15

Chapter

Probability

In the Chapter

In this chapter, you have studied the following points:

- An event for an experiment is the collection of some outcomes of the experiment.
- The empirical (or experimental) probability P(E) of an event E is given by

$$P(E) = \frac{\text{Number of trials in which E has happened}}{\text{Total number of trials}}$$

- The Probability of an event lies between 0 and 1 (0 and 1 inclusive).
- **Some basic definitions**

(i) Trial : A single performance of a random experiment is known as trial.

(ii) Sample space : The set consisting of all possible outcomes of a random experiment is known as sample space.

(iii) Event: A subset of sample space of a random experiment is called an event.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 15.1

Q.1. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

Ans. Total number of balls played by the batswoman = 30, Boundaries hit = 6.

P (Hit a boundary) $\frac{6}{30} = \frac{1}{5}$

P (did not hit a boundary)

$= 1 - \frac{1}{5} = \frac{4}{5}$ (Alternatively)

No. of balls in which she did not hit any boundary

$= 30 - 6 = 24.$

$\therefore$ P(she did not hit a boundary)

$$= \frac{\text{No. of balls in which she did not hit any boundary}}{\text{Total number of balls played}}$$

$= \frac{24}{30} = \frac{4}{5}.$

Q.2. 1500 families with 2 children were selected randomly, and the following data were recorded:

Number of girls in a family	2	1	0
Number of families	475	814	211

Compute the probability of a family, chosen at random, having

(i) 2 girls (ii) 1 girl (iii) No girl

Also check whether the sum of these probabilities is 1.

Ans. As there are 1500 families with 2 children or less, thus the total number of trials = 1500.

(i) There are 475 families having two girls.

Thus, the no. of favourable events -= 475.

$$\text{Hence, P (2 girls)} = \frac{\textit{No. of favourable events}}{\textit{Total No. of trials}}$$

$$= \frac{814}{1500} = \frac{407}{750}$$

(ii) There are 211 families having no girls.

Thus, the no. of favourable events = 211.

Hence probability (0 girls)

$$= \frac{No.of\ \ favourable\ events}{Total\ No.of\ trials}$$

$$= \frac{211}{1500}$$

Check : Sum of the above probabilities

$$= \frac{475}{1500} + \frac{814}{1500} + \frac{211}{1500}$$

$$= \frac{1500}{1500} = 1.$$

Q.3. Refer to Example 5, Section 14.4, Chapter 14. Find the probability that a student of the class was born in August.

Ans. Clearly from the histograph, six students were born in the month of August out of 40 students of a particular section of class IX.

Probability that a student of the class was born in August.

$$= \frac{Number\ of\ \ students\ born\ in\ August}{Total\ number\ of\ \ students}$$

$$= \frac{6}{40}$$

$$= \frac{3}{20}$$

Q.4. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

Ans. Three coins are tossed simultaneously 200 times.

Thus, the total number of trials = 200.

Two heads come up 72 times.

Thus, the number of favourable events = 72.

Hence, P (2 heads comes up)

$$= \frac{No.of\ \ favourable\ events}{Total\ No.of\ events}$$

$$= \frac{72}{200} = \frac{9}{25}$$

Q.5. An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the table below:

Monthly income (in Rs.)	Vehicles per family			
	0	1	2	Above 2
Less than 7000	10	160	25	0
7000 – 10000	0	305	27	2
10000 – 13000	1	535	29	1
13000 – 16000	2	469	59	25
16000 or more	1	579	82	88

Suppose a family is chosen. Find the probability that the family chosen is

(i) earning Rs. 10000 – 13000 per month and owning exactly 2 vehicles.

(ii) earning Rs.16000 or more per month and owning exactly 1 vehicle.

(iii) earning less than Rs.7000 per month and does not own any vehicle.

(iv) earning Rs.13000 – 16000 per month and owning more than 2 vehicles.

(v) owning not more than 1 vehicle.

Ans. Total number of families selected = 2400

(i) Number of families earning Rs. 10000 – 13000 per month and owning exactly 2 vehicles = 29.

