Chapter 7

Triangles

In the Chapter

In this chapter, you will be studying the following points :

- Two figures are congruent, if they are of the same shape and of the same size.
- Two circles of the same radii are congruent.
- Two squares of the same sides are congruent.
- If two triangles ABC and PQR are congruent under the correspondence $A \leftrightarrow P$, $B \leftrightarrow Q$ and $C \leftrightarrow R$, then symbolically, it is expressed as Δ ABC E $\cong \Delta$PQR.
- If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, then the two triangles are congruent (SAS Congruence Rule).
- If two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then the two triangles are congruent (ASA Congruence Rule).
- If two angles and one side of one triangle are equal to two angles and the corresponding side of the other triangle, then the two triangles are congruent (AAS Congruence Rule).
- Angles opposite to equal sides of a triangle are equal.
- Sides opposite to equal angles of a triangle are equal.
- Each angle of an equilateral triangle is of 60°.
- If three sides of one triangle are equal to three sides of the other triangle, then the two triangles are congruent (SSS Congruence Rule).
- If in two right triangles, hypotenuse and one side of a triangle are equal to the hypotenuse and one side of other triangle, then the two triangles are congruent (RHS Congruence Rule).
- In a triangle, angle opposite to the longer side is larger (greater).
- In a triangle, side opposite to the larger (greater) angle is longer.
- Sum of any two sides of a triangle is greater than the third side.
- **Congruence of Triangle :** Two triangles are congruent, if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.
  If ΔPQR is congruent to ΔABC, we write $\Delta PQR \cong \Delta ABC$.
- **Criteria for Congruence of Triangles**
  (i) SAS congruence rule : Two triangles are congruent, if two sides and the included angle of one triangle are equal to the sides and the included angle of the other triangle.
  (ii) ASA congruence rule : Two triangles are congruent, if two angles and the included sides of one triangle are equal to two angles and the included side of other triangle.
  (iii) AAS congruence rule : Two triangle are congruent, if any two pairs of angles and one pair of corresponding sides are equal.
  (iv) RHS congruence rule : If in two right triangles, hypotenuse and one side of a triangle are equal to the hypotenuse and one side of other triangle, then the two triangles are congruent.

- **Inequalities in a Triangle**

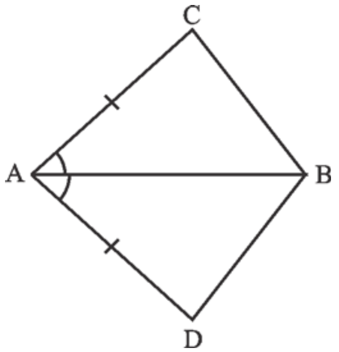
  (i) If two sides of a triangle are unequal, the angle opposite to the longer side is larger (or greater)

  (ii) In any triangle, the side opposite to the larger (or greater) angle is longer (converse of (i)).

  (iii) The sum of any two sides of a triangle is geater than the third side. i.e., AB + BC > CA.

EXERCISE 7.1

Q.1. In quadrilateral ACBD, AC = AD and AB bisects $\angle$ A (see Fig.). Show that ΔABC E $\cong$ ΔABD. What can you say about BC and BD?



Ans. In ΔABC and ΔABD, we have

AC = AD (Given)

$\angle$CAB = $\angle$DAB (AB bisects $\angle$A)

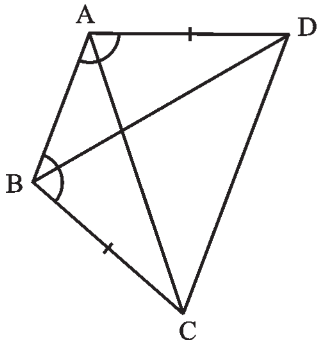
and AB = AB

$\therefore$ ΔABC = ΔABD

(By SAS congruence axiom)

$\therefore$ BC = BD (By CPCT)

Q.2. ABCD is a quadrilateral in which AD = BC and $\angle$DAB = $\angle$CBA (see figure). Prove that



(i) ΔABD $\cong$ ΔBAC **(ii) BD = AC**

(iii) $\angle$ABD = $\angle$BAC

Ans. In DABD and DBAC, we have

AD = BC (Given)

ΔDAB = ΔCBA (Given)

AB = AB (Common)

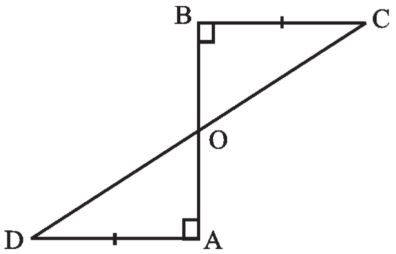
$\therefore$ ΔABD = ΔBAC

(By SAS congruence)

$\therefore$ BD = AC (CPCT)

and ΔABD = ΔBAC (CPCT) Proved.

