, $a_1 = 5, b_1 = -3, c_1 = 11$
$a_2 = -10, b_2 = 6, c_2 = -22$

$$\frac{a_1}{a_2} = \frac{5}{-10} \neq -\frac{1}{2}$$

$$\frac{b_1}{b_2} = -\frac{3}{6} = -\frac{1}{2}$$

and $\frac{c_1}{c_2} = \frac{11}{-22} = -\frac{1}{2}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given lines are consistent. So, the given pair of linear equations has infinitely many solutions and therefore it is consistent.

(*v*) $\frac{4}{3}x + 2y = 8; 2x + 3y = 12$

Hence, $a_1 = \frac{4}{3}, b_1 = 2, c_1 = 8$

$$\frac{a_1}{a_2} = \frac{4/3}{2} = \frac{2}{3}$$

$$\frac{b_1}{b_2} = \frac{2}{3}$$

and $\frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given lines are consistent. So the given pair of linear equations has infinitely many solutions and therefore it is consistent.

Q.4. Which of the following pairs of linear equations are consistent/inconsistent ? If Consistent, obtain the solution graphically :

(*i*) $x + y = 5, 2x + 2y = 10$

(*ii*) $x - y = 8; 3x - 3y = 16$

(*iii*) $2x + 2y - 6 = 0, 4x - 2y - 4 = 0$

(*iv*) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

Ans. (*i*) Wehave

$$x + y = 5$$

$$\Rightarrow \quad y = 5 - x$$

Thus, we have following table :

x	0	5
y	5	0

$$2x + 2y = 10$$

$$\Rightarrow \quad y = \frac{10 - 2x}{2}$$

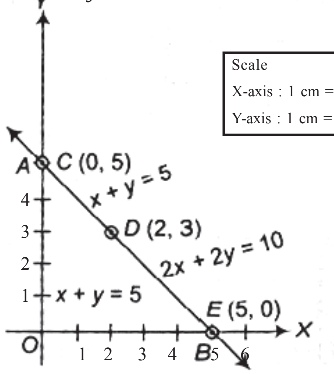
Thus, we have following table :

x	0	2	5
y	5	3	0

When we plot the graph of the equations we find that both the lines are coincident.

Hence, pair of linear equations has infinitely many solutions

$$y = 2x - 2$$



(*ii*) Given pair of lines are $x - y = 8, 3x - 3y = 16$

Here,

$a_1 = 1, b_1 = -1, c_1 = -8; a_2 = 3, b = -1, c_2 = -16$

Here, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$

and $\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines represented by the given equations are parallel.

Therefore, it has no solution. Hence, the given pair of lines is inconsistent.

(iii) We have,

$$2x + y - 6 = 0$$

$$\Rightarrow \quad y = 6 - 2x$$

x	0	3
y	6	0

and $4x - 2y - 4 = 0$

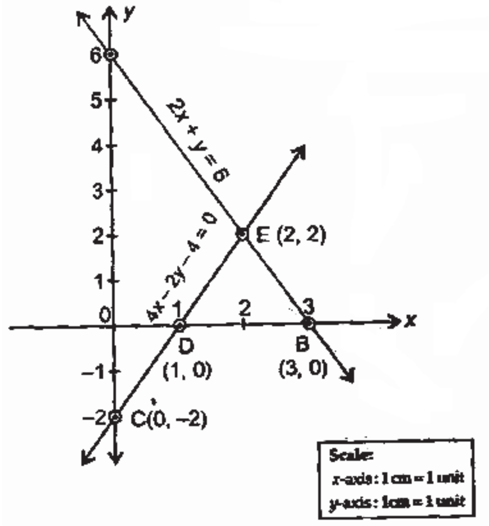
$$y = 2x - 0$$

Thus we have following table :

x	0	1
y	–2	0

When we plot graph of the equations, we find that both the lines intersect at point (2, 2).

Hence the solution of the given equation is $x = 2$, $y = 2$.



Therefore, the pair of equation is consistent at point (2, 2).

(*iv*) $2x - 2y - 2 = 0$

$y = 1$

Thus we have following table :

x	0	1
y	–1	0

$4x - 4y - 5 = 0$

$\Rightarrow \quad y = x - -\frac{5}{4}$

when $x = 0, y = -\frac{5}{4}$

when $x = \frac{5}{4}, y = 0$

x	0	$\frac{5}{4}$
y	$-\frac{5}{4}$	0

When we plot graph of the given equations, we find that both the lines never meet.

Hence lines are parallel and equations have no solution.

Q.5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Ans. Let the length of the garden be x m and width be y m.

Case I. $x = y + 4$

$\Rightarrow \quad x - y = 4$

Case II.

Half perimeter $= 36$

$\Rightarrow \quad x + y = 36$

So algebraic representation be

$x - y = 4$

$x + y = 36$

Graphical representation :

We have, $x - y = 0$

$\Rightarrow \quad x = 0$

Thus, we have following table

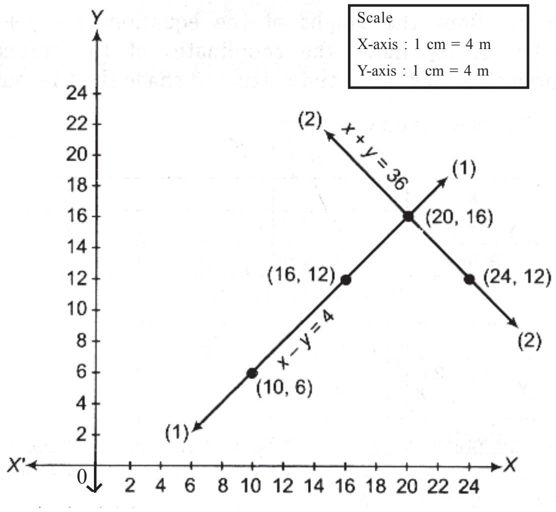
x	0	4	12
y	–4	0	8

$x + y = 0$

$\Rightarrow \quad x = 36 - y$

Thus, we have following table :

x	10	20	16
y	26	16	20



If we plot the graph of both the equations, we find that the two lines intersect at the point (20, 16). So, $x = 20, y = 16$ is the required solution of the given equation i.e., the length of the garden is 20 m and breadth is 16 m.

Q.6. Given the linear equations $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is :

(*i*) intersecting lines

(*ii*) parallel lines

(*iii*) coincident lines.

Ans. We have,

$2x + 3y - 8 = 0$

(i) Another linear equation in two variables such that the geometrical representation of the pair so formed is intersecting lines is

$3x - 2y - 8 = 0$

(*ii*) Another parallel lines to above line is

$4x - 6y - 22 = 0$

(*iii*) Another coincident line to above line is

$6x + 9y - 24 = 0$

Q.7. Draw the graph of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis and shade the triangular region.

Ans. We have,

$$x+y+1 = 0$$

$\Rightarrow$ $$y = x+1$$

Ans. We have,

Thus, we have following table :

x	2	–1	3
y	3	0	4

We have,

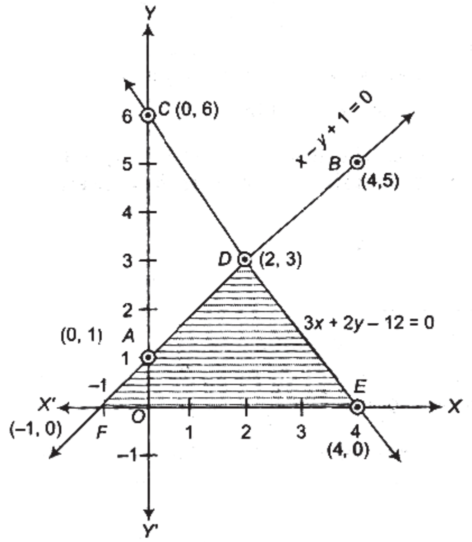
$$3x+2y+12 = 0$$

$\Rightarrow$ $$2y = 12 - x$$

$\Rightarrow$ $$y = \frac{12-3x}{2}$$

Thus, we have following table :

x	2	4	0
y	3	0	6



When we plot the graph of the given equations, we find that both the lines intersect at the pont (2, 3), therefore $x = 2$, $y = 3$ is the solution of the given system of equations.

Vertices of triangle are (2, 3), (–1, 0) and (4, 0).

EXERCISE 3.3

Q.1. Solve the following pairs of linear equations by the substitution method.

(*i*) $x+y=14$ (*ii*) $s-t=3$

$x-y = 4$ $\quad \frac{s}{3}+\frac{t}{2}=6$

(*iii*) $3x+y=3$ (*iv*) $0.2x+0.3y=1.3$

$9x-3y = 9$ $\quad 0.4x+0.5y=2.3$

(*v*) $\sqrt{2}x+\sqrt{3}y=0$ (*vi*) $\frac{3x}{2}-\frac{5y}{3}=-2$

$\sqrt{3}x+\sqrt{8}y=0$ $\quad \frac{x}{3}-\frac{y}{2}=\frac{13}{6}$

Ans. (*i*) $x+y = 14$

$x-y = 4$

The given pair of linear equations is

$$x+y = 14 \quad ...(i)$$

$$x-y = 4 \quad ...(ii)$$

From equation (*i*), we have

$$y = 14-x \quad ...(iii)$$

Substitute this value of y in equation (ii), we get

$$x-(14-x) = 4$$

$\Rightarrow$ $$x-14+x = 4$$

$\Rightarrow$ $$2x-14 = 4$$

$\Rightarrow$ $$2x = 4+14$$

$\Rightarrow$ $$2x = 18$$

$\Rightarrow$ $$x = \frac{18}{2} = 9$$

Substitute this value of x in equation (*iii*), we get

$$y = 14-9 = 5$$

Therefore the solution is

$$x = 9, y = 5$$

Verification. Substituting $x = 9$ and $y = 5$, we find that both the equations (*i*) and (*ii*) are satisfied as shown below

$$x+y = 9+5 = 14$$

$$x-y = 9-5 = 4$$

This verifies the solution.

Ans. (*ii*) We have

$$s-t = 3$$

$$\frac{s}{3}+\frac{t}{2} = 6$$

and $$\frac{s}{3}+\frac{t}{2} = 6$$

$$\Rightarrow \quad \frac{2s+3t}{6} = 6$$

$$\Rightarrow \quad 2s-3t = 36$$

The given pair of linear equations is

$$s-t = 3 \quad ...(i)$$

$$2s-3t = 36 \quad ...(ii)$$

From equation (i), we have

$$s = 3+t \quad ...(iii)$$

Substitute this value of s in equation (*ii*), we get

$$2s+3t = 36$$

$$\Rightarrow \quad 2(3+t)+3t = 36$$

$$\Rightarrow \quad 6+2t+3t = 36$$

$$\Rightarrow \quad 5t+6 = 36$$

$$\Rightarrow \quad 5t = 30$$

$$\Rightarrow \quad t = 6$$

Therefore, the solution is

$$s = 9, t=6$$

$$\frac{s}{3}+\frac{t}{2} = \frac{9}{3}+\frac{6}{2} = 3+3=6$$

This verifies the solution.

(*iii*) $3x-y = 3$

$9x-3y = 9$

The given pair of linear equation is

$$3x-y = 3 \quad ...(i)$$

$$9x-3y = 9 \quad ...(ii)$$

From eqn. (i), we have

$$y = 3x-3$$

Substitute this value of y in eq. (ii), we get

$$9x-(3x-3) = 9$$

$$\Rightarrow \quad 9x-9x+9 = 9$$

$$\Rightarrow \quad 9 = 9$$

which is true. Therefore equn. (i) and (ii) have infinitely many solutions.

Ans. (*iv*) $0.2x+0.3y = 1.3$

$0.4x-0.5y = 2.3$

The given system of linear equations

$$0.2x+0.3y = 1.3 \quad ...(i)$$

$$0.4x-0.5y = 2.3 \quad ...(ii)$$

From equation (*i*), we have

$$0.3y = 1.3-0.2x$$

$$\Rightarrow \quad y = \frac{1.3-0.2x}{0.3} \quad ...(iii)$$

Substituting this value of y in eqn. (ii), we get

$$0.4x+0.5\left(\frac{1.3-0.2x}{0.3}\right) = 2.3$$

$$\Rightarrow 0.21+0.65-0.1x = 0.69$$

$$\Rightarrow \quad 0.12x-0.1x = 0.69-0.65$$

$$\Rightarrow \quad 0.02x = 0.04$$

$$\Rightarrow \quad x = \frac{0.04}{0.02}=2$$

Substituting this vaue of x in eqn. (*iii*), we get

$$y = \frac{1.3=0.2(2)}{0.3}$$

$$= \quad = \frac{0.9}{0.3}=3$$

Therefore, the solution is $x=2, y=3$,

Verification, Substituting $x=2$ and $y=3$, we find that both the equations (*i*) and (*ii*) are satisfied as shown below :

$$0.2x+0.3y = (0.2)(2)+(0.3)(3)$$

$$= 0.4+0.9=1.3$$

$$0.4+0.5y = (0.4)(2)+(0.5)(3)$$

$$= 0.8+1.5=2.3$$

This verifies the solution.

