Chapter

13 Surface Areas and Volumes

In the Chapter

In this chapter, you will study the following points:

- Surface area of a cuboid = $2(lb + bh + hl)$
- Surface area of a cube = $6a^2$
- Curved surface area of a cylinder = $2\pi rh$
- Total surface area of a cylinder = $2\pi r(r + h)$
- Curved surface area of a cone = πrl
- Total surface area of a right circular cone = $\pi rl + \pi r^2$, i.e., $\pi r(l + r)$
- Surface area of a sphere of radius $r = 4\pi r^2$
- Curved surface area of a hemisphere = $2\pi r^2$
- Total surface area of a hemisphere = $3\pi r^2$
- Volume of a cuboid = $l \times b \times h$
- Volume of a cube = a^3
- Volume of a cylinder = $\pi r^2 h$
- Volume of a cone = $\frac{1}{3}\pi r^2 h$
- Volume of a sphere of radius $r = \frac{4}{3}\pi r^3$
- Volume of a hemisphere = $\frac{2}{3}\pi r^3$

[Here, letters l, b, h, a, r, etc. have been used in their usual meaning, depending on the context.]

NCERT TEXT BOOK QUESTIONS (SOLVED)

EXERCISE 13.1

Q.1. A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is opened at the top. Ignoring the thickness of the plastic sheet, determine:

(i) The area of the sheet required for making the box.

(ii) The cost of sheet for it, if a sheet measuring $1 m^2$ costs Rs 20.

Ans. For plastic box :

Length = 1.5 m

Breadth = 1.25 m

Height (depth) = 0.65 m

(i) The box is open at top, therefore area of sheet required

$= 2[lb + bh + hl] - lb$

$= 2[1.5 \times 1.25 + 1.25 \times 0.65 + 0.65 \times 1.5] - 1.5 \times 1.25$

$= 2[1.875 + 0.8125 + 0.975] - 1.875$
$= 2 \times 3.6625 - 1.875$
$= 7.325 - 1.875$
$= 5.45\,m^2$

(ii) Cost of the sheet = Rs. (5.45×20)
= Rs. 109. Ans.

Q.2. The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs.7.50 per m^2.

Ans. For room :

length = 5m
breadth = 4m
height = 3m

Area of four walls = $2(l + b) \times h$
= $2(5 + 4) \times 3 = 54m^2$

Area of ceiling = $l \times b = 5 \times 4 = 20m^2$

Total area to be white washed
= $54 + 20 = 74m^2$

Rate of white-washing = Rs. 7.50 per m^2

Cost of white-washing = Rs. (74×7.50)
= Rs. 555.

Q.3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs. 10 per m^2 is Rs. 15000, find the height of the hall. [Hint : Area of the four walls = Lateral surface area.]

Ans. length = l
breadth = b
height = h

Given, Perimeter = 250m
$\therefore$ $2(l + b) = 250$

Cost of painting the four walls = Rs. 15000

Rate of painting the four walls = Rs. 10 per m^2

Area of four walls = $\frac{15000}{10}\,m^2 = 1500m^2$

Also area of four walls= $2(l + b) \times h$

$\therefore$ $2(l + b) \times h = 1500$

or $250 \times h = 1500$

or $h = \frac{1500}{250}\,m = 6\,m$

Q.4. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

Ans. Area that can be painted by the container
= $9.375\,m^2$
= $9.375 \times 100 \times 100$
= $93750\,cm^2$

Area of the brick
= $2(22.5 \times 10 + 10 \times 7.5 + 75 \times 22.5)$
= $2(225 + 75 + 168.75)$
= $2 \times 468.75 = 937.5\,cm^2$

Number of bricks = $\frac{93750}{937.5}$ = 100 bricks.

Q.5. A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

(i) Which box has the greater lateral surface area and by how much?

(ii) Which box has the smaller total surface area and by how much?

Ans. We have l_1 for cubical box = 10 cm

For cuboidal box l = 12.5 cm
b = 10 cm
h = 8 cm

(i) Lateral surface area of cubical box
= $4t^2 = 4(10)^2$
= 4×100
= $400\,cm^2$

Lateral surface area of cuboidal box
= $4(l + b) \times h$
= $2(12.5 + 10) \times 8$
= $2(22.5) \times 8$
= $45 \times 8 = 360\,cm^2$

(Lateral surface area of cuboidal box) > (Lateral surface area of cuboidal box)
(400 > 360)

$\therefore$ Required Area = $(400 - 360)cm^2 = 40\,cm^2$

(ii) Total surface area of cubical box
= $2(l \times b + b \times h + h \times l)$
= $2(12.5 \times 10 + 10 \times 8 + 8 \times 12.5)$
= $2(125 + 80 + 100)$
= 2×305
= $610\,cm^2$

$\therefore$ (area of cuboidal box) > (Area of cubical box)
(610 > 600)

Required area = $(610 - 600)\,cm^2 = 10\,cm^2$

Q.6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.

(i) What is the area of the glass?

(ii) How much of tape is needed for all the 12 edges?

Ans. Dimension for herbarium are
l = 30 cm, b = 25 cm and h = 25 cm

Area of the glass = $2(l \times b + b \times h + h \times l)$
$= 2(30 \times 25 + 25 \times 25 + 25 \times 30)$
$= 2(750 + 625 + 750) = 2(2125)$
$= 4250 \text{cm}^2$

$\therefore$ Length of the taps
$= 4(l + b + h) = 4(30 + 25 + 25)$
[Herbarium is a shape of cuboid length = $2(l + b + h)$)
$= 4 \times 80 = 320$ cm.

Q.7. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs. 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.

Ans. For bigger box
length = 25 cm
breadth = 20 cm
height = 5 cm
Surface area of the box
$= 2(lb + bh + hl)$
$= 2(25 \times 20 + 20 \times 5 + 5 \times 25)$
$= 2(500 + 100 + 125)$
$= 2 \times 725 = 1450 \text{ cm}^2$

For smaller box
length = 15 cm
breadth = 12 cm
height = 5 cm
Surface area of the box
$= 2(lb + bh + hl)$
$= 2(15 \times 12 + 12 \times 5 + 15 \times 5)$
$= 2(180 + 60 + 72)$
$= 2 \times 315 = 630 \text{ cm}^2$
Total surface area of the two boxes
$= 1450 + 630 = 2080 \text{ cm}^2$
Surface area of cardboard required for overlapping.

$$= \frac{5}{100} \times 2080 = 104 \text{ cm}^2$$

Total surface required of one box of each kind
$= 2080 + 104 = 2184 \text{ cm}^2$
Total surface area of 250 boxes of each kind
$= 250 \times 2184 \text{ cm}^2$
Rate of 1000 cm^2 of cardboard = Rs. 4
Cost of total cardboard

$$= \frac{250 \times 2184 \times 4}{1000} = \text{Rs. } 2184.$$

Q.8. Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?

Ans. Length = 4m
Breadth = 3m
Height = 2.5m
Surface area of the shelter
$= 2(lb + bh + hl) - lb$
$= 2(4 \times 3 + 4 \times 2{,}5 + 2{,}5 \times 4) - 4 \times 3$
$= 2(12 + 7.5 + 10) - 12 = 2 \times 29.5 - 12$
$= 59 - 12 = 47 \text{ m}^2$

EXERCISE 13.2

Q.1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.