$\therefore$ P (the family chosen is earning Rs. 10000 – 13000 per month and owning exactly 2 vehicles)

$$= \frac{29}{2400}$$

(ii) Number of families earning Rs. 16000 or more per month and owning exactly 1 vehicle

= 579

$\therefore$ P (the family chosen is earning Rs. 16000 or more per month and owning exactly 1 vehicle)

$$= \frac{579}{2400} = \frac{193}{800}$$

(iii) Number of families earning less than Rs. 7000 per month and does not own any vehicle

= 10

$\therefore$ P (the family chosen is earning less than Rs. 7000 per month and does nto own any vehicle)

$$= \frac{10}{2400} = \frac{1}{240}$$

(iv) Number of families earning Rs. 13000 – 16000 per month and owning more than 2 vehicles = 25

∴ P (the family chosen is earning Rs. 13000 – 16000 per month and owning more than 2 vehicles)

$$= = \frac{25}{2400} = \frac{1}{96}$$

(v) Number of families owning not more than 1 vehicle.

= Number of families owning 0 vehicle + Number of families owning 1 vehicle

= (10 + 0 + 1 + 2 + 1)

+ (160 + 305 + 535 + 469 + 579)

= 14 + 2048 + 2062

∴ Probability that the family chosen owns not more than 1 vehicle = $\frac{2062}{2400} = \frac{1031}{1200}$

Q.6. Refer to Table 14.7, Chapter 14.

Marks	Number of students
0 - 20	7
20 - 30	10
30 - 40	10
40 - 50	20
50 - 60	20
60 - 70	15
70 - above	8
Total	**90**

(i) Find the probability that a student obtained less than 20% in the mathematics test.

(ii) Find the probability that a student obtained marks 60 or above.

Ans. Total number of students in mathematics is 90.

(i) Clearly, from the given table, the number of students who obtained less than 20% marks in the mathematics test = 7.

P (a student obtaining less than 20% marks)

$$= \frac{7}{90}$$

(ii) Clearly, from the given table, number of students who obtained marks 60 or above.

= (students in 60 – 70) + (students above 70)

= 15 + 8 = 23.

∴ P (students obtaining marks 60 and above)

$$= \frac{23}{90}$$

Q.7. To know the opinion of the students about the subject *statistics*, a survey of 200 students was conducted. The data is recorded in the following table.

Opinion	Number of students
like	**135**
dislike	**65**

Find the probability that a student chosen at random (i) likes statistics, (ii) does not like it.

Ans. Opinion of 200 students was taken in a survey.

Thus, the total number of events = 200.

(i) There are 135 students who like statistics.

∴ Number of favourable events = 135.

Hence, P (the students, who like statistics)

$$= \frac{\text{No. of favourable events}}{\text{Total No. of events}}$$

$$= \frac{135}{200} = \frac{27}{40}$$

(ii) There are 65 students who do not like statistics.

∴ Number of favourable events = 65.

Hence, P (the students, who does not like statistics)

$$= \frac{\text{No. of favourable events}}{\text{Total No. of events}}$$

$$= \frac{65}{200} = \frac{13}{40}$$

Q.8. Refer to Q.2, Exercise 14.2. What is the empirical probability that an engineer lives:

(i) less than 7 km from her place of work?

(ii) more than or equal to 7 km from her place of work?

(iii) within $\frac{1}{2}$ km from her place of work?

Ans. Total number of engineers live, n(S) = 40

(i) The number of engineers whose residence is less than 7 km from their place,

n(E) = 9

∴ The probability, that an engineer lives less than 7 km from their place of work

$$= \frac{n(E)}{n(S)} = \frac{9}{40}$$

(ii) The number of engineers whose residence is more than or equal to 7 km from their place of work, n(F) = 40 – 9 = 31

∴ The probability, that an engineer lives more than or equal to 7 km from their place of work

$$= \frac{n(E)}{n(S)} = \frac{31}{40}$$

(iii) The number of engineers whose residence within $\frac{1}{2}$ km from their place of work, i.e., n(G) = 0

∴ The probability, that an engineer lives within $\frac{1}{2}$ km from their place

$$= \frac{n(G)}{n(S)} = \frac{0}{40} = 0$$

Q.9. Activity : Note the frequency of two-wheelers, three-wheelers and four-wheelers going past during a time interval, in front of your school gate. Find the probability that any one vehicle out of the total vehicles you have observed is a two-wheeler.