(*v*) $\sqrt{2}x+\sqrt{3}y = 0$

$\sqrt{3}x-\sqrt{8}y = 0$

The given pair of linear eqn. is

$$\sqrt{2}x+\sqrt{3}y = 0 \quad ...(i)$$

$$\sqrt{3}x-\sqrt{8}y = 0 \quad ...(ii)$$

From eq. (ii), we have

$$\sqrt{3}x = \sqrt{8}y$$

$$\Rightarrow \quad x = \frac{\sqrt{8}}{\sqrt{3}}y \quad ...(iii)$$

Substituting this value of x in equation (i), we get

$$\sqrt{2}.\frac{\sqrt{8}}{\sqrt{3}}+\sqrt{3}y = 0$$

$$\Rightarrow \quad \frac{4}{\sqrt{3}}y+\sqrt{3}y = 0$$

$$\Rightarrow \quad \left(\frac{4}{\sqrt{3}}+\sqrt{3}\right)y = 0$$

$$\Rightarrow \quad y = 0$$

Substituting this value of y in eqn. (iii), we get

$$x = \frac{\sqrt{8}}{\sqrt{3}}(0)=0$$

Therefore, the solution is

$$x = 0, y = 0$$

Verification. Substituting $x = 0$ and $y = 0$, we find that both equations (*i*) and (*ii*) are satisfied as shown below :

$$\sqrt{2}x + \sqrt{3}y = \sqrt{2}(0) + \sqrt{3}(0) = 0$$

$$\sqrt{3}x - \sqrt{8}y = \sqrt{3}(0) - \sqrt{8}(0) = 0$$

This verifies the solution.

(*vi*) $$\frac{3x}{2} - \frac{5y}{3} = -2$$

$$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$$

We have

$$\frac{3x}{2} - \frac{5y}{3} = -2$$

$$\frac{9x - 10y}{6} = -2$$

$$9x - 10y = -12$$

and $$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$$

$\frac{141}{47}$ $9\left(\frac{13+3y}{2}\right)$

$$2x + 3y = 13$$

The given pair of linear equations is

$$9x - 10y = -12 \quad ...(i)$$

$$2x - 3y = 13 \quad ...(ii)$$

From (*ii*), we have

$$x = \frac{13 + 3y}{2} \quad ...(iii)$$

Substituting the value of x in (*i*), we get

$$9x - 10y = -12$$

$$\Rightarrow \quad -10y = -12$$

$$\Rightarrow \quad \frac{9(13 + 3y) - 20y}{2} = -12$$

$$\Rightarrow \quad 117 + 27y - 20y = -24$$

$$\Rightarrow \quad -47y = -24 - 117$$

$$\Rightarrow \quad y = \quad = 3$$

Substituting the value of y in (iii), we get

$$x = \frac{13 - 3y}{2}$$

$$x = \frac{13 - 3 \times 3}{2}$$

$$= \frac{13 - 9}{2} = \frac{4}{2}$$

Therefore, the solutions is

$$x = 2, y = 3$$

Verification. Substituting x = 2 and y = 3, we find that both the equations (i) and (ii) are satisfied as shown below :

$$\frac{3}{2}x - \frac{5y}{2} = \frac{3}{2}(2) - \frac{5}{2}(3) = 3 - 5 = -2$$

$$\frac{x}{3} + \frac{y}{2} = \frac{3}{2} + \frac{3}{2} = \frac{13}{6}$$

This verifies the solution

Q.2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of '*m*' for which $y = mx + 3$.

Ans. The given equations are

$$2x + 3y = 11 \quad ...(i)$$

$$2x - 4y = -24 \quad ...(ii)$$

From (*i*), we have

$$x = \frac{11 - 3y}{2} \quad ...(iii)$$

Putting the value of '*x*' in (*ii*), we get

$$\Rightarrow \quad 2\left(\frac{11 - 3y}{2}\right) - 4y = -24$$

$$\Rightarrow \quad \frac{22 - 6y - 8y}{2} - 4y = -24$$

$$22 - 14y = -48$$

$$-14y = -70$$

$$y = 5$$

Putting the value of y in (*iii*), we get

$$x = \frac{11 - 3y}{2} = \frac{11 - 15}{2}$$

$$\Rightarrow \quad x = \frac{-4}{2} = -2$$

Hence, the solution is $x = -2, y = 5$

It is given that $\quad y = mx + 3$

Putting the values of x and y in given condition we get

$$5 = m(-2) + 3$$

$$\Rightarrow \quad 5 = -2m + 3$$

$$\Rightarrow \quad -2m = 2$$

$$\Rightarrow \quad m = -1.$$

Q.3. From the pair of linear equations for the following problems and find their solutions by substitution method.

(*i*) The difference between two numbers is 26 and one number is three times the other. Find them.

(*ii*) The larger of two supplementary angles exceeds the smaller by 18 degree. Find them.

(*iii*) The coach of a cricket team buys 7 bats and 6 balls for Rs. 3800. Later, she buys 3 bats and 5 balls for Rs. 1750. Find the cost of each bat and each ball.

(*iv*) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs. 105 and for a journey of 15 km, the charge paid is Rs. 155. What are the fixed charges and the charge per km ? How much does a person has to pay for travelling a distance of 25 km ?

(*v*) Afraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the demoniator it becomes $\frac{5}{6}$. Find the fraction.

(*vi*) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages ?

Ans. (*i*) Let the two numbers be x and y

$\therefore$ By the given conditions,

$$x - y = 26$$

and $$x = 3y$$

Sinstituting $x = 3y$ in (*i*), we get

$$3y - y = 26$$

$$\Rightarrow \quad 2y = 26$$

$$\Rightarrow \quad y = 13$$

From (*ii*), $x = 3y = 3 \times 13 = 39$

Thus the numbers are 39 and 13.

(*ii*) Let the larger angle be *x* and smaller angle be *y*.

According to given conditions,

$$x + y = 180 \qquad ...(i)$$

$$\Rightarrow \quad (y+18)+y = 180 \qquad ...(ii)$$

$$\Rightarrow \quad y+18+y = 180$$

$$\Rightarrow \quad 2y = 180-18$$

$$\Rightarrow \quad 2y = 162$$

$$\Rightarrow \quad y = 81^o \qquad ...(iii)$$

Putting the value of (*iii*) in (*ii*)

$$x = y + 18$$

$$= 81 + 18 = 90^o$$

Thus, the angles are 81^o and 99^o.

(*iii*) Let the cost of each bat and each ball be *x* Rs. *x* and Rs. *y* respectively.

According to given conditions,

$$7x + 6y = 3800 \qquad ...(i)$$

$$3x + 5y = 1750 \qquad ...(ii)$$

From eqn. (*ii*) we get

$$3x + 5y = 1750$$

$$5y = 1750 - 3x$$

$$y = \frac{1750-3x}{5} \qquad ...(iii)$$

Substitue the value of eqn. (iii) in (i), we get

$$7x + 6y = 3800$$

$$7x + 6\left(\frac{1750-3x}{5}\right)x = 3800$$

$$\Rightarrow \quad \frac{35x + 6(1750-3x)}{5} = 3800$$

$$\Rightarrow \quad 35x + 10500 - 18x = 19000$$

$$\Rightarrow \quad 17x + 10500 = 19000$$

$$\Rightarrow \quad 17x = 19000 - 10500$$

$$\Rightarrow \quad 17x = 8500$$

$$\Rightarrow \quad x = \frac{8500}{17} = 500$$

Substituting the value of *x* in (iii) we get,

$$y = \frac{1750-3x}{5}$$

$$= \frac{1750-3\times 500}{5}$$

$$= \frac{1750-1500}{5} =$$

$$= \frac{250}{5} = 50$$

Hence, the cost of one bat = Rs. 500 and cost of one ball = Rs. 50.

(*iv*) Let fixed charge be Rs. *x* and charge per km be Rs. *y*

According to the given conditions,

$$x + 10y = 105 \qquad ...(i)$$

and $$x + 15y = 155 \qquad ...(ii)$$

Subtracting (*i*) from (*ii*), we get

$$(x+5y)-(x+10y) = 155-105$$

$$\Rightarrow \quad x + 15y - x - 10y = 50$$

$\Rightarrow \quad 5y = 50 \Rightarrow y = 10$

Putting the value of y in (*i*), we get

$$x + 10y = 105$$
$$\Rightarrow \quad x + 10(10) = 105$$
$$\Rightarrow \quad x + 100 = 105 \Rightarrow x = 5$$

$\therefore$ Fixed charge (x) = Rs. 5

and charge per km (y) = Rs. 10

Thus, charges for 25 km

$$= x + 25y = 5 + 25(10)$$
$$= 5 + 250 = \text{Rs. } 255$$

(*v*) Let the numerator of the fraction be x and denominator be y.

Then, Fraction $= \dfrac{x}{y}$

Case I. $\dfrac{x+2}{y+2} = \dfrac{9}{11}$

$$\Rightarrow \quad 11(x+2) = 9(y+2)$$
$$\Rightarrow \quad 11x + 22 = 9y + 18$$
$$\Rightarrow \quad 11x - 9y = -4 \qquad ...(i)$$

Case II. $\dfrac{x+3}{y+3} = \dfrac{5}{6}$

$$\Rightarrow \quad 6(x+3) = 5(y+3)$$
$$\Rightarrow \quad 6x + 18 = 5y + 15$$
$$\Rightarrow \quad 6x - 5y = -3 \qquad ...(ii)$$

Thus, we have following equations

$$\Rightarrow \quad 11x - 9y = -4$$
$$\Rightarrow \quad 6x - 5y = -3$$

From (ii), we have

$$\Rightarrow \quad 6x - 5y = -3$$
$$\Rightarrow \quad 6x = 5y - 3$$
$$\Rightarrow \quad x = \frac{5y-3}{6} \qquad ...(iii)$$

Substituting the value of (*iii*) in (*i*), we get

$$\Rightarrow \quad 11x - 9y = -4$$
$$\Rightarrow \quad 11\left(\frac{5y-3}{6}\right) - 9y = -4$$
$$\Rightarrow \quad \frac{11(5y-3) - 54y}{6} = -4$$
$$\Rightarrow \quad 55y - 33 - 54y = -24$$
$$\Rightarrow \quad y - 33 = -24$$
$$\Rightarrow \quad y = -24 + 33$$
$$\Rightarrow \quad y = 9$$

Now, substituting the value of y in (iii), we get

$$x = \frac{5y-3}{6}$$
$$\Rightarrow \quad x = \frac{5 \times 9 - 3}{6}$$
$$= \frac{45-3}{6} = \frac{42}{6} = 7$$

Hence, the required fraction is $\dfrac{7}{9}$.

(*vi*) Let the present ago of Jacob = x years and present age of his son = y years.

Case I.

5 years hence,

Age of Jacob = $(x + 5)$ years

and age of his son = $(y + 5)$ years

According to given conditions,

$$x + 5 = 3(y + 5)$$
$$\Rightarrow \quad x + 5 = 3y + 15$$
$$\Rightarrow \quad x - 3y = 10$$

Case II.

5 years ago,

Age of Jacob = $(x - 5)$ years

and age of his son = $(y - 5)$ years

According to given conditions,

$$x - 5 = 7(y - 5)$$
$$\Rightarrow \quad x - 5 = 7y - 35$$
$$\Rightarrow \quad x - 7y = -35 + 5$$
$$\Rightarrow \quad x - 7y = -30$$

Thus, we have following equations

$$x - 3y = 10 \qquad ...(i)$$
$$x - 7y = -30 \qquad ...(ii)$$

From (i), we have

$$x - 3y = 10$$
$$\Rightarrow \quad x = 3y + 10 \qquad ...(iii)$$

Substituting the value of x in (ii), we get

$$x - 7y = -30$$
$$\Rightarrow \quad 3y + 10 - 7y = -30$$
$$\Rightarrow \quad -4y + 10 = -30$$
$$\Rightarrow \quad -4y = -40$$
$$\Rightarrow \quad y = 10$$

Now, substituting the value of y in (iii), we get

$$x = 3y + 10$$
$$= 3(10) + 10$$
$$= 30 + 10 = 40$$

Hence,

Age of Jacob = 40 years

and Age of his son = 10 years

EXERCISE 3.4

Q.1. Solve the following pair of linear equations by the elimination method and the substitution method.