Ans. We have, height = 14 cm
Curved surface area of a right circular cylinder
$= 88 \text{ cm}^2$
$2\pi rh = 88$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 14 = 88$$

$$\Rightarrow \quad r = \frac{88 \times 7}{2 \times 22 \times 14}$$

$r = 1$ cm
Diameter = 2 × Radius = 2 × 1 = 2 cm

Q.2. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?

Ans. For cylindrical tank height = 1 m
Base diameter = 140 cm

$$\text{radius } r = \frac{140}{2} \text{ cm} = 70 \text{ cm} = 0.7 \text{ m}$$

Total surface area of the tank

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 0.7\,(1 + 0.7)$

$= 44 \times 0.1 \times 1.7\ m^2$

$= 7.48\ m^2$

Q.3. A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm (see Fig.). Find its

(i) inner curved surface area,

(ii) outer curved surface area,

(iii) total surface area.

Ans. Length = 77 cm

Inner diameter = 4 cm

Outer diameter = 4.4 cm

Inner radius r = 2 cm

Outer radius R = 2.2 cm

(i) Inner curved surface area

$= 2\pi rh = 2 \times \frac{22}{7} \times 2 \times 77$

$= 968\ cm^2$

(ii) Outer curved surface area

$= 2\pi Rh = 2 \times \frac{22}{7} \times 2.2 \times 77$

$= 1064.80\ cm^2$

(iii) Total surface area = Inner curved surface area + outer curved surface area + area of two bases

$= 968 + 1064.80 + 2\pi(R^2 - r^2)$

$= 968 + 1064.80 + 2 \times \frac{22}{7}(2.2^2 - 2^2)$

$= 968 + 1064.80 + 2 \times \frac{22}{7} \times 4.4 \times 0.4$

$= 2032.80 + 5.28$

$= 20.38.08\ cm^2$

Q.4. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2.

Ans. Diameter of the roller = 84 cm

Radius of the roller = 42 cm = 0.42 m

Length of the roller h = 120 cm = 1.2 m

Curved surface area $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 0.42 \times 1.2$

Area covered by the roller in 1 revolution

$= 3.168\ m^2$

Area covered in 500 revolutions

$= 500 \times 3.168$

$= 1584\ m^2$

Hence area of playground = 1584 m^2

Q.5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs. 12.50 per m^2.

Ans. For cylindrical pillar :

Diameter = 50 cm = 0.5 m

Radius = 0.25 m

Height = 3.5 m

Area of curved surface = $2\pi rh$

$= 2 \times \frac{22}{7} \times 0.25 \times 3.5$

$= 5.5\ m^2$

Rate of painting = Rs. 12.50 per m^2

Hence

Cost of painting = Rs. 12.50×5.5

= Rs. 68.75.

Q.6. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height.

Ans. We have, curved surface area of a right circular cylinder = 4.4m^2

$\therefore \quad 2\pi rh = 4.4$

$\Rightarrow \quad 2 \times \frac{22}{7} \times 0.7 \times h = 4.4$

$\Rightarrow \quad h = \frac{44}{44}$

$\Rightarrow \quad h = 1\ m$

Hence, the height of the right circular cylinder is 1 m.

Q.7. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find

(i) its inner curved surface area,

(ii) the cost of plastering this curved surface at the rate of Rs. 40 per m^2.

Ans. We have, inner diameter = 3.5 m

$\therefore \quad$ inner radius $= \frac{3.5}{2}$ m

and $h = 10$ m

(i) Inner curved surface area

$= 2\pi rh = 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 10 = 22 \times 5 = 110 m^2$

(ii) Cost of plastering per m^2 = Rs. 40

Cost of plastering 110m^2

= Rs. 40 × 110 × Rs. 4400.

Q.8. In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

Ans. Height (length) of the cylindrical pipe = 28 m

Diameter of the pipe = 5 cm

Radius of the pipe (r) $= \frac{5}{2}$ cm = 2.5 cm

= 0.025 m

Total radiating surface area

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 0.025 \times 28$

$= 176 \times 0.025$

$= 4.4\ m^2$

Q.9. Find

(i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

(ii) how much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank.

Ans. Diameter = 4.2 m

Radius = 2.1 m

Height = 4.5 m

(i) Lateral or curved surface area

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 2.1 \times 4.5$

$= 59.4\ m^2$

(ii) Total surface area of the tank

$= 2\pi rh\ (h + r)$

$= 2 \times \frac{22}{7} \times 2.1\ (4.5 + 2.1)$

$= 44 \times 0.3 \times 6.6 = 87.12\ m^2$

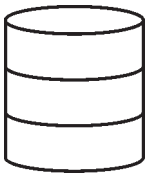
Let the actual area of steel used be x m^2. Since $\frac{1}{12}$ of the actual steel used was wasted, the area of steel which has gone into the tank.

$= \left(1 - \frac{1}{12}\right)$ of $x = \frac{11}{12}$ of x

$\therefore \quad \frac{11}{12}$ of $x = 87.12\ m^2$

$\Rightarrow \quad x = \frac{87.12 \times 12}{11} = 64.8 m^2$

Q.10. In Fig. , you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.



Ans. Given : $r = \frac{20}{2}$ cm = 10 cm

h = 30 cm

Since, a margin of 2.5 cm is used for folding it over the top and bottom so the total height of frame.

$h_1 = 30 + 2.5 + 2.5$

$h_1 = 35$ cm

$\therefore$ Cloth required for covering the lampshade = its curved surface area

$= 2\pi r\ (h_1) = 2 \times \frac{22}{7} \times 10(35)$

$= \frac{440}{7} \times 35 = 440 \times 5$

$= 2200\ cm^2$

Q.11. The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

Ans. For one penholder.

r = 3 cm

h = 10.5 cm

Surface area of a penholder

$= 2\pi rh + \pi r^2$

$= \pi r\ (2h + r)$

$= \frac{22}{7} \times 3 \times (2 \times 10.5 + 3)$

$= \frac{22}{7} \times 3 \times (24)\ cm^2$

Area of cardboard required for 35 boxes

$$= 35 \times \frac{22}{7} \times 3 \times 24$$

$$= 5 \times 22 \times 72 = 110 \times 72$$

$$= 7920 \text{ cm}^2$$

EXERCISE 13.3

Q.1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Ans. We have, diameter = 10.5 cm

$$\text{Radius } (r) = \frac{10.5}{2} = 5.25 \text{ cm}$$

and slant height $l = 10$ cm

$$\text{Curved surface area} = \pi rl = \frac{22}{7} \times 5.25 \times 10$$

$$= 165 \text{cm}^2$$

Q.2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Ans. Slant height l = 21 m

Diameter of base = 24 m

Radius of base = 12 m

Total surface area of the cone $= \pi r (l + r)$

$$= \frac{22}{7} \times 12 (21 + 12)$$

$$= \frac{22 \times 12}{7} \times 33$$

$$= 1244.57 \text{ m}^2.$$

Q.3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find (i) radius of the base and (ii) total surface area of the cone.

Ans. Curved surface area of cone = 308 cm^2

Slant height $l = 14$ cm

(i) Curved surface area $= \pi rl$

$\therefore \quad \pi rl = 308$

or $\frac{22}{7} \times r \times 14 = 308$

or $44r = 308$

or $r = \frac{308}{44} = 7$ cm

Hence radius of base = 7 cm

(ii) Total surface area of cone

$$= \pi r (l + r)$$

$$= \frac{22}{7} \times 7 (14 + 7)$$

$$= 22 \times 21 = 462 \text{ cm}^2$$

Q.4. A conical tent is 10 m high and the radius of its base is 24 m. Find

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs. 70.