Q.3. AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB (See fig.)



Ans. Given : AD and BC are equal perpendicular to a line-segment.

To prove : CD bisects AB or OA = OB.

Proof : $\angle$A = $\angle$B

But these are alternate interior angles.

$\therefore$ BC $\|$ DA

CD is their transversal.

$\therefore$ $\angle$C = $\angle$D (Alternate angles)

In ΔOBA and ΔOAD

BC = AD (Given)

$\angle$B = $\angle$A (Given)

$\angle$C = $\angle$D (Proved above)

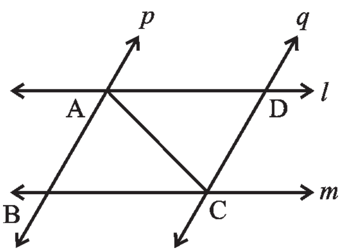
$\therefore$ ΔOBC = ΔOAD

(ASA congruence)

Hence OA = OB (CPCT)

Q.4. *l* and *m* are two parallel lines intersected by another pair of parallel lines *p* and *q* (See fig.). Show that

ΔABC $\cong$ ΔCDA



Ans. Given : *l* and *m* are two parallel lines intersected by another pair of parallel lines *p* and *q*.

To Prove : ΔABC $\cong$ ΔCDA

Proof : In DABC and DCDA,

$\angle BAC = \angle DCA$

[Alternate angles as AB || DC and AC is the transversal]

$\angle ACB = \angle CAD$

[Alternate angles as BC || DA and AC is the transversal]

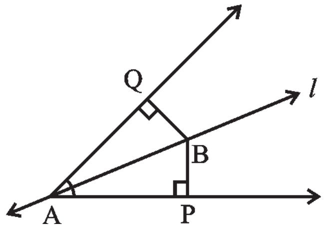
AC = CA (Common)

Hence $\Delta ABC = \Delta CDA$ (ASA rule)

Q.5. Line *l* is the bisector of an angle ∠A and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of ÐA. Show that :

(i) $\Delta APB \cong \Delta AQB$

(ii) BP = BQ or B is equidistant from the arms of ∠ A. (See fig.)



Ans. Line *l* is the bisector of an angle ∠ A and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of ∠ A.

To prove : (i) $\Delta APB \cong \Delta AQB$, (ii) BP = BQ.

Proof : In ΔAPB and ΔAQB,

AB = AB (Common)

$\angle BPA = \angle BQA$ (Given each = 90°)

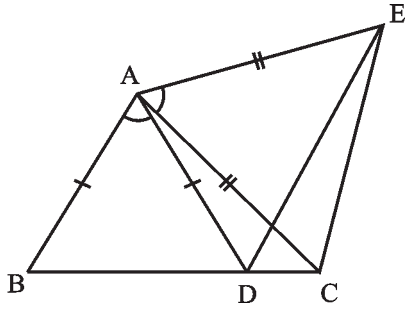
$\angle BAP = \angle BAQ$

(Given *l* is the bisector of ∠ A)

$\therefore \Delta APB \cong \Delta AQB$ (AAS rule)

Hence BP = BQ (CPCT)

Q.6. In Fig., AC = AE, AB = AD and ∠ BAD = ∠EAC. Show that BC = DE.



Ans. Given : As given in the figure

AC = AE

AB = AD

$\angle BAD = \angle EAC$

To Prove : BC = DE

Proof : $\angle BAD = \angle EAC$ (Given)

Adding ∠DAC to both sides, we get

$\angle BAD + \angle DAC = \angle EAC + \angle DAC$

$\angle BAC = \angle DAE$

Now in ΔBAC and ΔDAE,

$\angle BAC = \angle DAE$ (Proved above)

BA = DA (Given)

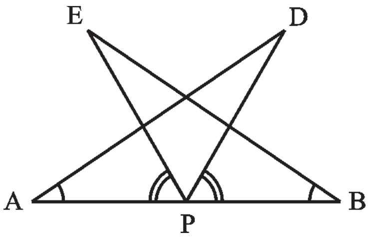
AC = AE

$\therefore \Delta BAC = \Delta DAE$ (SAS rule)