(*i*) $x + y = 5$ and $2x - 3 = 4$

(*ii*) $3x + 4y = 10$ and $2x - 2y = 2$

(*iii*) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

(*iv*) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x = \frac{y}{3} = 3$

Ans. Elimination Method :

(*i*)
$$x + y = 5 \quad ...(i)$$
$$2x - 3y = 4 \quad ...(ii)$$

For making the coefficeint of y in (*i*) and (*ii*) equal, we multiply (*i*) by 3 and adding, we get

$$\begin{aligned} 3x + 3y &= 15 \\ 2x - 3y &= 4 \\ \hline 5x &= 19 \end{aligned}$$

$$\Rightarrow \quad x = \frac{19}{5}$$

Now, putting the value of x in equation (*i*), we get

$$x + y = 5$$
$$\Rightarrow \quad \frac{19}{5} + y = 5$$
$$\Rightarrow \quad y = 5 - \frac{19}{5} = \frac{25-19}{5} = \frac{6}{5}$$

Hence, $x = \frac{19}{5}$, $y = \frac{6}{5}$

Substitution method :

We have following equations :

$$x + y = 5$$
$$2x - 3y = 4$$

From (*i*), we have

$$x + 5 = 5$$
$$\Rightarrow \quad x = 5 - y$$

Substituting the value of x in (*ii*), we get

$$2x - 3y = 4$$
$$\Rightarrow \quad 2(5 - y) - 3y = 4$$
$$\Rightarrow \quad 10 - 2y - 3y = 4$$
$$\Rightarrow \quad 10 - 5y = 4$$
$$\Rightarrow \quad -5y = 4 - 10$$
$$\Rightarrow \quad -5y = -6$$
$$\Rightarrow \quad y = \frac{6}{5}$$

Now, substituting the value of y in (*iii*), we get

$$x = 5 - y$$
$$\Rightarrow \quad x = 5 - \frac{6}{5}$$
$$\Rightarrow \quad x = \frac{25-6}{5} = \frac{19}{5}$$

Hence, $x = \frac{19}{5}$, $y = \frac{6}{5}$

(*ii*)
$$3x + 4y = 10$$
$$2x - 2y = 2$$

For making the coefficient of y in (i) and (ii) equal, we multiply (ii) by 2 and adding, we get

$$\begin{aligned} 3x + 4y &= 10 \\ 4x - 4y &= 4 \\ \hline 7x &= 14 \end{aligned}$$

$$\Rightarrow \quad x = \frac{14}{7} = 2$$

Now, putting the value of in (i), we get

$$3x + 4y = 10$$
$$\Rightarrow \quad 3(2) + 4y = 10$$
$$\Rightarrow \quad 6 + 4y = 10$$
$$\Rightarrow \quad 4y = 4$$
$$\Rightarrow \quad y = 1$$

Hence, $x = 2, y = 1$

Substitution method :

We have,

$$3x + 4y = 10 \quad ...(i)$$
$$2x - 2y = 2 \quad ...(ii)$$

From (*i*), we have

$$3x + 4y = 10$$
$$\Rightarrow \quad 3x = 10 - 4y$$
$$\Rightarrow \quad x = \frac{10-4y}{3} \quad ...(iii)$$

Substituting the value of (iii) in (ii), we get

$$2x - 2y = 2$$
$$\Rightarrow \quad 2\left(\frac{10-4y}{3}\right) - 2y = 2$$
$$\Rightarrow \quad \frac{2(10-4y) - 6y}{3} = 2$$
$$\Rightarrow \quad 20 - 8y - 6y = 6$$
$$\Rightarrow \quad -14y = 6 - 20$$
$$\Rightarrow \quad -14y = -14$$
$$\Rightarrow \quad y = 1$$

Now, substituting the value of y in (*iii*), we get

$$x = \frac{10-4y}{3} = \frac{10-4(1)}{3} = \frac{10-4}{3} = \frac{6}{3} = 2$$

Hence, $x = 2, y = 1$.

(*iii*) $3x - 5y = 4$ ...(*i*)

$9x - 2y = 7$ ...(*ii*)

For making the coefficient of x in (*i*) and (*ii*) equal, we multiply eqn. (*i*) by 3 and subtracting, we get

$$\begin{aligned} 9x - 15y &= 12 \\ 9x - 2y &= 7 \\ - \quad + \quad & \quad - \\ \hline -13y &= 5 \end{aligned}$$

$$y = \left(\frac{-5}{13}\right)$$

Now putting the value of y in (i), we get

$$3x - 5y = 4$$

$$\Rightarrow \quad 3x - 5\left(\frac{-5}{13}\right) = 4$$

$$\Rightarrow \quad 3x + \frac{25}{13} = 4$$

$$\Rightarrow \quad 3x = \frac{4}{1} - \frac{25}{13}$$

$$\Rightarrow \quad 3x = \frac{52-25}{13}$$

$$\Rightarrow \quad 3x = \frac{27}{13}$$

$$\Rightarrow \quad x = \frac{9}{13}$$

Hence, $x = \frac{9}{13}, y = \left(\frac{-5}{13}\right)$

Substitution method :

We have,

$3x - 5y = 4$ ...(*i*)

$9x - 2y = 7$ ...(*ii*)

From (*i*), we have

$$3x - 5y = 4$$

$$\Rightarrow \quad 3x = 4 + 5y$$

$$\Rightarrow \quad x = \frac{4+5y}{3} \qquad ...(iii)$$

Substituting the value of (iii) in (ii), we get

$$9x - 2y = 7$$

$$\Rightarrow \quad 9 \qquad -2y = 7$$

$\left(\frac{4+5y}{3}\right)$ $\frac{y}{3}$ $\frac{15}{39}$

$$\Rightarrow \quad \frac{9(4+5y) - 6y}{3} = 7$$

$$\Rightarrow \quad 36 + 45y - 6y = 21$$

$$\Rightarrow \quad 36 + 39y = 21 \Rightarrow 39y = -5$$

$$\Rightarrow \quad y = \qquad = \frac{-5}{13}$$

Now, substituting the value of y in (*iii*), we get

$$x = \frac{4 + 5\left(\frac{-5}{13}\right)}{3} = \frac{4 - \frac{25}{13}}{3} = \frac{\frac{52-25}{13}}{\frac{3}{1}} = \frac{27}{13} \times \frac{1}{3} = \frac{9}{13}$$

Hence, $x = \frac{9}{13}, y = \frac{-5}{13}$.

(*iv*) $$\frac{x}{y} + \frac{2y}{3} = -1$$

$$x - \frac{y}{3} = 3.$$

Considering equation

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$\Rightarrow \quad \frac{3x+4y}{6} = -1$$

$$\Rightarrow \quad 3x + 4y = -6$$

and $$x - \qquad = 3$$

$$\Rightarrow \quad \frac{3x-y}{3} = 3$$

$$\Rightarrow \quad 3x - y = 9$$

Now, we have following pairs of equations

$3x + 4y = -6$ ...(*i*)

$3x - y = 9$ ...(*ii*)

Since the coefficients of 'x' (*i*) and (*ii*) are equal. So simply by subtracting we can eliminate the variable i.e., x.

$$\begin{aligned} 3x + 4y &= -6 \\ 3x - y &= 9 \\ - \quad + \quad & = \quad - \\ \hline 5y &= -15 \end{aligned}$$

$$\Rightarrow \quad y = \frac{-15}{5}$$

$$\Rightarrow \quad y = -3$$

Now putting the value of y in eqn. (i), we get

$$3x+4y = -6$$
$$3x+4(-3) = -6$$
$$\Rightarrow 3x-12 = -6$$
$$\Rightarrow 3x = 6$$
$$\Rightarrow x = 2$$

Hence, $x=2, y=-3$.

Substitution method :

$$\frac{x}{2}+\frac{2y}{3} = -1$$
$$\Rightarrow 3x+4y = -6$$

and $$x-\frac{y}{3} = 3$$
$$\Rightarrow 3x-y = 9$$

Now, we have following pairs of equations

$$3x+4y = -6 \quad ...(i)$$
$$3x-y = 9 \quad ...(ii)$$

From (ii), we have

$$3x-y = 9$$
$$y = 3x-9 \quad ...(iii)$$

Substituting the value of (*iii*) in (*i*), we get

$$3x+4y = -6$$
$$3x+4(3x-9) = -6$$
$$3I+12x-36 = -6$$
$$15x = -6+36$$
$$15x = 30$$
$$x = 2$$

Now, substituting the value of x in (*iii*) we get

$$y = 3x-9$$
$$= 3(2)-9 = 6-9 = 3$$

Hence, $x=2, y=-3$

Q.2. From the pair of linear equation in the following problems and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It become $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction ?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu ?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw Rs. 2000. She asked the cashier to give her Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. Find how many notes of Rs. 50 and Rs. 100 she received.

(v) A leading library has a fixed charge for the first three days and an additional charge for each day thereafter. Sarita paid Rs. 27 for a book kept for seven days, while Susy paid Rs. 21 for the book she kept for five days. Find the fixed charge and the charge for each extra days.

Ans. (*i*) Let the numerator of the fraction be x and denominator be y. Then

$$\text{Fraction} = \frac{x}{y}$$

Then, according to given conditions,

Case I. $$\frac{x+1}{y-1} = 1$$
$$\Rightarrow x+1 = y-1$$
$$\Rightarrow x-y = -2$$

Case II.

$$\frac{x}{y+1} = \frac{1}{2}$$
$$\Rightarrow 2x = y+1$$
$$\Rightarrow 2x-y = 1$$

Thus, we have following equations

$$x-y = -2 \quad ...(i)$$
$$\Rightarrow 2x-y = 1 \quad ...(ii)$$

From (i), we have

$$x-y = -2$$
$$\Rightarrow x = y-2 \quad ...(iii)$$

Substituting the value of x in (ii), we get

$$2x-y = 1$$
$$\Rightarrow 2(y-2)-y = 1$$
$$\Rightarrow 2y-4-y = 1$$
$$\Rightarrow y-4 = 1$$
$$\Rightarrow y = 5$$

Now, substituting the value of y in (iii), we get

$$x = y-2$$
$$= 5-2 = 3$$

Hence, the required fraction = $\frac{3}{5}$.

(*ii*) Let the present ago of Nuri be x years and present age of Sonu be y years.

Case I.

5 years ago,

Age of Nuri = $(x-5)$ years

and Age of Sonu = $(y-5)$ years

According to the given conditions,

$$x - 5 = 3(y - 5)$$
$$\Rightarrow \quad x - 5 = 3y - 15$$
$$\Rightarrow \quad x - 3y = -10$$

Case II.

Ten yeras later,

Age of Nuri = $(x + 10)$ years

Age of Sonu = $(y + 10)$ years

According to the given conditions,

$$x + 10 = 2(y + 10)$$
$$x + 10 = 2y + 20$$
$$x - 2y = 10$$

Thus, we have following equations :

$$x - 3y = -10 \quad ...(i)$$
$$x - 2y = 10 \quad ...(ii)$$

From (i), we have

$$x - 3y = -10$$
$$x = 3y - 10 \quad ...(iii)$$

Substituting the value of (iii) in (ii), we get

$$x - 2y = 10$$
$$3y - 10 - 2y = 10$$
$$y = 20$$

Now, substituting the value of y in (iii), we get

$$x = 3y - 10$$
$$= 3(30) - 10$$
$$= 60 - 10 = 50$$

Hence, Age of Nuri = 50 years

and Age of Sonu = 20 years

(iii) Let the digit at Unit's place be x and digit at ten's place be y

Then, number = $10y + x$

Also, the number obtained by reversing the order of the digits = $10x + y$

According to the given condition,

$$x + y = 9$$
$$\text{And} \quad 9(10y + x) = 2(10x + y) \quad ...(i)$$
$$\Rightarrow \quad 90y + 9x = 20x + 2y$$
$$\Rightarrow \quad 11x - 88y = 0$$
$$\Rightarrow \quad x - 8y = 0 \quad ...(ii)$$

Subtracting question (ii) from equation (i) from equation (i), we get

$$9y = 9$$
$$\Rightarrow \quad y = \frac{9}{9} = 1$$

Substituting this value of y in equation (i), we get

$$x + 1 = 9$$
$$x = 9 - 1 = 8$$

Hence, the required number

$$= 10y + x$$
$$= 10(1) + 8$$
$$= 10 + 8 = 18$$

(iv) Suppose that Meena received x notes of Rs. 50 and y notes Rs. 100.