Ans. We have, $h = 10$ m

$r = 24$ m

(i) $l = \sqrt{r^2 + h^2}$

$\therefore \quad l = \sqrt{(24)^2 + 10^2}$

$= \sqrt{576 + 100}$

$= \sqrt{676}$

$= 26$ m

Hence, the slant height of the canvas tent is 26m

(ii) Canvas required to make the tent

= Curved surface area of tent

$= \pi rl$

$= \pi \times 24 \times 26 = 624\pi \text{ m}^2$

Cost of 1m^2 canvas = Rs. 70

Cost of 624πm^2 canvas = Rs. $70 \times 624\pi$

$$= \text{Rs. } 70 \times 624 \times \frac{22}{7}$$

$$= \text{Rs. } 10 \times 624 \times 22$$

$$= \text{Rs. } 137280$$

Hence, the cost of the canvas is Rs. 137280.

Q.5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use $\pi = 3.14$).

Ans. For conical tent, height = 8 m

and Cadius of base = 6m

Now, $l = \sqrt{r^2 + h^2}$

$l = \sqrt{6^2 + 8^2}$

$l = \sqrt{36 + 64}$

$= \sqrt{100} = 10$m

Area of tarpaulin = πrl
= $3.14 \times 6 \times 10 m^2$
= $188.4 m^2$

Given, width of tarpaulin = 3 m

$$\text{Length of tarpaulin} = \frac{188.4m}{3} = 62.8 \text{ m}$$

The extra material required for stitching margins and cutting is 20 cm = 0.2m

So, the total length of tarpaulin bought
= (62.8 + 0.2) m
= 63 m.

Q.6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs. 210 per 100 m^2.

Ans. We have, slant height, $l = 25$ m
and diameter = 14
$\therefore$ Radius, $r = 7$ m

Curved surface area of the conical tomb

$$= \pi rl = \frac{22}{7} \times 7 \times 25$$

$$= 22 \times 25 = 550 m^2$$

Cost of white washing per $100 m^2$ = Rs. 210

Cost of white washing per $1 m^2$ = Rs. $\frac{210}{100}$

Cost of white washing $550 m^2$ = Rs. $\frac{210 \times 550}{100}$
= Rs. 1155

Q.7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Ans. Base radius = 7 cm
Height = 24 cm

Let l be the slant height, then in ΔOAB,

$l^2 = 7^2 + 24^2$
$l^2 = 49 + 576$
$l = \sqrt{625}$
$l = 25$ cm

Area of the sheet required for 10 caps
= 10 × curved surface area of cone
= $10 \times \pi rl$

$$= 10 \times \frac{22}{7} \times 7 \times 25$$

$= 5500 \text{ cm}^2$

Q.8. A bus stop is barricated from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2, what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)

Ans. For one cone,

Diameter = 40 cm
Radius, r = 20 cm = 0.2 m
Height, h = 1 m

Slant height, $l = \sqrt{h^2 + r^2}$

$$= \sqrt{1^2 + (0.2)^2}$$

$$= \sqrt{1.04} = 1.02 \text{m}$$

Curved surface = πrl

$$= \frac{22}{7} \times 0.2 \times 1.02 m^2$$

$$= \frac{22}{7} \times 0.204 m^2$$

Curved surface area of such 50 hollow cones

$$= 50 \times \frac{22}{7} \times 0.204 \text{ m}^2$$

$$= \frac{224.4}{7} m^2 = 32.057 m^2$$

Rate of painting the cardboard = Rs. 12 per m^2

Hence, cost of painting the outer surface of all the cones
= Rs. (12 × 32.057)
= Rs. 384.68

EXERCISE 13.4

Q.1. Find the surface area of a sphere of radius: (i) 10.5 cm (ii) 5.6 cm (iii) 14 cm

Ans. (i) For the given sphere
r = 10.5 cm

Surface Area = $4 \pi r^2$

$$= 4 \times \frac{22}{7} \times 10.5 \times 10.5$$

$= 1386 \text{ cm}^2$ Ans.

(ii) $r = 5.6\text{ cm}$

Surface Area $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 5.6 \times 5.6$

$= 394.24\text{ cm}^2$

(iii) $r = 14\text{ cm}$

Surface Area $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 14 \times 14$

$= 2464\text{ cm}^2$

Q.2. Find the surface area of a sphere of diameter:

(i) 14 cm (ii) 21 cm (iii) 3.5 m

Ans. (i) We have, diameter = 14 cm

$\therefore$ Radius, $r = 7$ cm

Surface area of a sphere $= 4\pi r^2 = 4 \times \frac{22}{7} \times 7^2$

$= 4 \times 22 \times 7 = 616\text{ cm}^2$

(ii) We have, diameter = 21 cm

$r = \frac{21}{2} = 105$ cm

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times 105 \times 105$

$= 88 \times 1.5 \times 105$

$= 1386\text{ cm}^2$

(iii) We have, diameter 3.5 m

$\therefore$ $r = \frac{3.5}{2} = 1.75$ cm

$\therefore$ Surface area $= 4\pi r^2 = 4 \times \frac{22}{7} \times 1.75 \times 1.75$

$= \frac{26950}{7} = \mathbf{38.5\text{ cm}^2}$

Q.3. Find the total surface area of a hemisphere of radius 10 cm. (Use $\pi = 3.14$)

Ans. Here $r = 10$ cm

Total surface area of hemisphere

$= 3\pi r^2$

$= 3 \times 3.14 \times 10 \times 10\text{ cm}^2$

$= 942\text{ cm}^2$

Q.4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Ans. Let initial radius, $r_1 = 7$ cm

Area increases, $r_2 = 14$ cm

Surface area of initial balloon

$= 4\pi r_1^2 = 4 \times \frac{22}{7} \times 7 \times 7 = 88 \times 7$

$A_1 = 616\text{ cm}^2$

Surface area for increasing balloon

$= 4\pi r_2^2 = 4 \times \frac{22}{7} \times 14 \times 14 = 88 \times 28$

$A_2 = 2464\text{ cm}^2$

$\therefore$ Required ratio $= A_1 : A_2 = 616 : 2464 = 1 : 4$

Q.5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of Rs. 16 per 100 cm^2.

Ans. For bowl inner diameter = 10.5 cm

Inner radius $= \frac{10.5}{2}$ cm

Inner Surface area of the bowl

$= 2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{10.5}{2} \times \frac{10.5}{2}\text{ cm}^2$

$= 173.25\text{ cm}^2$

Rate of tin-plating = Rs. 16 per 100 cm^2

Hence cost of tin plating

$= \frac{16}{100} \times 173.25$

$=$ Rs. 27.72 **Ans.**

Q.6. Find the radius of a sphere whose surface area is 154 cm^2.

Ans. Surface area of a sphere = 154 cm^2

Let its radius = r cm

$\therefore$ $4 \times \frac{22}{7} \times r^2 = 154$

or $r^2 = \frac{154 \times 7}{4 \times 22} = \frac{49}{4}$

or $r = \sqrt{\frac{49}{4}} = \frac{7}{2} = 3.5$ cm

Hence radius of the sphere = 3.5 cm.

Q.7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Ans. Let diameter of moon $= 2r$

and diameter of earth = $4 \times 2r = 8r$
$\therefore$ Radius of moon = r
and radius of earth = $4r$
Surface area of moon S_1 = $4\pi r^2$
and surface area of earth S_2 = $4\pi(4r)^2$
= $64\pi r^2$

Now $\frac{S_1}{S_2} = \frac{4\pi r^2}{64\pi r^2}$

$= \frac{4}{64} = \frac{1}{16}$ or 1 : 16 Ans.