Ans. Do Yourself.

10. Activity : Ask all the students in your class to write a 3-digit number. Choose any student from the room at random. What is the probability that the number written by her/him is divisible by 3? Remember that a number is divisible by 3, if the sum of its digits is divisible by 3.

Ans. Do Yourself.

Q.11. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):

4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00

Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

Ans. There are 11 bags.

∴ Total numbr of events = 11

Favourable events are : 5.05, 5.08, 5.03, 5.06, 5.08, 5.04, 5.07

∴ Number of favourable events (i.e., bags contain more than 5 kg) = 7.

Hence requireded probability

$$= \frac{\text{No. of favourable events}}{\text{Total No. of events}}$$

$$= \frac{7}{11}$$

Q.12. In Q.5, Exercise 14.2, you were asked to prepare a frequency distribution table, regarding the concentration of sulphur dioxide in the air in parts per million of a certain city for 30 days. Using this table, find the probability of the concentration of sulphur dioxide in the interval 0.12 - 0.16 on any of these days.

Ans. Total number of days considered *i.e.*, total number of trials = 30.

P (concentration of sulphur dioxide in the interval 0.12 – 0.16 in a day)

$$= \frac{\text{No. of day in which the concentration was in the interval 0.12 – 0l.16}}{\text{Total no. of days}}$$

$$= \frac{2}{30} = \frac{1}{15}$$

Q. 13. In Q.1, Exercise 14.2, you were asked to prepare a frequency distribution table regarding the blood groups of 30 students of a class. Use this table to determine the probability that a student of this class, selected at random, has blood group AB.

Ans. Total no. of students = 30

P (a students has blood group AB)

$$= \frac{\text{No. of students which have the blood group AB}}{\text{Total No. of students}}$$

$$= \frac{3}{30} = \frac{1}{10}$$

Additional Questions

Q.1. As the number of tosses of a coin increases, the ratio of the number of heads to the total number of tosses will be $\frac{1}{2}$. Is it correct ? If not write the correct one.

Ans. No, as the number of tosses of a coin increases, the ratio of the number of heads to the total number of tosses will be nearer to $\frac{1}{2}$, not exactly $\frac{1}{2}$.

Q.2. Can the experimental probability of an event

be greater than 1 ? Justify your answer.

Ans. No. Since the number of trials in which the event can happen cannot be greater than the total number of trials i.e., the probability of an event lies between 0 and 1 (both inclusive).

Q.3. Can the experimental probability of an event be a negative number ? If not, why ?

Ans. No, since the number of trials in which the event can happen cannot be negative, and the total number of trials is always positive.

Q.4. A bag contains cards numbered from 1 to 100. A card is drawn at random from the bag. Find the probability that the :

(a) card bears a number which is a multiple of 5.

(b) card bears a number which is greater then or equal to 80.

Ans. Total no. of cards = 100

Multiple of 5 = 5, 10, 15, ... 100 = 20

(a) $\therefore$ P(multiple of 5) = $\frac{20}{100} = \frac{1}{5}$ **Ans.**

(b) Card bears greater than or equal to 80
= 80, 81, 82, 83, 84, 85, ... 100.
= 21

$\therefore$ P(a number > 80) = $\frac{21}{100}$ **Ans.**

Q.5. It is known that a box of 550 bulbs contains 22 defective bulbs. One bulb is taken out at random from the box. Find the probability of getting :

(i) Defective bulb, (ii) Good bulb.

Ans. Total no. of bulbs = 550

No. of defective bulbs = 22

$\therefore$ No. of good bulbs = 550 – 22 = 528

(i) P(Defective bulbs) = $\frac{22}{550} = \frac{2}{50} = \frac{1}{25}$ **Ans.**

(ii) P(Good bulb) = $\frac{528}{550}$ **= 0.96. Ans.**

Q.6. If the probability of winning a game is 0.6, what is the probability of losing it?