Then, according to the given conditions

$$x + y = 25 \quad ...(i)$$
$$\text{And} \quad 50x + 100y = 2000$$
$$\Rightarrow \quad x + 2y = 40 \quad ..(ii)$$

Subtracting this value of y in equation (i), we get

$$y = 15$$

Substituting this value of y in equation (i), we get

$$x + 15 = 25$$
$$\Rightarrow \quad x = 25 - 15 = 0$$

Hence, Meena received 10 notes of Rs. 50 and 15 notes of Rs. 100.

(v) Let the fixed charge be Rs. a and the charge for each extra day be Rs. b.

Then, according to the given conditions,

$$a + 4b = 27 \quad ...(i)$$

[Extra days = 7 – 3 = 4]

$$a + 2b = 21$$

[Extra days = 5 – 3 = 21]

Subtracting equation (ii) from equation (i), we get

$$2b = 6$$
$$b = \frac{6}{2} = 3$$

Substituting it is value of b in equation (ii), we get

$$a + 2(3) = 21$$
$$\Rightarrow \quad a + b = 21$$
$$\Rightarrow \quad a = 21 - 6 = 15$$

Hence, the fixed charges are Rs. 15 and the charge for each extra days is Rs. 3.

EXERCISE 3.5

Q.1. Which of the following pair of linear equations has unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it :

(i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

(*ii*) $2x+y=5$
$3x+9y=8$

(*iii*) $3x-5y=20$
$6x-10y=40$

(*iv*) $x-3y-7=0$
$3x-9y-2=0$

Ans.
$$x-3y-3 = 0$$
$$3x-9y-2 = 0$$

The given pair of linear equation is
$$x-3y-3 = 0$$
$$3x-9y-2 = 0$$

Here, $a_1 = 1, b_1 = -3, c_1 = -3$
$a_2 = 3, b_2 = -9, c_2 = -2$

We see that
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the given pair of linear equations has no solutions.

(ii)
$$2x+y = 5$$
$$3x+2y = 8$$

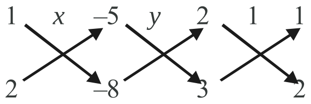
The given pair of linear equation is
$$2x+y = 5$$
$$3x+2y = 8$$
$$\Rightarrow \quad 2x+y-5 = 0$$
$$3x+2y-8 = 0$$

Here, $a_1 = 2, b_1 = 1, c_1 = -5$
$a_2 = 3, b_2 = 2, c_2 = -8$

We see that,
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence, the given pair of linear equations has a unique solution.

To solve the given equation by cross multiplication method, we draw the diagram below :



Then,
$$\frac{x}{(1)(-8)-(2)(-5)} = \frac{y}{(-5)(3)-(-8)(2)}$$
$$= \frac{1}{(2)(2)-(3)(1)}$$
$$\Rightarrow \quad \frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3}$$
$$\Rightarrow \quad \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$
$$\Rightarrow \quad x = 2, y = 1$$

Hence, the required solution of the given pair of linear equation is $x=2, y=1$.

(*iii*)
$$3x-5y = 20$$
$$6x-10y = 40$$

The given pair of linear equations is
$$3x-5y-20 = 0$$
$$\Rightarrow \quad 6x-10y-40 = 0$$

Here, $a_1 = 3, b_1 = -5, c_1 = -20$
$a_2 = 6, b_2 = -10, c_2 = -40$

What see that
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the given pair of linear equation has infinitely many solution.

(*iv*)
$$x-3y-7 = 0$$
$$3x-3y-15 = 0$$

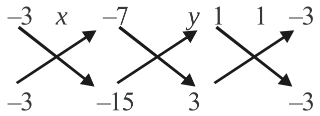
The given pair of linear equations is
$$x-3y-7 = 0$$
$$3x-3y-15 = 0$$

Here, $a_1 = 1, b_1 = -3, c_1 = -7$
$a_2 = 3, b_2 = -3, c_2 = -15$

We see that
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence, the given pair of linear equqtion has a unique solution.

To solve the given equations by cross multiplication method, we draw the diagram below :



Then,
$$\frac{x}{(-3)(-15)-(-7)(-3)}$$
$$= \frac{y}{(-7)(3)-(-15)(1)}$$
$$= \frac{y}{(1)(-3)-(3)(-3)}$$
$$\frac{x}{45-21} = \frac{y}{-21+15} = \frac{1}{-3+9}$$
$$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$
$$x = \frac{24}{6} = 4, y = -\frac{6}{6} = -1$$

Hence, the required method of the given pair of linear equatin is $x = 4, y = -1$,

Q.2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions ?

$$\begin{aligned} \mathbf{2x + 3y} &= \mathbf{7} \\ \mathbf{(a-b)\,x + (a+b)\,y} &= \mathbf{3a + b - 2} \end{aligned}$$

(ii) For which value of k will the following pair of linear equations have no solution ?

$$\begin{aligned} \mathbf{3x + y} &= \mathbf{1} \\ \mathbf{(2x-1)x + (k+1)\,y} &= \mathbf{2k + 1} \end{aligned}$$

Ans. (i) We have following equations

$$\begin{aligned} 2x + 3y &= 7 \\ (a-b)x + (a+b)y &= 3a + b - 2 \end{aligned}$$

Here, $a_1 = 2, b_1 = 3, c_1 = 7$

and $a_2 = a - b, b_2 = a + b,$

$c_2 = 3a + b - 2$

For having an infinite number of solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

4 1 3x
k 2 1

$$\Rightarrow \quad \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

From first two,

$$\begin{aligned} \frac{2}{a-b} &= \frac{3}{a+b} \\ \Rightarrow \quad 2(a+b) &= 3(a-b) \\ \Rightarrow \quad 2a + 2b &= 3a - 3b \\ \Rightarrow \quad a - 5b &= 0 \end{aligned}$$

and,

$$\begin{aligned} \frac{3}{a+b} &= \frac{7}{3a+b-2} \\ \Rightarrow \quad 3(3a+b-2) &= 7(a+b) \\ \Rightarrow \quad 9a + 3b - 6 &= 7a + 7b \\ \Rightarrow \quad 2a - 4b - 6 &= 0 \\ \Rightarrow \quad a - 2b - 3 &= 0 \end{aligned}$$

Thus, we have following equations

$a - 5b = 0$

$a - 2b - 3 = 0$

$$\begin{array}{ccccccc} -5 & a & 0 & b & 1 & 1 & -5 \\ -2 & & -3 & & 1 & & 2 \end{array}$$

Then

$$\frac{a}{(-5)(-3)-(-2)(0)} = \frac{b}{(0)(1)-(-3)(1)}$$

$$= \frac{1}{(1)(-2)-(1)(-5)}$$

$$\Rightarrow \quad \frac{a}{15} = \frac{b}{3} = \frac{1}{3}$$

$$\Rightarrow \quad a = \frac{15}{3} = 5$$

and $\quad b = \frac{3}{3} = 1$

Hence, $a = 5; b = 1$.

(*ii*) The given pair of linear equations

$$\begin{aligned} 3x + y &= 1 \\ (2k-1)\,x + (k-1)\,y &= 2k + 1 \\ \Rightarrow \quad 3x + y - 1 &= 0 \end{aligned}$$

Here, $(2x-1)\,x + (k-1)\,y - (2k+1) = 0$

$a_1 = 3, b_1 = 1, c_1 = -1$

$a_2 = 2k - 1, b_2 = k - 1, c_2 = -2k - 1$

For having no solution, we must have

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\begin{aligned} \Rightarrow \quad \frac{3}{2k-1} &= \\ \Rightarrow \quad 3(k-1) &= 2k - 1 \\ \Rightarrow \quad 3k - 3 &= 2k - 1 \\ \Rightarrow \quad 3k - 2k &= 3 - 1 \\ \Rightarrow \quad 2k &= 2 \end{aligned}$$

Hence, $k = 2$.

Q.3. Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$$\begin{aligned} 8x - 5y - 3 &= 9 \qquad \text{...(i)} \\ 3x + 2y &= 4 \qquad \text{...(ii)} \end{aligned}$$

Ans. The given pair of linear equtions is

$$\begin{aligned} 8x + 5y &= 9 \\ 3x + 2y &= 4 \end{aligned}$$

(*i*) By substitution method

From equation (*ii*), we have

$$2x = 4 - 3x$$

$$y = \qquad \text{...(iii)}$$

Substituting this value of y in equation (*i*), we get

$$\begin{aligned} 8x + 5\left(\frac{4-3x}{2}\right) &= 9 \\ 16x + 20 - 15x &= 18 \\ x + 20 &= 18 \end{aligned}$$

$$x = 18 - 20$$
$$x = -2$$

Substituting this value of x in equation (iii), we get

$$y = \frac{4 - 3(-2)}{2}$$
$$= \frac{4+6}{2} = \frac{10}{2} = 5$$

So, the solution of the given pair of linear equations is $x = -2$, $y = 5$.

(ii) By cross-multiplication method

Let us write the given pair of linear equations in

$$8x + 5y - 9 = 0$$
$$3x + 2y - 4 = 0$$

Solving the equations, we get

Then,

$$\frac{x}{(5)(-4)-(2)(-9)} = \frac{y}{(-9)(3)-(-4)(8)}$$
$$= \frac{1}{(8)(2)-(3)(5)}$$
$$= \frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15}$$
$$\Rightarrow \quad \frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$
$$\Rightarrow \quad x = -2 \text{ and } y = 5$$

Hence, the required solution of the given pair of linear equatins is $x = -2$, $y = 5$.

Q.4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method :

(*i*) A part of monthly charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs. 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs. 1180 as hostel charges. Find the fixed charge and the cost of food per day.

(*ii*) A fraction becomes when 1 is subtracted from the numerator and it becomes when 8 is added to its denominator. Find the fraction.

(*iii*) Yash scored 40 markes in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 markes been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test ?

(*iv*) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars ?

(*v*) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Ans. (i) Let the fixed hostel charges be Rs. *x* and the cost of food per day be Rs. *y* (Running charges)

Case I. Hostel charges of 'A'

Fixed charges be Rs x

and cost of food for 20 days = $20y$

According to question,

$$x + 20y = 1000$$

Thus, we have following equations

$$x + 20y = 1000$$
$$x + 26y = 1180$$

or
$$x + 20y - 1000 = 0$$
$$x + 26y - 1180 = 0$$

Now, by using cross multiplication method,

$$\begin{array}{cccccc} & x & & y & & 1 \\ 20 & & -1000 & & 1 & & 20 \\ 26 & & -1180 & & 1 & & 26 \end{array}$$

$$\Rightarrow \quad \frac{x}{(20)(-1180)-(26)(-1000)}$$
$$= \frac{y}{(-1000)(1)-(-1180)} = \frac{1}{(26)(1)-(20)(1)}$$
$$\Rightarrow \quad \frac{x}{-23600+26000} = \frac{y}{-1000+1180}$$
$$= \frac{1}{26-20}$$
$$\Rightarrow \quad \frac{x}{2400} = \frac{y}{180} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{x}{2400} = \frac{1}{6} \text{ and } \frac{y}{180} = \frac{1}{6}$$

$$x = 400 \text{ and } y = 300$$

Hence, fixed charges is Rs. 400 and cost of food per day is Rs. 30.

(*ii*) Let the numerator of the fraction be x and denominator be y. Then

$$\text{Fraction} = \frac{x}{y}$$

Case I. $$\frac{x-1}{y} = \frac{1}{3}$$

$$\Rightarrow \quad 3(x-1) = y$$
$$\Rightarrow \quad 3x-3 = y$$
$$\Rightarrow \quad 3x-y = 3$$
$$\Rightarrow \quad 3x-y-3 = 0$$

Case II.

$$\frac{x}{y+8} = \frac{1}{4}$$

$$\Rightarrow \quad 4x = y+8$$
$$\Rightarrow \quad 4x-y = 8$$
$$\Rightarrow \quad 4x-y-8 = 0$$

Thus, we have following equation

$$3x-y-3 = 0$$
$$4x-y-8 = 0$$

$$\begin{array}{ccccccc} -1 & x & -3 & y & 3 & 1 & -1 \\ -1 & & -8 & & 4 & & -1 \end{array}$$

Then,

$$\frac{x}{(-1)(-8)-(-3)(-1)} = \frac{y}{(-3)(4)-(-8)(-3)}$$

$$= \frac{1}{(3)(-1)-(-1)(4)}$$

$$\Rightarrow \quad \frac{x}{8-3} = \frac{y}{-12+24} = \frac{1}{-3+4}$$

$$\Rightarrow \quad \frac{x}{5} = \frac{y}{12} = \frac{1}{1}$$

$$\Rightarrow \quad \frac{x}{5} = \frac{1}{1} \text{ and } \frac{y}{12} = 1$$

$$\Rightarrow \quad x = 5 \text{ and } y = 12$$

Hence, numerator (x) of the fraction be 5 and denominator (y) be 12.