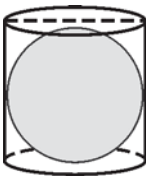
Q.8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Ans. For hemispherical bowl
Inner radius r = 5 cm
Outer radius R = inner radius + thickness
= (5 + 0.25) = 5.25 cm.
Outer curved surface area = $2\pi R^2$

$= 2 \times \frac{22}{7} \times 5.25 \times 5.25$

$= 173.25\ cm^2$

Q.9. A right circular cylinder just encloses a sphere of radius *r* (see Fig.). Find



(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in (i) and (ii).

Ans. (i) For sphere, radius = r
Surface area of sphere, $S_1 = 4pr^2$
(ii) For cylinder, Radius = r
Height = $2r$
[Diameter of sphere =height of the cylinder]
Curved surface area, $S_2 = 2pr h$
(iii) Requried ratio,

$\frac{S_1}{S_2} = \frac{4\pi r^2}{4\pi r^2}$

$= \frac{1}{1}$ or 1 : 1.

EXERCISE 13.5

Q.1. A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?

Ans. Volume of a match box
= 4 cm × 2.5 cm × 1.5 cm = $15\ cm^2$
Volume of a packet = $12 \times 15\ cm^3 = 180\ cm^3$

Q.2. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? ($1\ m^3 = 1000\ l$)

Ans. Volume of a cuboidal water tank
= 6 cm × 5 cm × 4.5
= $30 \times 4.5\ m^3 = 135 m^3$
= 135×1000 L = 135000L
($1m^3 = 1000L$)

Q.3. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

Ans. Volume of cuboidal vessel = $380\ cm^3$
Length = 10 m
Breadth = 8 m

Now Height = $\frac{Volume}{l \times b}$

$= \frac{380}{10 \times 8} = \frac{19}{4} = 4.75$ m.

Q.4. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs. 30 per m^3.

Ans. For tank :
Length = 8 m
Breadth = 6 m
Height (depth) = 3 m
Volume = $l \times b \times h$
= $8 \times 6 \times 3\ m^3 = 144\ m^3$
Rate of digging = Rs. 30 per m^3
Hence total cost of digging = Rs. 30×144
= Rs. 4320.

Q.5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Ans. Capacity of a cuboidal tank
= 50,000 litres

Volume of the tank = $\frac{50{,}000}{1000}\ m^3 = 50\ m^3$

Length = 2.5 m
Height (depth) = 10 m

$$\text{Breadth} = \frac{Volume}{l \times b}$$

$$= \frac{50}{2.5 \times 10} = 2 \text{ m.}$$

Q.6. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

Ans. Population = 4000
Consumption of water
= 150 litres per head per day
Total consumption of water per day
= 4000 × 150 litres

$$= \frac{4000 \times 150}{1000} \text{ m}^3$$

$= 600 \text{ m}^3$
Volume of tank = 20 × 15 × 6
$= 1800 \text{ m}^3$
Number of days the water of the tank will last

$$= \frac{1800}{600} = 3 \text{ days.}$$

Q.7. A godown measures 40 m × 25 m × 15 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.

Ans. Volume of godown = 40 × 25 × 10
$= 10000 \text{ m}^3$
Volume of one wooden crate
= 1.5 × 1.25 × 0.5
$= 0.9375 \text{ m}^3$

$$\text{Number of crates} = \frac{10{,}000}{0.9375}$$

= 10.666.66 crates.
Hence, the maximum number of wooden crates that can be stored in the godown is 10666.

Q.8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Ans. Volume of bigger cube
= 12 × 12 × 12 cm^3
= 1728 cm^3
Number of smaller cubes = 8

$$\text{Volume of one smaller cube} = \frac{1728}{8} \text{ cm}^3$$

$\Rightarrow \quad a^3 = 216 \text{ cm}^3$
Side of smaller cube (a)

$$= \sqrt[3]{216} \text{ cm}^3 = 6 \text{ cm}$$

Surface area of bigger cube
$= 6a^2$
$= 6 \times (12)^2$
$= 6 \times 144$
$= 864 \text{ cm}^2$
Surface of 8 small cubes = $6 \times (6)^2$
$= 216 \text{ cm}^2$

$$\text{Radio} = \frac{864}{216} = \frac{1}{4} \text{ or } 1 : 4.$$

Q.9. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

Ans. Height (depth) = 3m
Breadth = 40 m
Rate of water flown = 2 km per hour
= 2000 m per hour

$$= \frac{2000}{60} \text{ m per minutes}$$

Hence volume of water

$$= 3 \times 40 \times \frac{2000}{60} \text{ m}^3$$

$= 4000 \text{ m}^3$.

EXERCISE 13.6

Q.1. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? (1000 cm^3 = 1*l*)

Ans. For cylindrical vessel,
Height = 25 cm
Circumferecne of base = 132 cm
$\therefore \quad 2\pi r = 132$

$$\text{or} \quad \frac{2 \times 22 \times r}{7} = 132$$

$$r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

Volume of the cylindricxal vessel = $\pi r^2 h$

$$= \frac{22}{7}(21)^2 \times 25$$

$$= \frac{22}{7} \times 21 \times 21 \times 25$$

$$= 22 \times 63 \times 25$$

$$= 34650 \text{ cm}^3$$

Volume of water in litres

$$= \frac{34650}{1000} \text{ litres}$$

$$= 34.65 \text{ litres}$$

Q.2. The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm3 of wood has a mass of 0.6 g.

Ans. For cylindrical vessel, inner diameter = 24 cm and outer diameter = 28 cm

$$\therefore \quad \text{Inner radius } r = \frac{24}{2} = 12 \text{ cm}$$

$$\text{Outer radius R} = \frac{28}{2} = 14 \text{ cm}$$

Height (length) of pipe = 35 cm.

Volume of the wood

$$= \pi R^2 h - \pi r^2 h$$

$$= \pi h (R^2 - r^2)$$

$$= \frac{22}{7} \times 35 (14^2 - 12^2)$$

$$= \frac{22}{7} \times 35 (14 + 12)(14 - 12)$$

$$= \frac{22}{7} \times 35 \times 26 \times 2 = 5720 \text{ cm}^3$$

Mass of 1 cm^3 = 0.6 g

$\therefore$ Required mass = 0.6 × 5720 gm

= 3432 g = 3.432 kg.

Q.3. A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

Ans. (i) We have, $h = 15$ cm

$l = 5$ cm, b = 4 cm, h = 15 cm

Volume of cuboidical body = $l \times b \times h$

$= 5 \times 4 \times 15 = 300 \text{ cm}^3$ ...(i)

(ii) We have, Diameter = 7 cm

$$\text{Radius, } r = \frac{7}{2} \text{ cm}$$

Height, $h = 10$ cm

Then, volume of a plastic cylinder

$$= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10$$

$$= 77 \times 5 = 385 \text{ cm}^3$$

$\therefore$ From Eqs. (i) and (ii), we see that volume of a plastic cylinder has greater capacity and its capacity is 385 – 300 = 85 cm^3 is more than the tin can.