Hence BC = DE (CPCT)

Q.7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB (See fig.). Show that

(i) $\Delta DAP = \Delta EBP$ (ii) AD = BE



Ans. In ΔDAP and ΔEBP, we have

AP = BP

[P is the mid-point of line segment AB]

$\angle PAD = \angle PBE$ (Given)

$\angle EPB = \angle DPA$

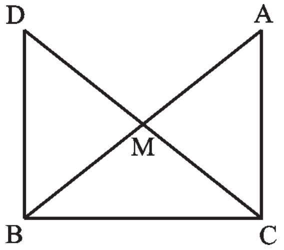
$[\angle EPA = \angle DPB \Rightarrow \angle EPA + \angle DPE = \angle DPB + \angle DPE]$

$\therefore \Delta DPA = \Delta EPB$

[By ASA congruence]

$\Rightarrow$ AD = BE [CPCT] Proved.

Q.8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig.). Show that:



(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$

Ans. Given : A right angles triangle, right angles at C. M is the mid-point of AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B.

To prove : (i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$

Proof : (i) In ΔAMC and ΔBMD

AM = BM (Given)

CM = DM (Given)

$\angle AMC = \angle BMD$

(Vertically opposite angles)

$\therefore$ $\Delta AMC \cong \Delta BMD$ (SAS rule)

(ii) $\angle DBC$ is a right angle.

$\angle CAM = \angle DBM$ (CPCT)

Also $\angle CAM + \angle MBC = 90°$ [Since $\angle C = 90°$]

$\therefore$ $\angle DBM = \angle MBC = 90°$

$\angle DBC = 90°$

or

(iii) $\Delta DBC \cong \Delta ACB$

In ΔDBC and ΔACB

BC = BC (Common)

DB = AC ($\Delta BMD \cong \Delta AMC$, CPCT)

$\angle DBC = \angle ACB = 90°$ (Proved above)

Hence $\Delta DBC \cong \Delta ACB$ (SAS rule)

(iv) $CM = \frac{1}{2} AB$

Since

$\Delta DBC \cong \Delta ACB$

$\therefore$ DC = AB (CPCT)

Thus $\frac{1}{2} DC = \frac{1}{2} AB$

or CM = AM

[M is mid-point of AB and DC]

Hence $\boxed{CM = \frac{1}{2} AM}$ (Proved)

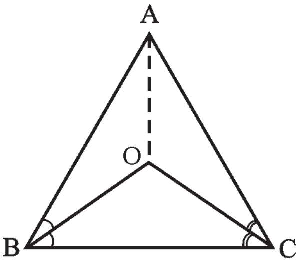
EXERCISE 7.2

Q.1. In an isosceles triangle ABC, with AB = AC, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that :

(i) OB = OC (ii) AO bisects $\angle A$

Ans. Given : An isosceles ΔABC, with AB = AC.

The bisectors of $\angle B$ and $\angle C$ intersect each other at O. A is joined to O.



To Prove : (i) OB = OC

(ii) AO bisector $\angle A$

Proof : (i) $\angle ABC = \angle ACB$

$\frac{1}{2} \angle ABC = \frac{1}{2} \angle ACB$

$\therefore$ $\angle OBC = \angle OCB$

Thus OB = OC

[isosceles Δ theorem]

(ii) In ΔAOB and ΔAOC,

OB = OC (Proved above)

AB = AC (Given)

AO = AO (Common)

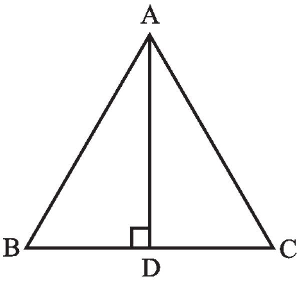
Thus $\Delta AOB = \Delta AOC$ (SAS rule)

$\therefore$ $\angle AOB = \angle OAC$ (CPCT)

Hence AO bisects $\angle A$.

Q.2. In ΔABC, AD is the perpendicular bisector of BC (see Fig.). Show that ΔABC is an isosceles triangle in which AB = AC.

Ans. Given : In ΔABC, Ad is the perpendicular bisector of BC,



To prove : ΔABC is an isosceles triangle with $AB = AC$.