So fraction $= \dfrac{5}{12}$

(*iii*) Let the number of correct answers of Yash be x and number of wrong answres be y. Then according to question :

Case I. He gets 40 marks if 3 marks are given for correct answer and 1 mark is deducted for incorrect answers.

$$3x-y = 40$$

Case II. He gets 50 marks if 4 marks are given for correct answer and 2 marks are deducted for incorrect answers.

$$4x-2y = 50$$

Thus, we have following equations

$$3x-y = 0$$
$$4x-2y-50 = 0$$

Now, using cross mulitiplication method,

$$\begin{array}{ccccccc} -1 & x & -40 & y & 3 & 1 & -1 \\ -2 & & -50 & & 4 & & -2 \end{array}$$

$$\frac{x}{(-1)(-50)-(-2)(-40)}$$

$$= \frac{1}{(3)(-2)-(4)(-1)}$$

$$\Rightarrow \quad \frac{x}{50-80} = \frac{y}{-160+150} = \frac{1}{-6+4}$$

$$\Rightarrow \quad \frac{x}{-30} = \frac{y}{-10} = \frac{1}{-2}$$

$$\Rightarrow \quad \frac{x}{-30} = \frac{1}{-2} \text{ and } \frac{y}{-10} = \frac{1}{-2}$$

$$\Rightarrow \quad x = \frac{-30}{-2} \text{ and } y = \frac{-10}{-2}$$

$$\Rightarrow \quad x = 15 \text{ and } y = 5$$

Hence, total number of questions

= Number of correct answer + Number of incorrect answers

$= 15 + 5 = 20.$

(*iv*)

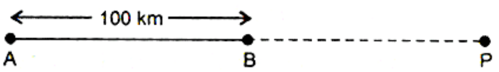


Fig. 1

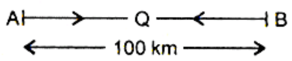


Fig. 2

Let the speed of the car starting from point A be x km/hr and speed of the car starting from point B be y km/hr.

Case I. When they travel in the same direction, let they meet at point P (Fig. 1).

So, AP = distance covered by first car from point A to P
= speed × time
= $x \times 5 = 5x$

and BP = distance covered by second car from point B to P
= speed × time
= $y \times 5 = 5y$

Therefore, AB = AP – BP

$$\Rightarrow \quad 100 = 5x - 5y$$
$$\Rightarrow \quad 5x - 5y = 100$$
$$\Rightarrow \quad x - y = 20$$

Case II. (Fig. 2)

When they travel in opposite direction, let they meet at point Q.

So. AQ = Distance covered by first car car from point B to Q
= speed × time
= $x \times 1 = x$

and BQ = Distance covered by second car from point B to Q
= speed × time
= $y \times 1 = y$

Threrefore AB = AQ + BQ

$$100 = x + y$$
$$x + y = 100$$

Thus, we have following eqn.

$$x - y = 20$$
$$x + y = 100$$
$$\Rightarrow \quad x - y - 20 = 0$$
$$x + y - 100 = 0$$

Now,

$$\begin{array}{ccccccc} -1 & x & -20 & y & 1 & 1 & -1 \\ +1 & & -100 & & 1 & & +1 \end{array}$$

$$\frac{x}{(-1)(-100)-(-20)(+1)} = \frac{y}{(-20)(-1)-(-100)(1)} = \frac{1}{(1)(+1)-(1)(-1)}$$

$$\Rightarrow \quad \frac{x}{100+20} = \frac{y}{-20+100} = \frac{1}{+1+1}$$

$$\Rightarrow \quad \frac{x}{120} = \frac{y}{80} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{x}{120} = \frac{1}{2} \text{ and } \frac{y}{80} = \frac{1}{2}$$

$$\Rightarrow \quad x = 60 \text{ and } y = 40$$

Hence, speed of the car that starts from point A = 60km/hr

Speed of the car that starts from point B = 40 km/hr.

(v) Let the dimension (*i.e.*, the length and the breadth) of the rectangle be x units and y units respectively.

Then, area of the rectangle

= length × bredth

= xy square units

According to the question,

$$xy - 9 = (x-5)(y+3)$$
$$xy - 9 = xy + 3x - 5y - 15$$
$$3x + 3y - 61 = 0$$

and

$$xy + 67 = xy + 2x + 3y + 6$$
$$2x + 3y - 61 = 0$$

Thus, we have following equation

$$3x - 5y - 6 = 0$$
$$2x + 3y - 61 = 0$$

Then,

$$\begin{array}{ccccccc} -5 & x & -6 & y & 3 & 1 & -5 \\ 3 & & -61 & & 2 & & 3 \end{array}$$

$$\frac{x}{(-5)(-61)-(3)(-6)} = \frac{y}{(-6)(2)-(61)(3)} = \frac{1}{(3)(3)-(2)(-5)}$$

$$\frac{x}{305+18} = \frac{y}{-12+183} = \frac{1}{9+10}$$

$$= \frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$$x = \frac{323}{19}\ 17 \text{ and } y = 9$$

Hence, the dimensions (i.e, the length and the breadth) of the rectangle are 17units and 9 units respectively.

EXERCISE 3.6

Q.1. Solve the following pair of equations by reducing them to a pair of linear equations :

(*i*) $\frac{1}{2x}+\frac{1}{3y}=0$ (*ii*) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$ $\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=0$

(*iii*) $\frac{4}{x}+3y=14$ (*iv*) $\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{3}{x}-4y=23$ $=1$

(*v*) $\frac{7x-2y}{xy}=5$ (*vi*) $6x+3y=6xy$

$\frac{8x+7y}{xy}=15$ $2x+4y=5xy$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}+\frac{1}{2(3x-y)}=\frac{-1}{8}$

Ans. (i) Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$

Then, the given system of equation becomes

$$\frac{1}{2}u+\frac{1}{3}v = 2$$
$$\Rightarrow \quad 3u+2v = 12 \quad ...(i)$$
and
$$\frac{1}{3}u+\frac{1}{2}v = \frac{13}{6}$$
$$\Rightarrow \quad \frac{2u+3v}{6} = \frac{13}{6}$$
$$\Rightarrow \quad 2u+3v = 13 \quad ...(ii)$$

Thus, we have two equations

$$2u+2v = 12$$
$$2u+3v = 13$$

From (*i*), wehave

$$u = \frac{12-2v}{3}$$

Putting the value of 'u' in (*ii*), we get

$$2\left(\frac{12-2v}{3}\right)+3v = 13$$
$$\Rightarrow \quad 24-6u+9v = 39$$
$$\Rightarrow \quad 5v = 39-24$$
$$\Rightarrow \quad 5v = 15$$
$$\Rightarrow \quad v = 3$$

Putting the value of 'v' in (*ii*), we get

$$u=\frac{12-2v}{3} = \frac{12-6}{3}=\frac{6}{3}$$
$$\Rightarrow \quad u = 2$$

Now $u = 2$

$$\Rightarrow \quad \frac{1}{x} = 2$$
$$\Rightarrow \quad x = \frac{1}{2}$$

and $v = 3$

$$\Rightarrow \quad \frac{1}{y} = 3$$
$$\Rightarrow \quad y = \frac{1}{3}$$

Hence, $x=\frac{1}{2}$, $y=\frac{1}{3}$ is the solution of the given equations.

(*ii*) Let $\frac{1}{\sqrt{x}}=u$ and $\frac{1}{\sqrt{y}}=v$

Then the given system of equation becomes

$$2u+3v = 2 \quad ...(i)$$
$$4u-6v = -1 \quad ...(ii)$$

For making the co-efficient of 'u' in (*i*) and (*ii*) equal, we multiply (*i*) by 2 and then subtracting

$$\begin{array}{rcrcl} 4u & + & 6v & = & 4 \\ 4u & - & 9v & = & -1 \\ - & + & & & + \\ \hline & & 15v & = & 5 \end{array}$$

$$\Rightarrow \quad v = \frac{5}{15}=\frac{1}{3}=v=\frac{1}{3}$$

Putting the value of 'v' in (*i*), we get

$$2u+3v = 2$$
$$\Rightarrow \quad 2u+3\frac{1}{2} = 2$$
$$\Rightarrow \quad 2u = 2-1$$
$$\Rightarrow \quad 2u = 1$$
$$\Rightarrow \quad u = \frac{1}{2}$$

Now $u = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2}$

$\Rightarrow \sqrt{x} = 2$

$\Rightarrow x = 4$ (squaring both side)

and $v = \frac{1}{3}$

$\Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3}$

$\Rightarrow \sqrt{y} = 3$ (by squaring both side)

$\Rightarrow y = 9$

Hence, $x = 4$, $y = 9$ is the solution of the given equation.

(*iii*) Let $\frac{1}{x} = a$

Thus, the given equation becomes

$4a + 3y = 14$ ...(*i*)

$3a - 3y = 23$ ...(ii)

For making the coefficient of '*y*' in (*i*) and (*ii*), we multiply (*i*) by 4 and (*ii*) by 3 and then adding, we get

$$\begin{aligned} 16a + 12y &= 56 \\ 9a - 12y &= 69 \\ \hline 25a &= 125 \end{aligned}$$

$\Rightarrow a = \frac{125}{25}$

$\Rightarrow a = 5$

Putting the value of '*a*' in (i), we get

$4a + 3y = 14$

Þ $4(5) + 3y = 14$

Þ $20 + 3y = 14$

Þ $3y = -6$

Þ $y = -2$

Now $a = 5$

Þ $\frac{1}{x} = 5$

Þ $x = \frac{1}{5}$

Hence, $x = \frac{1}{5}$ and $y = -2$ is the solution of the given equation.

(iv) Let $\frac{1}{x-1} = u$ and $\frac{1}{y-2} = v$

Thus, the given equaion becomes

$5u + v = 2$ ...(i)

$6u - 3v = 1$ (iii)

For making the coefficient of '*v*' in (*i*) and (*ii*) equal, we multiply (*i*) by 3 and adding, we get

$$\begin{aligned} 15u + 3v &= 6 \\ 6u - 3v &= 1 \\ \hline 21u &= 7 \end{aligned}$$

$\Rightarrow u = \frac{7}{21} = \frac{1}{3}$

Putting the value of *u* in (*i*), we get

$5u + v = 2$

$\Rightarrow 5\left(\frac{1}{3}\right) + v = 2$

$\Rightarrow \frac{5}{3} + v = 2$

Now,

$u = \frac{1}{3}$

$\Rightarrow \frac{1}{x-1} =$

$\Rightarrow x - 1 = 3$

$\Rightarrow x = 4$

and $v = \frac{1}{3}$

$\Rightarrow \frac{1}{y-2} = \frac{1}{3}$

$\Rightarrow y - 2 = 3$

$\Rightarrow y = 5$

Hence, $x = 4$, $y = 5$.

(*v*) Considering equation

$\frac{7x - 2y}{xy} = 5$

$\Rightarrow 7x - 2y = 5xy$

Dividing bot sides by xy, we get

$\frac{7x}{xy} - \frac{2y}{xy} = \frac{5xy}{xy}$

$\Rightarrow \frac{7}{y} - \frac{2}{x} = 5$

Considering equation

$\frac{8x + 7y}{xy} = 15$

$$\Rightarrow \quad 8x+7y = 15xy$$

Dividing both sides by xy, we get

$$= \frac{15xy}{xy}$$

$$\Rightarrow \quad \frac{8}{y}+\frac{7}{x} = 15 \qquad ...(ii)$$

Let $\frac{1}{x} = u$, $\frac{1}{y} = v$. Then the given system of equation bcomes.

$$7v-2u = 5 \qquad ...(iii)$$
$$8v+7u = 15 \qquad ...(iv)$$

For making the coefficient of 'u' in (iii) and (iv) equal, we multiply (iii) by '7' and (iv) by '2' and then adding

$$49v-14u = 35$$
$$16v+14u = 30$$
$$65v+0 = 65$$
$$\Rightarrow \quad v = 1$$

Putting the value of 'v' in (iii), we get

$$7v-2u = 5$$
$$\Rightarrow \quad 7(1)-2u = 5$$
$$\Rightarrow \quad -2u = -2$$
$$\Rightarrow \quad u = 1$$

$\frac{8x}{xy}+\frac{7y}{xy}$

Now, $\quad u = 1$

$$\Rightarrow \quad \frac{1}{x} = 1$$
$$\Rightarrow \quad x = 1$$

and $\quad v = 1$

$$\Rightarrow \quad \frac{1}{y} = 1$$
$$\Rightarrow \quad y = 1$$

Hence, $x = 1$, $y = 1$ is the solution of the given equation.