Q.4. If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm, then find (i) radius of its base (ii) its volume. (Use $\pi = 3.14$)

Ans. We have, lateral surface of a cylinder = 94.2 cm^2 and $h = 5$ cm

$$\therefore \quad 2\pi rh = 94.2$$

$$\Rightarrow \quad 2 \times 3.14 \times r \times 5 = 94.2$$

$$\Rightarrow \quad r = \frac{94.2}{31.4}$$

$$\Rightarrow \quad r = 3 \text{ cm}$$

(i) Hence, radius of base, $r = 3$ cm

(ii) Volume of a cylinder $= \pi r^2 h = 3.14(3)^2 \times 5$

$= 3.14 \times 9 \times 5$

$= 141.3 \text{ cm}^3$

Q.5. It costs Rs. 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of Rs. 20 per m^2, find

(i) inner curved surface area of the vessel,

(ii) radius of the base,

(iii) capacity of the vessel.

Ans. We have, cost to paint the inner curved surface = Rs. 2200

Cost to paint per m^2 = Rs. 20

Inner curved surface area

$$= \frac{\text{Cost to paint the inner curved surface}}{\text{Cost to paint per m}^2}$$

$$2\pi rh = \frac{Rs.2200}{Rs.20} = 110 \text{m}^2$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 10 = 110$$

$$r = \frac{110 \times 7}{2 \times 220}$$

$$r = \frac{7}{4} = 1.75 \text{ m}$$

(i) Inner curved surface area of the vessel = $110 m^2$

(ii) Hence, radius of the base is 1.75 m

(iii) Capacity of the vessel = Volume of the vessel

$= \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 10 = \frac{77}{8}$

$= 96.25\ m^3$

Q.6. The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?

Ans. Capacity of a closed cylindrical vessel = 15.4 L

$= 15.4 \times 1000\ cm^3$ $(1L = 1000\ cm^2)$

$\therefore \quad \pi r^2 h = 15400\ cm^3$

$\pi r^2 \times 100 = 15400$

$r^2 = 154 \times \frac{7}{22}$ $(h = 1m = 100\ cm)$

$\Rightarrow \quad r^2 = 7 \times 7$

$\Rightarrow \quad r = 7\ cm$

Total surface area of closed cylindrcial vessel = $2\pi r(r + h)$

$= 2 \times \frac{22}{7} \times 7(7 + 100)$

$= 44 \times 107 = 4708\ cm^2$

The metal sheet must be required = $\frac{4708}{100 \times 100}\ m^2$

$(1\ cm = \frac{1}{100}\ m)$

$= 0.4708\ m^2$

Q.7. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

Ans. Diameter of pencil = 7 mm = 0.7 cm

Diameter of graphite = 1 mm 0.1 cm

Radius of pencil R = $\frac{0.7}{2}$ cm = 0.35 cm

Radius of graphite $r = \frac{0.1}{2}$ cm = 0.05 cm

Length $h = 14$ cm

Volume of graphite $= \pi r^2 h$

$= \frac{22}{7} \times (0.05)^2 \times 14$

$= \frac{22}{7} \times 0.0025 \times 14$

$= 22 \times 0.0025 \times 2\ cm^2$

$= 0.11\ cm^3$

Volume of wood $= \pi R^2 h - \pi r^2 h$

$= \frac{22}{7} \times h\,(R^2 - r^2)$

$= \frac{22}{7} \times 14\,[(0.35)^2 - (0.05)^2]\ cm^3$

$= \frac{22}{7} \times 14\,(0.4 \times 0.3)\ cm^3$

$= 44 \times 0.12\ cm^3$

$= 5.28\ cm^3$

Q.8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

Ans. Diameter of Base = 7 cm

Radius of Base, $r = \frac{7}{2}$ cm

Height, $h = 4$ cm

Volume $= pr^2h$

$= \frac{22}{7}\left(\frac{7}{2}\right)^2 \times 4\ cm^3$

$= 154\ cm^2$

Soup needed for 250 patients

$= (250 \times 154)\ cm^3$

$= 38500\ cm^3$

$= \frac{38500}{1000}$ litres

= 38.5 litres.

EXERCISE 13.7

Q.1. Find the volume of the right circular cone with (i) radius 6 cm, height 7 cm (ii) radius 3.5 cm, height 12 cm

Ans. For cone :

(i) Radius = 6 cm, Height = 7 cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 7$$

$$= \frac{1}{3} \times \frac{22}{7} \times 36 \times 7$$

$$= 264 \text{ cm}^3$$ **Ans.**

(ii) Radius = 3.5 cm, Height = 12 cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12 \text{cm}^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times 12.25 \times 12 \text{ cm}^3$$

$$= 22 \times 1.75 \times 4 \text{ cm}^3$$

$$= 154 \text{ cm}^3$$

Q.2. Find the capacity in litres of a conical vessel with (i) radius 7 cm, slant height 25 cm (ii) height 12 cm, slant height 13 cm

Ans. (i) We have, $r = 7$ cm and $l = 25$ cm

We know that $l^2 = r^2 + h^2$

$$\Rightarrow \quad h = \sqrt{t^2 - r^2} = \sqrt{25^2 - 7^2}$$

$$= \sqrt{625 - 49} = \sqrt{576} = 24 \text{ cm}$$

Capacity of conical vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$$

$$= 22 \times 7 \times 8 = 1232 \text{ cm}^3$$

$$= \frac{1232}{1000} \text{ L} = 1.232 \text{ L}.$$

(ii) We have, $h = 12$ cm and $l = 13$ cm

we know that, $l^2 = h^2 + r^2$

$$\Rightarrow \quad r = \sqrt{l^2 - h^2} = \sqrt{13^2 - 12^2}$$

$$= \sqrt{169 - 144} = \sqrt{25} = \mathbf{5 \text{ cm}}$$

Capacity of conical vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12$$

$$= \frac{22 \times 100}{7} \text{ cm}^3$$

$$= \frac{22 \times 100}{7 \times 1000} \text{ L} = 0.314\text{L}$$

$$\left(1\text{cm}^3 = \frac{1}{1000}\text{L}\right)$$

Q.3. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use π = 3.14)

Ans. We have, volume of a cone = 1570 cm^3

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 1570$$

$$\Rightarrow \quad \frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570$$

$$\therefore \quad r^2 = \frac{1570}{3.14 \times 5} = \frac{15700}{157}$$

$$r = 10 \text{ cm}$$

Hence, radius of the base = 10 cm.

Q.4. If the volume of a right circular cone of height 9 cm is 48πcm^3, find the diameter of its base.

Ans. We have, volume of a right circular cone = 48π cm^3

$$\therefore \quad \frac{1}{3}\pi r^2 h = 48\text{p} \Rightarrow \frac{1}{3} r^2 \times 9 = 48$$

(h = 9 cm, given)

$$\Rightarrow \quad r^2 = 16 - r = 4 \text{ cm}$$

Hence, diameter of the base = $2r = 2 \times 4 = 8$ cm.