Proof : In ΔABD and ΔACD,

$BD = CD$ (AD bisects BC)

$\angle ADB = \angle ADC = 90°$ [$Ad \perp BC$]

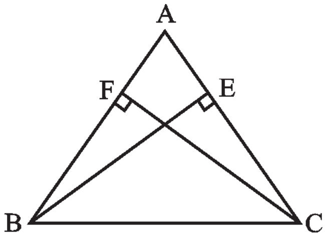
$AD = AD$ (Common)

$\therefore \Delta ABD \cong \Delta ACD$ (SAS rule)

Therefore, $AB = AC$ (CPCT)

Hence ΔABC is an isosceles triangle.

Q.3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to sides AC and AB respectively (See fig.). Show that these altitudes are equal.



Ans. Given : An isosceles ΔABC in which altitude BE and CF are drawn to sides AC and AB respectively.

To prove : Altitude BE = Altitude CF.

Proof : In ΔABE and ΔACF,

$\angle A = \angle A$ (Common)

$\angle AEB = \angle AFC = 90°$ (Given)

$AB = AC$ (Given)

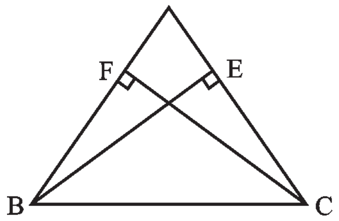
So, $\Delta ABE \cong \angle ACF$ (AAS rule)

Hence, $BE = CF.$ (CPCT)

Q.4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see fig.). Show that

(i) $\Delta ABE \cong \Delta ACG.$

(ii) $AB = AC$ or ΔABC is an isosceles triangle.



Ans. Given : A ΔABC in which altitude BE and CF to sides AC and AB are equal.

To Prove : (i) $\Delta ABE \cong \Delta ACF$

(ii) $AB = AC$ or ΔABC is an isosceles triangle.

Proof : In ΔABE and $\Delta ACF.$

$BE = CF$ (Given)

$\angle A = \angle A$ (Common)

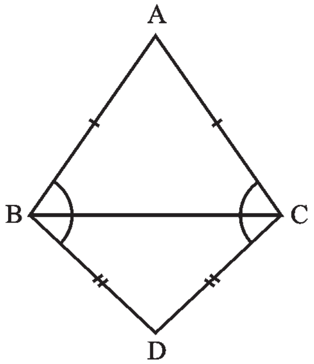
$\angle AEB = \angle AFC = 90°$ (Given)

So, $\Delta ABE = \Delta ACF$ (AAS rule)

(ii) $AB = AC$ (CPCT)

Hence ΔABC is an isosceles triangle.

Q.5. ABC and DBC are two isosceles triangles on the same base BC (See fig.). Show that $\angle ABD = \angle ACD$.



Ans. Given ΔABC and ΔDBC are two isosceles triangles on the same base BC.

To prove : $\angle ABD = \angle ACD$

Proof : In ΔABC,

$AB = AC$ (given)

$\therefore \angle ABC = \angle ACB$ ... (i)

(isosceles Δ theorem)

Again in ΔBDC,

$BD = DC$ (given)

$\therefore \angle DBC = \angle DCB$ ... (ii)

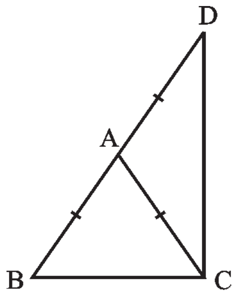
(isosceles Δ theorem)

Adding (i) and (ii), we get

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

Hence, $\angle ABD = \angle ACD$. Proved.

Q.6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB. Show that $\angle BCD$ is a right angle. (See fig.).



Ans. Given : DA BC is an isosceles triangle in which $AB = AC$.

Side BA is produced to D such that AD = AB.

To prove : $\angle BCD = 1$ rt. angle.

Proof : In ΔABC,

$AB = AC$

$\therefore \quad \angle ABC = \angle ACB$

[Angles opposite to equal sides]

Also $\angle ACD = \angle ADC$

[Angles opposite to equal sides of ΔADC]

Now

$\angle BAC + \angle CAD = 180°$ [Linear pair]

Also $\angle CAD = \angle ABC + \angle ACB$

$= 2\angle ACB$

[Exterior angles of ΔABC]

and $\angle BAC = \angle ACD + \angle ADC$

[Exterior angles of ΔABC]

$= 2\angle ACD$

Therefore,

$\angle BAC + \angle CAD = 2[\angle ACB + \angle ACD]$

$= 2\angle BCD$

or $2\angle BCD = 180°$

Hence $\angle BCD = 90°$ Proved.