Ans. Let the probability of winning game
= P(E) = 0.6

Then the probability of not winning (losing) it = P(not E)

$\therefore$ P(not E) = 1 – P(E)
= 1 – 0.6 = 0.4.

Q.7. 1000 tickets of a lottery were sold and there are 5 prizes on these tickets. If Saket has purchased one lottery ticket, what is the probability of winning a prize?

Ans. Total number of ticks (outcomes)
= 1000

Number of favourable outcomes (No. of prize)
= 5

Hence p(for winning a prize) = $\frac{5}{1000}$ = 0.005

Q.8. On a page of a telephone directory, there are 200 telephone numbers. The frequency distribution of the digits at their units place is given below :

Unit digits	Frequency
0	**22**
1	**26**
2	**22**
3	**22**
4	**20**
5	**10**
6	**14**
7	**28**
8	**16**
9	**20**

Without looking at the page, a number is chosen at random from the page. What is the probability that the digit at the units place of the number chosen is greater than 6.

Ans. The digit at the unit palce which is greater than 6 = 7, 8, 9

Frequency of (7 + 8 + 9) = 28 + 16 + 20 = 64

$\therefore$ Required probability = $\frac{64}{200} = \frac{8}{25}$ **Ans.**

Q.9. A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Distance (in km)	**Frequency**
less than 4000	20
4000 to 9000	210
9001 to 14000	325
more than 14000	445

If you buy a tyre of this company, what is the probability that :

(i) it will need to be replaced before it has covered 4000 km?

(ii) it will last more than 9000 km?

(iii) it will need to be replaced after it has covered somewhere between 4000 km and 14000 km?

Ans. : (i) The total number of trials = 1000.

The frequency of a tyre that needs to be replaced before it covers 4000 km is 20.

So, P(tyre to be replaced before it covers 4000 km) = $\frac{20}{1000} = 0.02$

(ii) The frequency of a tyre that will last more than 9000 km is 325 + 445 = 770

So, P(tyre will last more than 9000 km) = $\frac{770}{1000} = 0.77$

(iii) The frequency of a tyre that requires replacement between 4000 km and 14000 km is 210 + 325 = 535.

So, P(tyre requiring replacement between 4000 km and 14000 km) = $\frac{535}{1000} = 0.535$

Q.10. The percentage of marks obtained by a student in the monthly unit tests are given below:

Unit test	I	II	III	IV	V
Percentage of marks obtained	69	71	73	68	74

Based on this data, find the probability that the student gets more than 70% marks in a unit test.

Ans. The total number of unit tests held is 5.

The number of unit tests in which the student obtained more than 70% marks is 3.

So, P(scoring more than 70% marks) = $\frac{3}{5} = 0.6$

Multiple Choice Questions

Q.1. A coin is tossed 100 times with the following frequencies:

Head– 45, Tail 55

Then the probability of getting a head will be:

(a) 0.45 (b) 0.55

(c) 0.40 (d) 0.50

Ans. (a) 0.45

Q.2. If P (event E) = 0.47, then P (not E) is:

(a) 0.53 (b) 0.55

(c) 0.47 (d) 0.50

Ans. (a) 0.53

Q.3. A box contain 6 black balls and 4 red balls, then the probability of a black balls is:

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{4}{5}$ (d) $\frac{1}{5}$

Ans. (b) $\frac{3}{5}$

Q.4. In the throw of a dice in a game of snakes and ladder, the probability of getting an even number is:

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$

(c) $\frac{4}{5}$ (d) $\frac{2}{5}$

Ans. (a) $\frac{1}{2}$

Q.5. A dice is thrown once, what is the probability of setting an even prime is:

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{1}{6}$ (d) $\frac{1}{5}$

Ans. (c) $\frac{1}{6}$

Q.6. In the previous table, probability of getting 3 head is:

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{6}$ (d) $\frac{1}{5}$

Ans. (d) $\frac{1}{5}$

Q.7. Probability of getting 3 or 5 in a single throw of dice is:

(a) $\frac{3}{5}$ (b) 1

(c) $\frac{1}{3}$ (d) $\frac{1}{5}$

Ans. (c) $\frac{1}{3}$