Q.5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

Ans. We have, diameter = 3.5 m

$$\therefore \quad \text{radius, } r = \frac{3.5}{2} \text{ m}$$

and $h = 12$ m

Capacity of a conical pit $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{3.5}{2}\right)^2 \times 12$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 12$$

$$= 22 \times 0.5 \times 3.5$$

$$= 38.5 \text{ m}^3 \qquad (1 \text{ m}^3 = 1\text{kL})$$

$$= 38.5 \text{ kL}$$

Q.6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find
(i) height of the cone (ii) slant height of the cone (iii) curved surface area of the cone

Ans. For right circular cone :

Volume = 9856 cm^3
Diameter of the base = 28 cm
Radius of the base r = 14 cm

(i) $\therefore \quad \frac{1}{3}\pi r^2 h = 9856$

$$\frac{1}{3}\times\frac{22}{7}(14)^2 h = 9856$$

$$h = \frac{9856\times 7\times 3}{22\times 14\times 14}\text{ cm}$$

$$= 48\text{ cm.}$$

(ii) For slant height (l)

$$l^2 = h^2 + r^2$$
$$l^2 = (48)^2 + (14)^2$$
$$l = \sqrt{2304+196}\text{ cm}$$
$$= \sqrt{2500}\text{ cm} = 50\text{ cm}$$

Hence, $l = 50$ cm

(iii) Curved surface area of the cone

$$= \pi rl$$
$$= \frac{22}{7}\times 14\times 50\text{ cm}^2$$
$$= 2200\text{ cm}^2$$

Q.7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

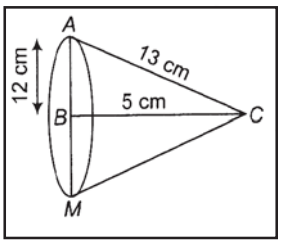
Ans. If we revolve the right ΔABC about the side AB (= 12 cm), we get a cone as shown in the figure :

Height of the cone h = 12 cm
Radius of the cone r = 5 cm
$\therefore$ Volume of the solid cone so obtained

$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi (5)^2\times 12\text{ cm}^3$$
$$= 100\text{ p cm}^3$$

Q.8. If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Ans. If we revolve the right ΔABC about the side BC (=5 cm), we get a cone as shown in the figure:



Height of the cone, h = 5 cm
Radius of the cone, r = 12 cm
$\therefore$ Volume of the solid cone so obtained

$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi (12)^2\times 5\text{ cm}^3$$
$$= 240\pi\text{ cm}^3$$

Now, ratio of the volumes obtained in Q.7 and Q.8.

$$= \frac{100\pi}{240\pi} = \frac{5}{12} = 5:12$$

Q.9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Ans. We have, d = 10.5 m, $r = \frac{10.5}{2} = 5.25m$ *and* h = *3m*

$$l = \sqrt{r^2+h^2}$$
$$= \sqrt{(5.25)^2+3^2}$$
$$= \sqrt{27.5625+9}$$
$$= \sqrt{36.5625}$$
$$= 6.046$$
$$= 6.05\text{ m}$$

Volume of the heap of cone of wheat

$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\times\frac{22}{7}\times 5.25\times 5.25\times 3$$
$$= 86.625\text{ m}^2$$

Area of the canvas required = curved surface area of the heap

$$= \pi rl$$
$$= \frac{22}{7}\times 5.25\times 6.05\text{m}^2$$
$$= 99.825\text{ m}^2$$

EXERCISE 13.8

Q.1. Find the volume of a sphere whose radius is (i) 7 cm (ii) 0.63 m

Ans. (i) r = 7 cm

Volume of sphere

$$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{88 \times 49}{3} \times \frac{4312}{3}$$

$$= 1437\frac{1}{3}\ \text{cm}^3$$

$$= 1437.3\ \text{cm}^3$$

(ii) $r = 0.63$ m

$$\text{Volume of sphere} = \frac{4}{3} \times pr^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{63}{100} \times \frac{63}{100} \times \frac{63}{100}\ \text{m}^3$$

$$= 1.047816\text{m}^3 = 1.05\text{m}^3 \text{ (Approx.)}$$

Q.2. Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm (ii) 0.21 m

Ans. (i) For sphere,

Diameter = 28 cm

Radius = 14 cm

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$$

$$= \frac{88 \times 392}{3} = \frac{34496}{3}$$

$$= 11498\frac{2}{3}\ \text{cm}^3$$

$$= 11498.67\ \text{cm}^3$$

(ii) Diameter = 0.21 m

$$\text{Radius } r = \frac{0.21}{2}\text{ m} = 0.105\text{ m}$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.105)^3\ \text{m}^3$$

$$= 0.004851\ \text{m}^3$$

Q.3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm³?

Ans. Diameter = 4.2 cm

$$\text{Radius } r = \frac{4.2}{2}\text{ cm} = 2.1\text{ cm}$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times \frac{21}{10}\ \text{cm}^3$$

$$= \frac{38808}{1000} = 38.808\ \text{cm}^3$$

Mass of 1 cm^3 = 8.9 gm

Hence total mass = 38.808 × 8.9 gm

= 345.3912 gm

= 345.39 g. (Approx)

Q.4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Ans. Let diameter of earth = $8x$

and diameter of moon $= \frac{1}{4} \times 8x = 2x$

Now radius of earth = $4x$

and radius of moon = x

$$\text{Volume of moon } V_1 = \frac{4}{3}\pi (x)^3$$

$$\text{Volume of earth } V_2 = \frac{4}{3}\pi (4x)^3$$

$$\text{Now required ratio} = \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi (x)^3}{\frac{4}{3}\pi (4x)^3}$$

$$= \frac{1}{64} \text{ or } 1 : 64.$$

Q.5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

Ans. Diameter = 10.5 cm

$\Rightarrow$ Radius, r = 5.25 cm

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 5.25$$

$= 303.1875\ cm^3$

Hemispherical bowl can hold milk $= \frac{303.1875}{1000} L$

$\left(1cm^3 = \frac{1}{1000} L\right)$

$= 0.303$ L (Approx.)

Q.6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Ans. For hemispherical tank

Inner radius r = 1 m - 100 cm

Outer radius R = (100 + 1) = 101 cm

Volume of the iron-sheet

$$= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(R^3 - r^3)$$

$$= \frac{2}{3}\times\frac{22}{7}[(101)^3 - (100)^3]$$

$$= \frac{44}{21}[1030301 - 1000000]$$

$$= \frac{44}{21}[30301] = \frac{1333244}{21}\ cm^3$$

$= 63487.809\ cm^3$

Volume $= \frac{63487.809}{100\times100\times100}\ m^3$

$\left[1cm^3 = \frac{1}{100}\times\frac{1}{100}\times\frac{1}{100} m^3\right]$

$= 0.063487809\ m^3$

$= 0.06348\ m^3$ (Approx.)

Q.7. Find the volume of a sphere whose surface area is 154 cm².

Ans. Surface area = $154\ cm^2$

$\therefore \quad 4\pi r^2 = 154\ cm^2$

$$4\times\frac{22}{7}\times r^2 = 154\ cm^2$$

$$r^2 = \frac{154\times7}{4\times22} = \frac{49}{4}$$

$$r = \sqrt{\frac{49}{4}} = \frac{7}{2}\ cm$$

Now volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^3$

$$= \frac{4}{3}\times\frac{22}{7}\times\frac{7\times7\times7}{2\times2\times2}$$

$$= \frac{11\times49}{3} = \frac{539}{3}$$

$= 179\frac{2}{3}\ cm^3$

Q.8. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of Rs. 498.96. If the cost of white-washing is Rs. 2.00 per square metre, find the

(i) inside surface area of the dome,

(ii) volume of the air inside the dome.

Ans. (i) Rate of white-washing

= Rs. 2 per sq. m

Cost of white-washing = Rs. 498.96

Inside surface area of the hemisphere

$= \frac{498.96}{2} = 249.48 m^2.$

Now inside surface area of hemisphere = $2\pi r^2$.