Q.7. ABC is a right angled triangle in which $\angle A = 90°$ and AB and AC. Find $\angle B$ and $\angle C$.

Sol. Given : A right angled triangle in which

$\angle A = 90°$

$AB = AC$

Required : To find the measures of $\angle B$ and $\angle C$.

Proof : In ΔABC,

$AB = AC$ (Given)

$\therefore \quad \angle B = \angle C$

[Angles opposite to equal sides of ΔABC]

Now, $\angle A + \angle B + \angle C = 180°$

or $90° + 2\angle B = 180°$

or $2\angle B = 180° - 90°$

or $2\angle B = 90°$

or $\angle B = 45°$

As $\angle B = \angle C$

Hence $\angle B = \angle C = 45°$

Q.8. Show that the angles of an equilateral triangle are 60° each.

Sol. Given : An equilatral triangle ABC.

Required : $\angle A = \angle B = \angle C = 60°$

Proof : In ΔABC,

$AB = AC$ (Given)

$\therefore \quad \angle B = \angle C$ ...(i)

[Angles opposite to equal sides of ΔABC]

Again $BC = AC$ (Given)

$\angle A = \angle C$ ...(ii)

(Angles opposite to equal sides of ΔABC)

From (i) and (ii), we get

$\angle A = \angle B = \angle C$

But $\angle A + \angle B + \angle C = 180°$

[By ASP of a Δ]

or $3\angle A = 180°$

or $\angle A = 60°$

Hence, $\angle A = \angle B = \angle C = 60°$ Proved.

EXERCISE 7.3

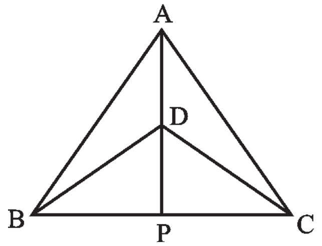
Q.1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig.). If AD is extended to intersect BC at P, show that

(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$.

(iv) AP is the perpendicular bisector of BC.



Ans. Given : ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and B on the same side of BC.

AD is extended to intersect BC at P.

To Prove :

(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$

(iv) AP is the perpendicular bisector of BC.

Proof : (i) ΔABD and ΔACD.

$AB = AC$ (given)

$BD = CD$ (given)

$AD = AD$ (common)

$\therefore \quad \Delta ABD \cong \Delta ACD$ (SSS rule)

(ii) $\angle BAD = \angle CAD$ (CPCT)

or $\angle BAP = \angle CAP$

$\therefore$ AD or AP bisects $\angle A$

In ΔABP and ΔACP,

$\angle BAP = \angle CAP$ (proved above)

$AB = AC$ (given)

$AP = AP$ (common)

$\therefore \quad \Delta ABP = \angle ACP$ (SAS Rule)

$\therefore \quad \angle APB = \angle APC$ (CPCT)

But $\angle APB + \angle APC = 180°$ (Linear pair)

$2\angle APB = 180°$

$\Rightarrow \quad \angle APB = 90°$

$\therefore \quad AP \perp BC$

or $\quad BP = PC$ (CPCT)

Hence AP is the perpendicular bisector of BC.

... (iv) **Proved.**

(iii) In ΔBPD and ΔCPD,

$PD = PD$ (common)

$BD = CD$ (given)

$BP = CP$ (Proved above)

$\therefore \quad \Delta BPD = \Delta CPD$ (SSS rule)

Thus $\quad \angle BDP = \angle CDP$ (CPCT)

As $\quad BP = CP$ (Proved above)

$\therefore$ AP bisects $\angle A$ as well as $\angle D$.

Q.2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that :

(i) AD bisects BC (ii) AD bisects $\angle A$

Ans. Given : An isosceles ΔABC in which AB = AC and $AD \perp BC$.

To prove : (i) AD Bisects BC.

(ii) Ad bisects $\angle A$.