(vi) Considering equation

$$6x-5y = 6xy$$

Dividing both sides by xy, we get

$$\frac{6x}{xy} + \frac{3y}{xy} = \frac{6xy}{xy}$$

$$\Rightarrow \quad \frac{6}{y}+\frac{3}{x} = 6 \qquad ...(i)$$

Considering equation

$$2x+4y = 5xy$$

Dividing both sides by xy, we get

$$\frac{8x}{xy}+\frac{7y}{xy} = \frac{15xy}{xy}$$

$$\Rightarrow \quad \frac{8}{y}+\frac{7}{x} = 15 \qquad ...(ii)$$

Let $\frac{1}{x} = u$, $\frac{1}{y} = v$. Then the given system of equation bcomes.

$$6v+3u = 6 \qquad ...(iii)$$
$$2v+4u = 5 \qquad ...(iv)$$

For making coefficient of u in (iii) and (iv) equal, we multiply (iii) by 4 and (iv) by 3 and by subtracting, we get

$$\begin{aligned} 24v + 12u &= 24 \\ 6v + 12u &= 15 \\ - \quad - \quad & \quad - \\ 18v \quad &= 9 \end{aligned}$$

$$\Rightarrow \quad v = \frac{9}{18} = \frac{1}{2}$$

Putting the value of v in (iii), we get

$$6v+3u = 6$$
$$\Rightarrow \quad 6\left(\frac{1}{2}\right)+3u = 6$$
$$\Rightarrow \quad 3u = 6$$
$$\Rightarrow \quad u = 1$$

Now, $u = 1$	and $v = \frac{1}{2}$
$\Rightarrow \frac{1}{x} = 1$	$\Rightarrow \frac{1}{y} = \frac{1}{2}$
$\Rightarrow x = 1$	$\Rightarrow y = 2$

Hence, $x=1, y=2$

(vii) Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$

Then the given system of equation becomes

$$10u+2v = 4 \qquad ...(i)$$
$$15u-5v = -2 \qquad ...(ii)$$

For making the coefficient of 'v' in (i) and (ii), equal we multiply (i) by 5 and (ii) by 2 and then adding, we get

$$50u+10v = 20$$
$$30u-10v = -4$$
$$80u = 16$$

$$\Rightarrow \quad u = \frac{16}{80} = \frac{1}{5}$$

Putting the value of u in (i), we get

$$10v+2v = 4$$

$$\Rightarrow \quad 10\left(\frac{1}{5}\right)+2v = 4$$
$$\Rightarrow \quad 2+2v = 4$$
$$\Rightarrow \quad 2v = 2$$
$$\Rightarrow \quad v = 1$$

Now, $u = \frac{1}{5}$ and $v = 1$

$$\Rightarrow \quad \frac{1}{x+y} = \frac{1}{5} \qquad \Rightarrow \quad \frac{1}{x-y} = 1$$
$$\Rightarrow \quad x+y = 5 \qquad \Rightarrow \quad x-y = 1 \qquad ..(iv)$$

Thus, we have
$$x+y = 5$$
$$\Rightarrow \quad x = 5-y \qquad ...(v)$$

Substituting the value of x in (*iv*), we get
$$x-y = 1$$
$$\Rightarrow \quad 5-y-y = 1$$
$$\Rightarrow \quad 5-2y = 1$$
$$\Rightarrow \quad -2y = 1-5 \Rightarrow -2y = -4$$
$$\Rightarrow \quad y = 2$$

Now, substituting the value of y in (v), we get
$$x = 5-y$$
$$= 5-2 = 3$$

Hence, $x=3, y=2$

(*viii*) Let $\frac{1}{(3x+y)} = u$ and $\frac{1}{(3x-y)} = v$

Then the given system of equations become
$$u+v = \frac{3}{4}$$
and
$$\frac{u}{2}-\frac{v}{2} = \frac{-1}{8}$$
$$\Rightarrow \quad u-v = \frac{-1}{4}$$

Thus, we have following equations
$$u+v = \frac{3}{4} \qquad ...(i)$$
$$u-v = \frac{-1}{4} \qquad ...(ii)$$

Adding equations (i) and (ii), we get
$$2u = \frac{3}{4}+\left(\frac{-1}{4}\right)$$
$$\Rightarrow \quad 2u = \frac{3-1}{4} = \frac{2}{4}$$
$$\Rightarrow \quad u = \frac{1}{4}$$

Putting the value of 'u' in eqn. (i), we get
$$u+v = \frac{3}{4}$$
$$\Rightarrow \quad \frac{1}{4}+v = \frac{3}{4}$$
$$\Rightarrow \quad v = \frac{3}{4}-\frac{1}{4}$$
$$\Rightarrow \quad v = \frac{2}{4}$$
$$\Rightarrow \quad v = \frac{1}{2}$$

Now, $u = \frac{1}{4}$
$$\Rightarrow \quad \frac{1}{3x+y} = \frac{1}{4}$$
$$\Rightarrow \quad 3x+y = 4 \qquad ...(iii)$$

and $v = \frac{1}{2}$
$$\Rightarrow \quad \frac{1}{3x+y} = \frac{1}{2}$$
$$\Rightarrow \quad 3x-y = 2 \qquad ...(iv)$$

Adding (*iii*) and (*iv*), we get
$$6x = 6$$
$$\Rightarrow \quad x = 1$$

Putting the value of x in (iii), we get
$$3x+y = 4$$
$$\Rightarrow \quad 3(1)+y = 4$$
$$\Rightarrow \quad y = 1$$

Hence, $x = 1$, $y = 1$ is the solution of the given equation.

Q.2. Formulate the following problems as a pair of linear equations and hence find their solutions :

(*i*) Ritu can row downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(*ii*) 2 women and 5 men can together finish a piece of embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work and also that taken by 1 man alone.

(*iii*) Rohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Ans. Let the speed of the boat in still water $= x$ km/hr

and speed of the current $= y$ km/hr

$= (x-y)$ km/hr

now, the speed of the boat downstream $= (x+y)$ km/hr

Case I. Time taken to cover 20 km downstream

$$= \frac{20}{x+y}$$

$$\Rightarrow \quad 2 = \frac{20}{x+y}$$

$$\Rightarrow \quad x+y = 10 \quad ...(i)$$

Case II. Time taken to cover 4 km upstream

$$= \frac{4}{x-y}$$

$$\Rightarrow \quad 2 = \frac{4}{x-y}$$

$$\Rightarrow \quad x-y = 2 \quad ...(ii)$$

From (*i*), we have

$$x = 10-y$$

Putting the value of '*x*' in (*ii*), we get

$$10-y-y = 2$$

$$\Rightarrow \quad 10-2y = 2$$

$$\Rightarrow \quad -2y = -8$$

$$\Rightarrow \quad y = 4$$

Now, substituting the value of y in (i), we get

$$x+y = 10$$

$$\Rightarrow \quad x+y = 10$$

$$\Rightarrow \quad x = 6$$

Hence, speed of rowing her boat in still water $= 6$ km/hr

and speed of the current $=$ 4km/hr.

(*ii*) Let 1 woman can finish embroidery work in x days

and 1 man can finish embroidery work in y days

$\therefore$ 1 woman's 1 day's work $= \frac{1}{x}$

and 1 man's 1 day's work $= \frac{1}{y}$

Case I. 2 women's 1 day's work $= \frac{2}{x}$

and 5 men's 1 day's work $= \frac{5}{y}$

According to question $\frac{3}{x}+\frac{6}{y} = \frac{1}{3}$

Taking $\frac{1}{x} = u$, $\frac{1}{y} = v$, we have

$$2u+5v = \frac{1}{4}$$

$$3u+6v = \frac{1}{3}$$

For making the coefficient of '*u*' we multiply equation (*i*) by 3 and equation (*ii*) by '2' and then subracting, we get

$$6u + 15v = \frac{3}{4}$$

$$6u + 12v = \frac{2}{3}$$

$$- \quad - \quad\quad -$$

$$3v = \frac{3}{4} - \frac{2}{3}$$

$$\Rightarrow \quad 3v = \frac{9-8}{12}$$

$$\Rightarrow \quad 3v = \frac{1}{12}$$

$$\Rightarrow \quad v = \frac{1}{36}$$

Putting the value of '*v*'in (*i*), we get

$$2u+5v = \frac{1}{4}$$

$$\Rightarrow \quad 2u+\frac{5}{36} = \frac{1}{4}$$

$$\Rightarrow \quad 2u = \frac{1}{4}-\frac{5}{36}$$

$$\Rightarrow \quad 2u = \frac{9-5}{30}$$

$$\Rightarrow \quad 2u = \frac{4}{36}$$

$$\Rightarrow \quad 2u = \frac{1}{9}$$

$$\Rightarrow \quad u = \frac{1}{18}$$

Now, $\quad u = \frac{1}{18}$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{x}$$

$$\Rightarrow \quad x = 18$$

and $$v = \frac{1}{36}$$

$$\Rightarrow \quad \frac{1}{y} = \frac{1}{36}$$

$$\Rightarrow \quad y = 36$$

Hence, I woman alone can finish the work in 18 days and 1 man alone can finish the work in 36 days.

(iii) Let the speed of the train and the bus be x km/hour and y km/hour respectively.

Case I. When she travels 100 km by train and the remaining (300 – 100)km, i.e., 200 km by bus, the time taken is 4 hours 10 minutes, *i.e*, $\frac{25}{6}$ hours.

$$\therefore \quad \frac{60}{x} + \frac{240}{y} = 4 \qquad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\Rightarrow \quad \frac{1}{x} + \frac{4}{y} = \frac{1}{15} \qquad \text{...}(i) \text{ [Dividing by 60]}$$

Case II. When she travels 100 km by train and the remaining (300 – 100) km, i.e., 200 km by bus, the time taken is 4 hours 10 minutes, i.e., $\frac{25}{6}$ hours.

$$\therefore \quad \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$$

$$\Rightarrow \quad \frac{4}{x} + \frac{8}{y} = \frac{1}{6} \qquad \text{...}(ii)$$

[Dividing by 25]

Multiplying equation (*i*) by 2, we get

$$\frac{2}{x} + \frac{8}{y} = \frac{2}{15} \qquad \text{...}(iii)$$

Subtracting equation (*iii*) from equation (*ii*), we get

$$x = \frac{1}{6} - \frac{2}{15} = \frac{1}{30}$$

$$\Rightarrow \quad x = 60$$

Substituting this value of *x* in equation (iii), we get

$$\frac{2}{60} + \frac{8}{y} = \frac{2}{15}$$

$$\Rightarrow \quad \frac{1}{30} - \frac{8}{y} = \frac{2}{15}$$

$$\Rightarrow \quad \frac{8}{y} = \frac{2}{15} - \frac{1}{30} = \frac{1}{10}$$

$$\Rightarrow \quad y = 80$$

So, the soltion of the equations (i) and (ii) is x = 60 and y = 80.

Hence, the speed of the train is 60 km/hour and the speed of the bus is 80 km/hour.

EXERCISE 3.7 (Optional)

Q.1. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Ans. Let the ages of Ani and Biju be *x* years and *y* years respectively. Then, according to the question,

$$x - y = \pm 3 \qquad \text{...(i)}$$

Age of Ani's father Dharam = 2*x* years

Age of Biju's sister = $\frac{y}{2}$ years.

According to the question,

$$2x - \frac{y}{2} = 30$$

$$\Rightarrow \quad 4x - y = 60 \qquad \text{...(ii)}$$

Case I. When $x - y = 3$ ...(iii)

On substracting Eq. (iii) from Eq. (i), we get

$$3x = 57 \quad \Rightarrow x = 19 \text{ yr}$$

On putting $x = 19$ in Eq. (iii), we get

$$19 - y = 3 \quad \Rightarrow y = 16 \text{ yrs}$$

$$\therefore \quad x = 19 \text{ yrs and } y = 16 \text{ yrs}$$

Case II. When $x - y = -3$ ...(iv)

On subtracting Eq. (iv) from Eq. (ii), we get

$$\therefore \quad 3x = 60 + 3$$

$$\Rightarrow \quad 3x = 63$$

$$\Rightarrow \quad x = 21$$

On putting $x = 21$ in Eq. (iv), we get

$$21 - y = -3 \Rightarrow y = 24 \text{ yr.}$$

Hence, age of Ani is 19 yr and ageof Biju is 16 yr or age of Ani is 21 yr and age of Biju is 24 yr.