(ii) $\therefore \quad 2\pi r^2 = 249.48$

$$2\times\frac{22}{7}\times r^2 = 249.48$$

$$r^2 = \frac{249.48\times7}{2\times22} = 5.67\times7$$

$$r^2 = 39.69$$

$$r = \sqrt{39.69} = 6.3\ cm$$

Volume of air inside the dome

$$= \frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{7}\times(6.3)^3$$

$= 523.9\ m^3$

Q. 9. Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S' . Find the (i) radius r 2 of the new sphere, (ii) ratio of S and S' .

Ans. (i) For smallest sphere,

radius = r

$\therefore$ For one sphere S = $4\pi r^2$

Volume of one smaller sphere = $\frac{4}{3}\ 4\pi r^3$

Now volume of 27 smaller spheres

$$= 27 \times \frac{4}{3}\ 4\pi r^2 = 36\ \pi r^3$$

$\therefore$ Volume of one big sphere = $36\ \pi r^3$

Given : Radius of one big sphere = r

$\therefore$ Its volume = $\frac{4}{3}\pi r^3$

$$\therefore \quad \frac{4}{3}\pi r^3 = 36\ \pi r^3$$

$$r'^3 = 27 r^3$$

$$\Rightarrow \quad (r')^3 = (3r)^3$$

$$r' = 3r \Rightarrow \frac{r}{r'} = \frac{1}{3}$$

For new sphere,

Radius r' = 3r

(ii) Now S = $4\pi r^2$

and S' = $4\pi r^2$

$$\therefore \quad \frac{S}{S'} \times \frac{4\pi r^2}{4\pi r'^2} = \left(\frac{r}{r'}\right)^2 = \left(\frac{1}{3}\right)^2$$

$$\frac{S}{S'} = \frac{1}{9} \text{ or } 1 : 9$$

Q.10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

Ans. For spherical capsule :

Diameter = 3.5 mm

Radius $r = \frac{3.5}{2}$ mm = 1.75 mm

$$= \frac{175}{100}\text{ mm} = \frac{7}{4}\text{ mm}$$

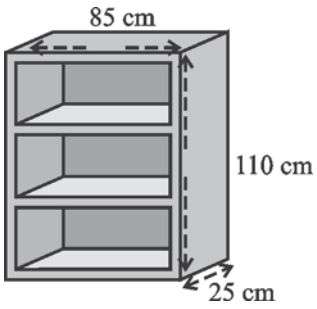
$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{4}\right)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{4}\left(\frac{7}{4}\right)^2 = \frac{22}{3} \times \frac{49}{16} = \frac{539}{24}$$

= 22.46 mm^3 (Approx.)

EXERCISE 13.9 (Optional)

Q.1. A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see Fig.). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.



Ans. External surface area to be polished

= [(110 × 85) + 2(110 × 25) + 2(85 × 25) + 2(110 ×5) + 4(75 ×5)]

= (9350 + 5500 + 4250 + 110 + 1500)cm^2

= 21700 cm^2

$\therefore$ Expenses requried for polishing @ 20 paise per cm^2

= 21700 × 20 paise

= Rs. $\frac{21700 \times 20}{100}$ = Rs. 4240

Internal surface area to be painted

= [2(20 × 90) + 6(75 × 20) + (75 × 90)]

= (3600 + 9000 + 6750) cm^2

= 19350 cm^2

$\therefore$ Expenses required for painting @ 10 paise per cm^2

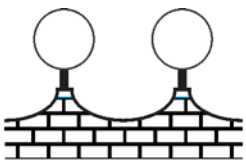
= 19350 × 10 paise

= Rs. $\frac{19350 \times 10}{100}$ = Rs. 1935

$\therefore$ Total expenses required for polishing and painting the surface of the bookshelf

= Rs. 4340 + Rs. 1935 = Rs. 6275. Ans.

Q.2. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in Fig. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is



to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.

Ans. It is obvious, we have to subtract the cost of the sphere that is resting on the supports while calculating the cost of silver paint.

Surface area to be silver paint
= 8 (Curved surface area of the sphere
– Area of circle on which sphere is resting)
$= 8\,(4\pi R^2 - \pi r^2)$

$= 8\pi(4R^2 - r^2)$ (Given, $R = \frac{21}{2}$ cm and $r = 1.5$ cm)

$$= 8\pi\left(4\times\left(\frac{21}{2}\right)^2 - (15)^2\right)$$

$$= 8\pi\left(4\times\frac{441}{4} - 2.25\right)$$

$= 8\pi\,(438.75)$ cm^2

Therefore, the cost of silver paint at the rate of 25 paise per cm^2

$$= \left(8\times\frac{22}{7}\times 438.75\times\frac{25}{100}\right)$$

$$= \frac{19305}{7} = \text{Rs. } 2757 \text{ (Approx)}$$

Hence, surface area to be black painted
= 8 × Curved surface area of cylinder
$= 8 \times 2\pi rh$

$$= 8 \times 2 \times \frac{22}{7} \times 1.5 \times 7$$

$= 528$ cm^2

∴ Cost of black paint at the rate of 5 paise per cm^2

$$= \left(528\times\frac{5}{100}\right) = \text{Rs. } 26.40$$

Hence, total costing of painting = 2757.86 + 26.40
= Rs. 2784.26 (Approx.)

Q.3. The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

Ans. Let diameter = $2r$

∴ Radius = r

$$\therefore \text{Now diameter} = 2r - \frac{25}{100}\times 2r$$

$$= 2r\left(1-\frac{25}{100}\right) = 2r\left(\frac{75}{100}\right)$$

$$= 2r\times\frac{3}{4} = \frac{3r}{2}$$

$$\therefore \text{Now radius} = \frac{3r}{2} + 2 = \frac{3r}{4}$$

Original curved surface area
$= 4\pi(r)^2 = 4\pi r^2$

New curved surface area

$$= 4\pi\left(\frac{3r}{4}\right)^2 = \frac{9\pi r^2}{4}$$

Decrease in curved surface area

$$= 4\pi r^2 - \frac{9\pi r^2}{4} = \frac{7\pi r^2}{4}$$

Hence percentage decrease

$$= \frac{\frac{7\pi r^2}{4}}{4\pi r^2}\times 100\% = \frac{7}{16}\times 100\%$$

$$= \frac{175}{4}\% = 43.75\%$$

Additional Questions

Q.1. A storage tank is in the form of a cube. When it is full of water, the volume of water is 15.625 m^3. If the present depth of water is 1.3 m, find the volume of water already used from the tank.

Ans. Volume of cubical tank = 15.625 m^2
depth = 1.3 m
Let, side of cube = x cm

$x^3 = 15.625$ m^2
$\Rightarrow$ $x^3 = (2.5)^3$
$x = 2.5$ cm
Now, height of water = 1.3 m
Height of water used = 2.5 – 1.5 = 1.2 m
Volume of water used = $1.2 \times 2.5 \times 2.5 = 7.5$ m^3

Q.2. Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimensions 16 cm × 8 cm × 8 cm. When 16 spheres as packed the box is filled with preservative liquid. Find the volume of this liquid. Give your answer to the nearest integer. [Use $\pi = 3.14$]

Ans. Volume of cuboidal box = $l \times b \times h$

$= 16 \times 8 \times 8 = 1024\ cm^3$

Volume of sphere

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2)^3 = \frac{4}{3}\pi \times 8$$

$$= \frac{32\pi}{3} = 33.49\ cm^3$$

Volume of 16 sphere = 535.84 cm^3

Volume of liquid = volume of box – volume of 16 spheres

$\therefore$ Volume of liquid = 1024 – 535.84

$= 488.16\ m^3$

or $= 488\ cm^3$

Q.3. A shopkeeper has one spherical laddo of radius 5 cm. With the same amount of material, how many laddoos of radius 2.5 cm can be made ?