Proof : In ΔABC,

$AB = AC$ (Given)

$\therefore \quad \angle B = \angle C$

(Angles opposite to equal sides)

Now in ΔABD and ΔACD,

$AD = AD$ (common)

$AB = AC$

$\angle ADB = \angle ADC$

[Each = 90°, as $AD \perp BC$ given]

Thus $\Delta ABD = \Delta ACD$ (RHS rule)

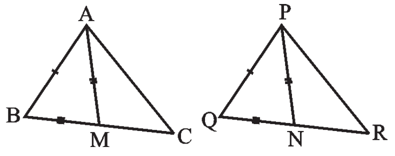
So $\quad BD = CD$

and $\angle BAD = \angle CAD$ (CPCT)

Hence, (i) AD bisects BC and (ii) AD bisects $\angle A$.

Q.3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (See fig.). Show that :

(i) $\Delta ABM = \Delta PQN$ (ii) $\Delta ABC = \Delta PQR$.



Ans. Given : Two sides AB and BC and median AM of ΔABC are respectively equal to sides PQ and QR and median PN of ΔPQR.

To prove :

(i) $\Delta ABM = \Delta PQN$

(ii) $\Delta ABC = \Delta PQR$

Proof : (i) $BC = QR$ (given)

$\frac{1}{2}BC = \frac{1}{2}QR$ or $BM = QN$

In ΔABM and ΔPQN,

$BM = QN$ (proved above)

$AB = PQ$ (given)

$AM = PN$ (given)

Thus $\Delta ABM = \Delta PQN$ (SSS rule)

So $\quad \angle B = \angle Q$ (CPCT)

(ii) Again in ΔABC and ΔPQR

$AB = PQ$ (given)

$BC = QR$ (given)

$\angle B = \angle Q$ (proved above)

Thus $\Delta ABC = \Delta PQR$ (SAS rule)

Q.4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Sol. Given : A ΔABC, in which

$BE \perp AC$

$CF \perp AB$

and $\quad BE = CF$

To prove : ΔABC is an isosceles triangle.

Proof : In ΔBFC and ΔCEB

$\angle BFC = \angle CEB$ (Each = 90° given)

$BC = BC$ (Common)

$CF = BE$ (Given)

$\therefore \quad \Delta BCF = \Delta CBE$ (RHS rule)

So $\quad \angle CBF = \angle BCE$ (CPCT)

As two angles of a triangle ABC are equal therefore sides opposite to them are equal i.e., AB = AC.

Hence ΔABC is an isosceles triangle.

Q.5. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC and show that ∠B = ∠C.

Ans. Given : An isosceles ΔABC in which

$AB = AC$

$AP \perp BC$

To prove : $\angle B = \angle C$

Proof : In ΔAPB and ΔAPC,

$AB = AC$ (given)

$AP = AP$ (common)

$\angle APB = \angle APC$

(Each = 90° Given $AP \perp BC$)

Thus $\quad \Delta APB = \Delta APC$ (RHS rule)

Hence $\quad \angle B = \angle C.$ (CPCT)

EXERCISE 7.4

Q.1. Show that in a right angles triangle, the hypotenuse is the longest side.

Ans. Given : ΔABC in which $\angle A$ = rt. angle.

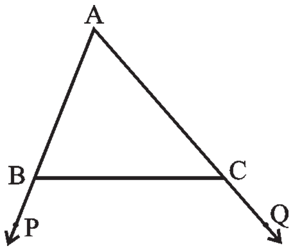
To prove : BC is the long side.

Proof : $\angle A > \angle B$ and $\angle A > \angle C$. Therefore $BC > AC$ and $BC > AB$.

[Sides opposite to bigger angles is longer]

Hence BC is the longest side.

Q.2. In Fig., sides AB and AC of ΔABC are extended to points P and Q respectively, Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.



Ans. Given : Sides AB and AC of ΔABC are extended to points P and Q such that

$\angle PBC < \angle C < \angle B$

To prove : $AC > AB$

Proof : $\angle PBC < \angle QCB$

But $\angle PBC + \angle ABC = 180°$

$\Rightarrow \angle PBC = 180° - \angle ABC$

and $\angle QCB + \angle BCA = 180°$

$\Rightarrow \angle QCB = 180° - \angle BCA$.

[By linear pair axiom]

$\therefore 180° - \angle ABC < 180° - \angle BCA$

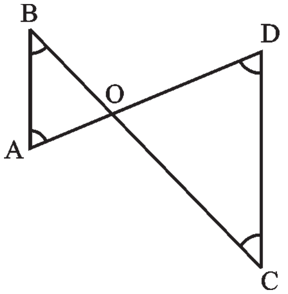
or $-\angle ABC < -\angle BCA$

or $\angle ABC > \angle BCA$

Hence $AC > AB$

[Side opposite to bigger angle is longer]

Q.3. In fig. $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.