Q.2. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten. I shall be six as rich as you." Tell me what is the amount of their (respective) capital?

Ans. Let the amounts of thier respective capitals be Rs. *x* and Rs. *y* respectively.

Then, according to the question,

$$x + 100 = 2(y - 100)$$

$$\Rightarrow \quad x - 2y = -300 \qquad \text{...}(ii)$$

$$\text{and} \quad 6(x - 10) = y + 10$$

$$\Rightarrow \quad 6x - y = 70 \qquad \text{...}(ii)$$

From eqution (*i*), we have

$$x = 2y - 300 \qquad \text{...}(iii)$$

Substitute the value of x in equation (*ii*), we get

$$6(2y-300)-y = 70$$
$$\Rightarrow \quad 12y-1800-y = 70$$
$$\Rightarrow \quad 11y = 1870$$
$$\Rightarrow \quad y = \frac{1870}{11} = 170$$

Substituting the value of y in equation (*iii*), we get

$$x = 2(170)-300$$
$$x = 340-300 = 40$$

So, the solution of the equtions (*i*) and (*ii*) is $x = 40$ and $y = 170$. Hence, the amounts of their respective capitals are Rs. 40 and Rs. 170 respectively.

Q.3. A train covered a certain distance at a uniform speed. If the train would have been 10km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Ans. Let the actual speed of the train be x km/hr and the actual time taken be y hours. Then,

$$\text{Distance covered} = (xy)\text{ km} \qquad ...(i)$$

Case I. If the speed is increased by 10 km/hr then time of journey is reduced by 2 hrs. *i.e.* when speed is $(x+10)$km/hr, time of journey is $(y-2)$ hrs.

$$\therefore \quad \text{Distance covered} = (x+10)(y-2)$$
$$\Rightarrow \quad xy = xy-2x+10y-20 \quad [\text{Using } (i)]$$
$$\Rightarrow \quad 2x-10y = -20$$

Case II. When the speed is reduced by 10 km/hr then time of journey is by 3 hrs. i.e. when speed is $(x-10)$ km/hr, time of journey is $(y+3)$ hrs.

$$\therefore \quad \text{Distance covered} = (x-10)(y+3)$$
$$xy = (x-10)(y+3) \quad [\text{Using } (i)]$$
$$\Rightarrow \quad xy = xy+3x-10y-30$$
$$\Rightarrow \quad -3x+10y = -30$$

Thus, we have following eqn.

$$2x-10y = -20 \qquad ...(i)$$
$$-3x+10y = -30 \qquad ...(ii)$$

Since the coefficient of y in both the equations are same, so we can eliminate it directly by adding i.e.

$$\begin{aligned} 2x-10y &= -20 \\ -3x+10y &= -30 \\ \hline -x &= -50 \\ x &= 50 \end{aligned}$$

Putting the value of x in (i), we get

$$2x-10y = -20$$
$$\Rightarrow \quad 2(50)-10y = -20$$
$$\Rightarrow \quad 100-10y = -20$$
$$\Rightarrow \quad -10y = -120$$
$$\Rightarrow \quad y = 12$$

Hence, the distance covered by the train

$$= (xy)\,km$$
$$= (50\times 12)\,km$$
$$= 600\,km.$$

Q.4. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in row, there would be 2 rows more. Find the number of students in the class.

Ans. Let the total no. students in each row be x and the total no. of rows be y

$\therefore$ Total no. of sutdents in the class $= xy$ ...(*i*)

Case I. If 3 students are extra in a row, there would be 1 row less, i.e.

total no. of students in each row $= (x+3)$

and the total no. of rows $= (y-1)$

$\therefore$ Total no. students in the class

$$= (x+3)(y-1) \qquad ...(ii)$$

Comparing (*i*) and (*ii*), we get

$$xy = (x+3)(y-1)$$
$$\Rightarrow \quad xy = xy-x+3y-3$$
$$\Rightarrow \quad x-3y = -3$$

Case II. If 3 students are less in a row, there would be 2 rows more, i.e.

Total no. of students in each row $= (x-3)$

and the total no. of rows $= (y+2)$

$\therefore$ Total no. of students in the class

$$= (x-3)(y+2) \qquad ...(iii)$$

Comparing (*i*) and (*iii*), we get

$$xy = (x-3)(y+2)$$
$$\Rightarrow \quad xy = xy+2x-3y-6$$
$$\Rightarrow \quad -2x+3y = -6$$

Thus, we have following equations

$$x-3y = -3 \qquad ...(iv)$$
$$-2x+3y = -6 \qquad ...(v)$$

From (*iv*), we have

$$x - 3y = -3$$

$\Rightarrow \quad x = 3y - 3 \quad ...(vi)$

Substituting the value of (vi) in (υ), we get

$$-2x + 3y = -6$$

$\Rightarrow \quad -2(3y-3) + 3y = -6$

$\Rightarrow \quad -6y + 6 + 3y = -6$

$\Rightarrow \quad -3y = -12$

$\Rightarrow \quad y = 4$

Now, substituting the value of 'y' in (vi), we get

$$x = 3y - 3$$
$$= 3(4) - 3 = 12 - 3 = 9$$

Hence, total no. of students in the class

$$= xy$$
$$= 9 \times 4 = 36.$$

Q.5. In a ΔABC, ∠C = 3 ∠B = 2 (∠A + ∠B). Find the three angles.

Ans. Let $\angle A = x^o$, $\angle B = y^o$. Then,

$$\angle C = 3\angle B \Rightarrow C = 3y^o$$

We have, $\angle C = \angle 3\,B = 2\,(\angle A + \angle B)$. ...($i$)

Now, 0 $\quad 3\angle B = 2(\angle A + \angle B)$ From (i)

$\Rightarrow \quad 3y = 2(x + y)$

$\Rightarrow \quad y = 2x$

Since A, B and C are angles of a triangle

$\therefore \quad \angle A + \angle B + \angle C = 180^o$

$\Rightarrow \quad x + y + 3y = 180$

$\Rightarrow \quad x + 4y = 180$

Putting $y = 2x$ in equation (ii), we get

$$x + 4(2x) = 180$$

$\Rightarrow \quad x + 8x = 180$

$\Rightarrow \quad 9x = 180$

$\Rightarrow \quad x = 20^o$

Putting the value of x equation (i), we get in $y = 40^o$

Hence, $\quad \angle A = 20^o, \angle B = 40^o$

and $\quad \angle C = 3y^o = (3 \times 40^o) = 120^o$

Q.6. Draw the graph of the equtions $5x - y = 5$ and $3x - y = 3$. Determine the coordiantes of the vertices of the triangle formed by these lines and the y axis.

Ans. The given equations are

$5x - y = 5 \quad ...(i)$

$3x - y = 3 \quad ...(ii)$

For equation (i), we have

$$y = 5x - 5$$

Thus, we have following table :

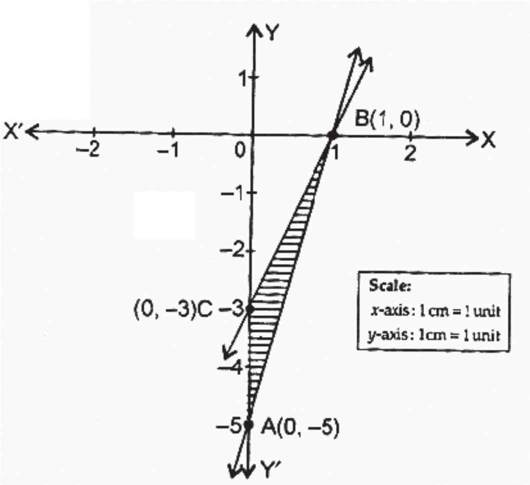
x	0	1
y	–5	0

For equation (ii), we have

$$y = 3x - 3$$

Thus, we have following table :

x	0	1
y	–3	0



Q.7. Solve the following pair of linear equations:

(i) $px + qy = p - q$
$qx - py = p + q$

(ii) $ax + by = c$
$bx + ay = 1 + c$

(iii) $\dfrac{x}{a} - \dfrac{y}{b} = 0$
$ax + by = a^2 + b^2$

(iv) $(a - b)x + (a + b)\,y = a^2 - 2ab - b^2$
$(a + b)\,(x + y) = a^2 + b^2$

(v) $152x - 378y = -74$
$-378x + 152y = -604$

Ans. We have

$px - qy = p - q \quad ...(i)$

$qx - py = p + q \quad ...(ii)$

From (i), we get

$$x = \frac{p - q - qy}{p} \quad ...(iii)$$

Putting the value of 'x' in (ii), we get

$\Rightarrow \quad q\left(\dfrac{p - q - qy}{p}\right) - \dfrac{py}{1} = p + q$

$\Rightarrow \quad \dfrac{q(p - q - qy) - p^2 y}{p} = p + q$

$\Rightarrow \quad qp - q^2 - q^2 y - p^2 y = p(p + q)$

$\Rightarrow \quad -q^2y - p^2y = p(p + q) - dp + q^2$

$\Rightarrow \quad -y(q^2 + y^2) = p^2 + pq - dp + q^2$

$$\Rightarrow \quad -y = \frac{p^2+q^2}{p_2+q^2} = 1$$

$\Rightarrow \quad y = -1$

Putting thte value of 'y' in (iii), we get

$$x = \frac{p-q+q(-1)}{p}$$

$$\Rightarrow \quad x = \frac{p-q+q}{p} = \frac{p}{p}$$

$\Rightarrow \quad x = 1$

Hence, the solution is

$x = 1, y = -1$

(ii) We have,

$ax + by = c \quad ...(i)$

$bx + ay = 1 + c \quad ...(ii)$

From (i), we have

$$x = \frac{c-by}{a} \quad ...(iii)$$

Putting the value of (iii) in (ii), we get

$$b\left(\frac{c-by}{a}\right) + ay = (1+c)$$

$$\Rightarrow \quad \frac{bc-b^2y}{a} + ay = (1+c)$$

$$\Rightarrow \quad \frac{bc-b^2y+a^2y}{a} = (1+c)$$

$bc - b^2y + a^2y = a(1+c)$

$\Rightarrow \quad -b^2y + a^2y = a(1 + c) - bc$

$\Rightarrow \quad y(-b^2 + a^2) = a + ac - bc$

$$\Rightarrow \quad y = \frac{a+ac-bc}{a^2-b^2}$$

Putting the value of 'y' in (iii), we get

$$x = \frac{\frac{c}{1} - b\left(\frac{a+ac-bc}{a^2-b^2}\right)}{a}$$

$$= \frac{\frac{c(a^2-b^2)-b(a+ac-bc)}{a^2-b^2}}{\frac{a}{1}}$$

$$= \frac{a^2c-b^2c-ab-abc+b^2c}{a^2-b^2} \times \frac{1}{a}$$

$$= \frac{a^2c-ab-abc}{a(a^2-b^2)}$$

$$= \frac{a(ac-b-bc)}{a^2-b^2}$$

$$\Rightarrow \quad x = \frac{ac-b-bc}{a^2-b^2}$$

Hence, the solution is

$$x = \frac{ac-b-bc}{a^2-b^2}$$

and $$y = \frac{a+ac-bc}{a^2-b^2}$$

(iii) $\frac{x}{a} - \frac{y}{b} = 0 \quad ...(i)$

$ax + by = a^2 + b^2 \quad ...(ii)$

The given pair of their equations is

$$\frac{y}{b} = \frac{x}{a}$$

$$\Rightarrow \quad y = \frac{b}{a}x \quad ...(iii)$$

Substituting the value of y in equation (ii), we get

$$ax + b\left(\frac{b}{a}x\right) = a^2 + b^2$$

$$ax + \frac{b^2}{a}x = a^2 + b^2$$

$a^2x + b^2x = a(a^2 + b^2)$

$x(a^2 + b^2) = a(a^2 + b^2)$

$$x = \frac{a(a^2+b^2)}{a^2+b^2} = a$$

Substituting the value of x in (iii), we get

$$y = \frac{b}{a}x$$

$$y = \frac{b}{a} \times a - b$$

Hence, $x = a, y = b$.