Ans. Radius of big spherical laddoo R = 5 cm

Volume of laddo $= \frac{4}{3}\pi R^3$

$$= \frac{4}{3} \times 5^3 \pi = \frac{500\pi}{3}\ cm^3$$

Volume of each small laddoo

$$= \frac{4}{3}\pi \times (2.5)^3$$

$$= \frac{4}{3}\pi \times 15.625 = \frac{62.5\pi}{3}\ cm^3$$

Number of laddoos = $\frac{\frac{500\pi}{3}}{\frac{62.5\pi}{3}} = \frac{500}{62.5} = 8$

Number of laddoo = 8.

Q.4. A school provides milk to the students daily in a cylindrical glasses of diameter 7 cm. If the glass is filled with milk upto an height of 12 cm, find how many litres of milk is needed to serve 1600 students.

Ans. Diameter of glass = 7 cm

Radius of glass = $\frac{7}{2}$ cm

Milk contained = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12$$

$= 462\ cm^3$

Milk required for 1600 students

$= 462 \times 1600\ cm^2$

= 739.2 litres

Q.5. A cylindrical roller 2.5 m in length, 1.75m in radius when rolled on a road was found to cover the area of 5500 m^2. How many revolutions did it make ?

Ans. Length of roller = height of roller

(h) = 2.5 m

radius (r) = 1.75 m

$\therefore$ Curved surface area of cylindrical roller

$= 2\pi rh$

$$= \left(2 \times \frac{22}{7} \times 1.75 \times 2.5\right) m^2$$

$= (2 \times 22 \times 0.25 \times 2.5) m^2$

$= 44 \times 0.625) m^2 = 27.5\ m^2$

$\therefore$ Area covered in 1 revolution = 27.5 m^2

Thus, No. of revolutions

$$= \frac{\text{Total area}}{\text{Area covered in 1 revolution}}$$

$$= \frac{5500}{27.5} = 200$$

Hence, the road roller will make 200 complete revolutions.

Q.6. A cube of side 4 cm contains a sphere touching its sides. Find the volume of the gap in between.

Ans. Volume of gap in between = Volume of cube volume of sphere

$$= a^3 - \frac{4}{3}\pi r^3 \quad \left(r = \frac{a}{2} Given\right)$$

Volume of gap $= (4)^3 - \frac{4}{3} \times \frac{22}{7} \times 2^3$

$$= 64 - \frac{4}{3} \times \frac{22}{7} \times 8$$

$= 64 - 33.52 = 30.48\ cm^3$

Q.7. 30 circular plates, each of radius 14 cm and thickness 3 cm are placed one above the another to form a cylindrical solid. Find :

(i) the total surface area
(ii) volume of the cylinder so formed
Ans. Radius of each circular plate (r) = 14 m
Thickness of each plate = 3 cm
$\therefore$ Height of cylinder so formed
$h = 30 \times 3 = 90$ cm
(i) Total surface area = $2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 14 \times (90 + 14)$$

$$= 44 \times 2 \times 104 = 9152 \text{ cm}^2$$

(ii) Volume of cylinder = pr^2h

$$= 2 \times \frac{22}{7} \times 14 \times 14 \times 90$$

$$= 22 \times 2 \times 14 \times 90$$

$= (44 \times 14 \times 90)\text{ cm}^3 = 55440\text{ cm}^3$ **Ans.**

Q.8. A hemispherical bowl is made of steel 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bow. (use $\pi = \frac{22}{7}$)

Ans. Thickness of the bowl

$$= 0.25 \text{ cm} = \frac{25}{100} = \frac{1}{4} \text{ cm}$$

The inner radius of the bowl = 5 cm
$\therefore$ Outer radius = Inner radius + Thickness

$$= \left(5 + \frac{1}{4}\right) = 5\frac{1}{4}\text{cm} - \frac{21}{4}\text{cm}$$

Curved surface area= $2\pi r^2$

$$= \left(2 \times \frac{22}{7} \times \frac{21}{4} \times \frac{21}{4}\right) \text{cm}^2$$

$$= \frac{693}{4} \text{ cm}^2 = 173.25 \text{ cm}^2$$

Q.9. If slant height of a cone is 21 m and diameter of base is 24 m, then find its total surface area.

Ans. We have slant height (l) = 21 m

and radius (r) = $\frac{24}{2}$ m = 12 m

$\therefore$ Total surface area of the cone = $\pi r(l + r)$

$$= \frac{22}{7} \times 12(21 + 12)\text{ m}^2$$

$$= \frac{22}{7} \times 12 \times 33\text{ m}^3$$

$$= 12.44.57\text{ m}^2$$

Q.10. Surface area of a sphere is 154 cm^2 Find its radius.

Ans. Let the radius of the sphere be r cm.
Then, surface area of the sphere = $4\pi r^2$
So, $4\pi r^2 = 154$

$$\Rightarrow \quad r^2 = \frac{154}{4\pi} = \frac{154 \times 7}{4 \times 22} = \frac{49}{4}$$

$$\Rightarrow \quad r = \frac{7}{2} \text{ cm.}$$

Multiple Choice Questions

Q.1. Three solid cubes of side 4 cm each are joined to form a cuboid. Then its surface area is:
(a) 288 cm^2 (b) 200cm^2
(c) 192cm^2 (d) 224cm^2
Ans. (d) 224cm^2

Q.2. The number of planks of dimensions 4 m × 5 m × 2 m that can be stored in a pit which is 40 m long, 12 m wide and 160 m deep is:
(a) 1900 (b) 1920
(c) 1800 (d) 1840
Ans. (b) 1920

Q.3. In a cylinder, radius is doubled and height is halved, curved surface area will be:
(a) halved (b) doubled
(c) same (d) four times
Ans.(c) same

Q.4. The volume of a cube whose surface area is 54 cm^2 is:
(a) 9 cm^3 (b) 27 cm^3
(c) 3 cm^3 (d)82 cm^3
Ans. (b) 27 cm^3

Q.5. The radii of cylinders of the same height are in the ratio 4:5, the ratio of their volumes is:
(a) 16:25 (b) 4:5
(c) 64:125 (d) 25:16
Ans. (a) 16:25

Q.6. The surface area of sphere of diameter 14 cm is:
(a) 616cm^2 (b) 632cm^2

(c) 742cm^2 (d) 246cm^2

Ans. (a) 616cm^2

Q.7. If the volume of a sphere is numerically equal to its surface area, then diameter of the sphere is:

(a) 6cm (b) 8cm

(c) 10cm (d) 4cm

Ans. (a) 6cm

Q.8. The radius of largest sphere that is carved out of a cube of side 7 cm is:

(a) 7 cm (b) 12 cm

(c) 13 cm (d) 3.5 cm

Ans. (d) 3.5 cm

Q.9. The sheet required to make closed cylindrical vessel of hight 1 m and diameter 140 cm is:

(a) 7.48 m^2 (b) 7.43 m^2

(c) 5.48 m^2 (d)6.49 m^2

Ans. (a) 7.48 m^2

Q.10.If the areas of three adjacent faces of a cuboid are 8 cm^2, 18 cm^2 and 25 cm^2, then volume of the cuboid is:

(a) 70 cm^3 (b) 60 cm^3

(c) 80 cm^3 (d) 90 cm^3

Ans. (b) 60 cm^3