Sol. Given : As in the figure $\angle B < \angle A$ and $\angle C < \angle D$.

To prove : $AD < BC$

Proof : $\angle B < \angle A$

or $\angle A > \angle B$

$\therefore OB > OA$ ...(i)

[Side opposite to bigger angle is longer]

Also $\angle C < \angle D$

or $\angle D > \angle C$

$\therefore OC > OD$ ...(ii)

[Side opposite to bigger angle is longer]

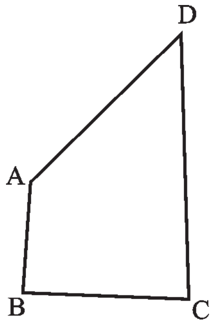
Adding (i) and (ii), we get

$OB + OC > OA + OD$

$BC > AD$

Hence $AD < BC$

Q.4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (See fig.). Show that $\angle A > \angle C$ and $\angle B > \angle D$.



Ans. Given : A quadrilateral ABCD in which AB and CD are respectively the smallest and longest side.

To prove : $\angle A > \angle C$ and $\angle B > \angle D$

Construction : Join B and D and A to C.

Proof : In ΔABC

$BC > AB$ (given)

Therefore

$\angle BAC > \angle BCA$ ...(i)

[longer side has the greater angle opposite to it]

Also in ΔACD,

$CD > AD$ (given)

Therefore

$\angle CAD > \angle ACD$ ...(ii)

Adding (i) and (ii), we get

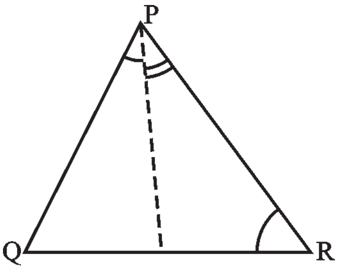
$\angle BAC + \angle CAD > \angle BCA + \angle ACD$

$\angle BAD > \angle BCD$

$\angle A > \angle C$

Similarly $\angle B > \angle D$

Q.5. In Fig. PR > PQ and PS bisects < QPR. Prove that ∠PSR > ∠PSQ.



Ans. Given : $PR > PQ$

To prove : $\angle PSR > \angle PSQ$

Proof : $PR > PQ$ (Given)

$\angle PQR > \angle PRQ$ ...(i)

[Angle opposite to longer side is greater]

Also, $\angle QPS = \angle RPS$ ...(ii)

[PS bisects ∠QPR]

Adding (i) and (ii), we get

$\angle PQR + \angle QPS = \angle PRS + \angle RPS)$ ...(iii)

Similarly, in ΔPRS,

$\Rightarrow \angle PQS + \angle QPS > \angle PRS + \angle RPS$

$\Rightarrow \angle PSR > \angle PSQ$

(By exterior angle property)

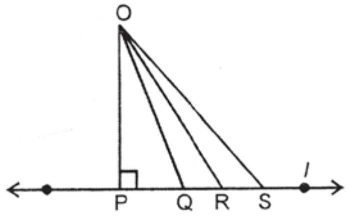
$\angle PSR = \angle PQS + \angle QPS$

and $\angle PSQ = \angle PSR + \angle RPS$

$\Rightarrow \angle PSR = \angle PSQ$ **Proved**

Q.6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Ans. We have a line $\overleftrightarrow{l}$ and O is a point not on $\overleftrightarrow{l}$. $OP \perp \overleftrightarrow{l}$.



We have to prove that OP < OQ, OP < OR and OP < OS.

In ΔOPQ,

$\angle P = 90$

∴ ∠Q is an acute angle (i.e., ∠Q < 90°)

∴ $\angle Q < \angle P$

Hence, $OP < OQ$

[Side opposite to greater angle is longer]

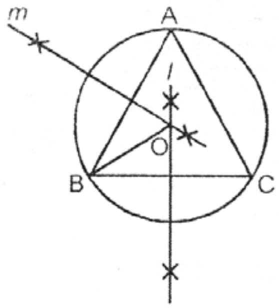
Similarly, we can prove that OP is shorter than QR, OS etc. **Proved.**

EXERCISE 7.5 (OPTIONAL)

Q.1. ABC is a triangle. Locate a point in the interior of ΔABC which is equidistant from all the vertices of ΔABC.