(iv) The given system of equations may be written as

$(a - b)x + (a + b)y = a^2 - 2ab - b^2$

$(a + b)x + (a + b)y = a^2 + b^2$

$(a - b)x - (a + b)x = -2ab - 2b^2$

$\Rightarrow \quad x[(a - b) - (a + b)] = -2ab - 2b^2$

$\Rightarrow \quad x[(a - b - a - b)] = -2ab - 2b^2$

$\Rightarrow \quad x(-2b) = -2ab - 2b^2$

$\Rightarrow \quad x = a + b$

Putting the value of x in (ii), we get

$(a + b)x + (a + b)y = a^2 + b^2$

$\Rightarrow \quad (a + b)(a + b) + (a + b)y = a^2 + b^2$

$\Rightarrow \quad (a + b)y = (a^2 + b^2) - (a + b)^2$

$\Rightarrow \quad (a + b)y = (a^2 + b^2) - (a^2 + b^2 + 2ab)$

$\Rightarrow \quad (a + b)y = a^2 + b^2 - (a^2 + b^2 + 2ab)$

$$\Rightarrow \quad (a+b)y = -2ab$$

$$\Rightarrow \quad y = \frac{-2ab}{a+b}$$

Hence,

$$x = a\,b \text{ and } y = \frac{-2ab}{a+b}$$

(*v*) $152x - 378y = -74$
$-378x + 152y = -604$

The given pair of linear equations is

$$152x - 378y = -74 \quad ...(i)$$
$$-378x + 152y = -604 \quad ...(ii)$$

Adding equation (*i*) and equation (*ii*), we get

$$-226x - 226y = -678$$
$$x + y = 3 \quad ...(iii)$$

[Dividing throughout by –226]

Subtracting equation (*ii*) from equation (*i*), we get

$$530x - 530y = 530$$
$$\Rightarrow \quad x - y = 1$$

[Dividing throughout by –226]

Adding equation (iii) and equation (iv), we get

$$2x = 4$$

$$\Rightarrow \quad x = \frac{4}{2} = 2$$

Subtracting equation (*iv*) from equation (*iii*), we get

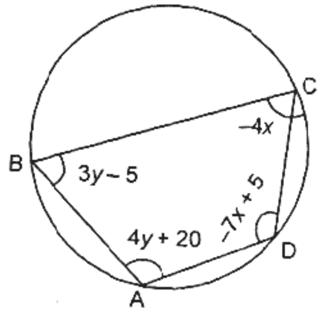
$$2y = 2$$

$$\Rightarrow \quad y = \frac{2}{2} = 1$$

Hence, the solution of the given pair of linear equations is $x = 2, y = 1$.

Q.8. ABCD is cyclic quadrilateral (See Fig.). Find the angles of the cyclic quadrilateral.

Ans. We know that the opposite angles of a cyclic quadrilateral are supplementary, therefore,



$$\Rightarrow \quad 4y + 20 + 4x = 180^o$$
$$\Rightarrow \quad 4x + 4y = 160^o$$
$$\Rightarrow \quad x + y = 40^o \quad ...(i)$$

[Dividing throughout y 4]

and $\angle A + \angle C = 180^o$

$$\Rightarrow \quad 3y - 5 + 7x + 5 = 180^o$$
$$\Rightarrow \quad 7x + 3y = 180^o \quad ...(ii)$$

From equation (*i*), we have

$$y = 40 - x \quad ...(iii)$$

Substituting this value of y in equation (ii), we get

$$7x + 3(40 - x) = 180^o$$
$$\Rightarrow \quad 7x + 120 - 3x = 180^o$$
$$\Rightarrow \quad 4x = 60$$
$$\Rightarrow \quad x = \frac{60}{4} = 15^o$$

Substituting $x = 15$ in the equation (*iii*), we get

$$y = 40 - x$$
$$= 40 - 15 = 20^o$$

Hence, required angles be

$$\angle A = 4y + 20 = 4 \times 25 + 20 = 120^o$$
$$\angle B = 3y - 5 = 3 \times 25 - 5 = 75 - 5 = 70^o$$
$$\angle C = 4x = 4 \times 15 = 60^o$$
$$\angle D = 7x + 5 = 7 \times 15 + 5$$
$$= 105 + 5 = 110^o$$

Additional Questions

Q.1. For all real values of *c*, the pair of equations

$$\mathbf{x - 2y = 8}$$
$$\mathbf{5x - 10y = c}$$

have a unique solution. Justify whether it is true or false.

Ans. Given equations are

$$x - 2y = 8$$

and $\quad 5x - 10y = c$

Here $\quad \frac{a_1}{a_2} = \frac{1}{5}$

$$\frac{b_1}{b_2} = \frac{1}{5}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

which is not the case of unique solution.

$\therefore$ Given statement is false.

Q.2. The line represented by $x = 7$ is parallel to the x-axis. Justify whether the statement is true or not.

Ans. The line represented by $x = 7$ is of the form $x = k$, which is the form of a line parallel to y - axis.

$\therefore$ Given statement is false.

Q.3. Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 60y = 5$. Check whether the paths cross each other or not.

Ans. Two straight paths are represented by :

$$x - 3y = 2 \text{ and } -2x + 6y = 5$$

or, $$x - 3y - 2 = 0 \text{ and } -2x + 5y - 5 = 0$$

The paths will cross each other if the equations intersect.

Now, $$\frac{a_1}{a_2} = \frac{1}{-2} = -\frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}$$

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$\therefore$ The paths will not intersect *i.e.*, paths will not cross each other.

Q.4. Write a pair of linear equations which has the unique solution $x = -1, y = 3$. How many such pairs can you write ?

Ans. One of such pairs can be $x - y = -4$

and $2x + 3y = 7$

We can write infinitely many pairs

Q.5. Write an equation of a line passing through the point representing solution of the pair of linear equations $x + y = 2$ and $2x - y = 1$. How any such lines can we find ?

Ans. Given equations are :

$$x + y = 2 \quad \text{...(i)}$$

and $$2x - y = 1 \quad \text{...(ii)}$$

$\therefore$ $$x + y = 2 \quad \text{(i)} \times 1$$

$$2x - y = 1 \quad \text{(ii)} \times 1$$

Adding $3x = 3$

$\Rightarrow$ $x = 1$

Substituting the value of x in (i), we get $y = 1$.

$\therefore$ $x = 1$ and y - 1 is the solution of the given pair of linear equations.

Now, to write equation of a line we must get relationship between x and y for the point (1, 1) which is $x = y$.

Likewise we can write many such equations by multiplying both sides of the above equations by any non-zero constant.

Q.6. The angles of a triangle are x, y and 40°. The difference between the two angles x and y is 30°. Find x and y.

Ans. The angles of a triangle are x , y and 40°.

therefore, $x + y = 40° = 180°$ by angle sum properly of a Δ.

Hence, we obtain corresponding equation as

$$x + y = 140 \quad \text{...(i)}$$

Also, the difference between the two angles x and y is 30°.

Therefore, we obtain the following equation :

$$x - y = 30 \quad \text{... (ii)}$$

Adding (i) and (ii), we get

$$2x = 170$$

$$\Rightarrow \quad x = \frac{170}{2}$$

$$= 85$$

Subtracting (ii) from (i), we get

$$2y = 110$$

$$\Rightarrow \quad y = 55$$

$\therefore$ $x = 85°$ and $y = 55°$.

Q.7. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Ans. Let the age of the father is x years and the sum of the ages of his two children is y years.

$\therefore$ According to the questions :

$$x = 2y \quad \text{... (i)}$$

After 20 years, the age of the father will be $x + 20$ and the sum of ages of his two children will be $y = 40$.

$\therefore$ According to the questions :

$$x + 20 = y + 40$$

$$\Rightarrow \quad x - y = 20 \quad \text{... (ii)}$$

Substituting the value of x from (i) and (ii), we get

$$2y - y = 20$$

$$\Rightarrow \quad y = 20$$

Substituting the value of y in (i), we get

$$x = 2 \times 20$$

$$= 40$$

$\therefore$ The age of the father is 40 years.

Q.8. There are some students in the two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A and B. But if 20 students are sent from B to A, the

number of students in A become double the number of students in B. Find the number of students in the two halls.

Ans. Let the number of students in hall A be x and the number of students in hall B be y.

$\therefore$ According to the question,

$$x - 10 = y + 10$$

$\Rightarrow$ $x = 20 + y$ ...(i)

and $2(y - 20) = x + 20$

$\Rightarrow$ $2y - 60 = x$ ...(ii)

Equating the two values of x, we get

$$20 + y = 2y - 60$$

$\Rightarrow$ $y = 80$

Substituting the value of y in (i), we get

$$x = 100$$

$\therefore$ Number of students in hall A is 100 and the number of students in hall B is 80.

Q.9. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid Rs. 22 for a book kept for six days, while Anand paid Rs. 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.

Ans. Let the fixed charge be Rs. x for two days.

Let the additional charge for each day thereafter be Rs. y per day.

Now, Latika paid Rs., 22 for a book kept for six day.

$\therefore$ $x + 4y = 22$ ... (i)

and Anand paid Rs. 16 for a book kept for four days.

$\therefore$ $x + 2y = 16$ ...(ii)

Subtracting (ii) from (i), we get

$2y = 6$

$\Rightarrow$ $y = 3$

Substituting the value of y in (i), we get

$x + 4 \times 3 = 22$

$\Rightarrow$ $x = 10$

$\therefore$ Fixed charges = Rs. 10

and charge per day after two days = Rs. 3.

Q.10. For which values of *a* and *b*, will the following pair of linear equations have infinitely many solutions ?

$x + 2y = 1$

$(a - b)\, x + (a + b)\, y = a + b - 2$

Ans. Given equations are $x + 2y = 1$

or $x + 2y - 1 = 0$ ....(i)

and $(a - b)\, x + (a + b)y = a + b - 2$

or $(a - b)x + (a + b)y - a - b + 2 = 0$ ...(ii)

$\therefore$ $a_1 = 1, b_1 = 2, c_1 = -1$

$a_2 = a - b_2 = a + b,\ c_2 = -a - b + 2$

Now, for infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_2}{c_2}$$

$$\Rightarrow \quad \frac{1}{a-b} = \frac{2}{a+b} = \frac{-1}{-a-b+2}$$

$\Rightarrow$ $a + b = 2a - 2b;\ -2a - 2b + 4 = -a - b;$

$-a - b + 2 = -a + b$

$\Rightarrow$ $a - 3b = 0;\ -a - b + 4 = 0;\ -2b + 2 = 0$

$\Rightarrow$ $a - 3b = 0;\ a + b = 4;\ b = 1$

$\Rightarrow$ $a = 3;\ b = 1$

$\therefore$ For infinitely many solutions $a = 3$ and $b = 1$.

Multiple Choice Questions

Q.1. Two lines is given to be parallel. The equation of one of the lines is $4x + 3y = 14$. The eqution of the second line can be

(a) $3x + 4y = 14$ (b) $8x + 6y = 28$

(c) $12x + 9y = 42$ (d) $-12x = 9y$

Ans. (d)

Q.2. If $x = a, y = b$ is the solution of th pair of equation $x - b = 2$ and $x + y = 4$, then the respective values of a and b are :

(a) 3, 5 (b) 5, 3

(c) 3, 1 (d) $-1, -3$

Ans. (c)

Q.3. The pair of equations :

$$3x + 4y = 18$$

$$4x + \frac{16}{3}y = 25$$

(a) No solution

(b) Unique solution

(c) Infinitely many solution

(d) Can not say anything.

Ans. (c)

Q.4. If $31x + 43y = 117$ and $43x + 31y = 105$, then the value of $(x + y)$ is :

(a) -3 (b) $\frac{1}{3}$

(b) $-\frac{1}{3}$ (d) 3

Ans. (d).

Q.5. If $19x - 17y = 55$ and $17x - 19y = 53$, then the value of $(x - y)$ is :

(a) -3 (b) $\frac{1}{3}$

(b) 3 (d) 5

Ans. (d)

Q.6. If $(6, k)$ is a solution of the equation $3x + y - 22 = 0$, then the value of k is :

(a) –4 (b) 4

(c) 3 (d0 –3

Ans. (b)

Q.7. Rs. 4,900 were divided among 150 children. If each girl get Rs. 50 and a boy get Rs. 25, then the number of boys is :

(a) 100 (b) 102

(c) 104 (d) 105

Ans. (c)

Q.8. If the pair of equations $2x + 3y = 5$ and $5x + \frac{15}{2}y = k$ represent two coincident lines, then the value of k is :

(a) $\frac{-25}{2}$ (b) – 5

(c) $\frac{25}{2}$ (d) $\frac{-5}{2}$

Ans. (c)

Q.9. If $3x - 5y = 1$, $\frac{2x}{x-y} = 4$, then the value of $(x + y)$ is

(a) 3 (b) –3

(c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

Ans. (a)

Q.10. If $2x + 3y = 0$ and $4x - 3y = 0$, then $x + y$ equal:

(a) 0 (b) –1

(c) 1 (d) 2

Ans. (a)