Ans. Step of Construction :

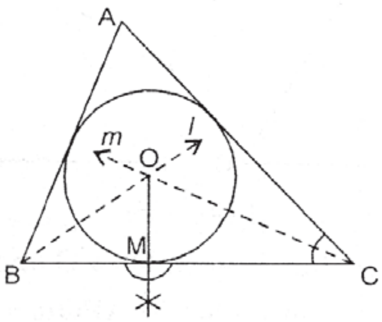
1. Draw any ΔABC.
2. Draw *l* the perpendicular bisector of BC.
3. Draw *m* the perpendicular bisector of AB.
4. Let *l* and *m* meet at O.



Now O is the required point.

(Now with O as centre and OB as radius we can draw a circle which will pass through A, B and C.)

Q.2. In a triangle, locate a point in its interior which is equidistant from all the sides of the triangle.

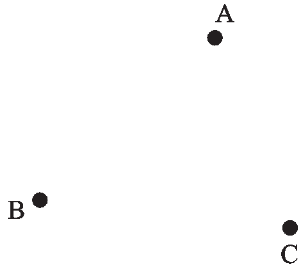


Ans. Steps of Construction :

1. Draw a ΔABC.
2. Draw *l*, the bisector of ∠B.
3. Draw *m*, the bisector of ∠C.
4. Let the bisectors of ∠B and ∠C meet at O.

(Now draw OM ⊥ BC and with O as centre and OM as radius draw a circle. The circle will touch the sides of the triangle and the radius which are equidistant from the centre.)

Q.3. In a huge park, people are concentrated at three points (see fig.)



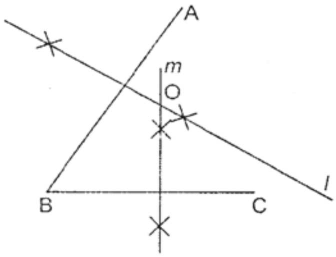
A : where there are different slides and swings for children

B : near which a man-made lake is situated.

C : which is near to a large parking and exit. Where should an ice-cream parlour be set-up so that maximum number of persons can approach it?

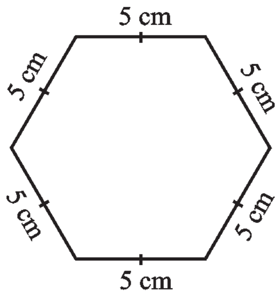
[**Hint :** The parlour should be equidistant from A, B and C]

Ans. Steps of Construction :

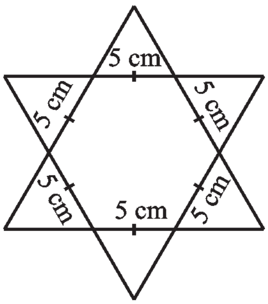


1. Join A and B and C.
2. Draw the right bisector l and AB and right bisector m of BC.
3. Let the right bisectors l and m meet at O.
4. O is equidistant from A, B and C.

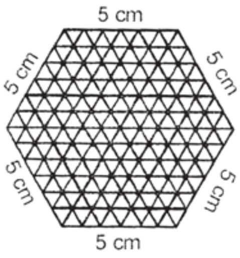
Q.4. Complete the hexagonal and star shaped Rangolies [see Fig. (i) and (ii)] by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?



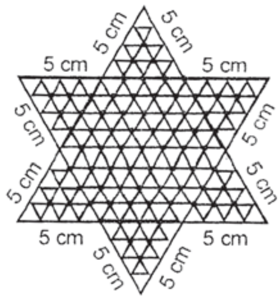
(i)



(ii)



(*i*)



(*ii*)

No. of triangle = 150.

Obviously, Fig. (ii) has more triangles.

Additional Questions

Q.1. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent". Is the statement true? why?

Ans. No, angles must be included angles.

Q.2. "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent." Is the statement true ? Why?

Ans. No, sides must be corresponding sides.

Q.3. Is it possible to construct a triangle with length of its sides as 4 cm, 3 cm and 7 cm? Give reason for your answer.

Ans. No, because sum of two sides = the third side

i.e., $4 + 3 = 7$

But sum of two sides must be greater than the third side.

Q.4. Is it possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm? Give reason for your answer.

Ans. No.

$$9 + 7 < 17$$

Sum of the two sides is less than the 3rd side.

Q.5. Is it possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm? Give reason for your answer.

Ans. Yes.

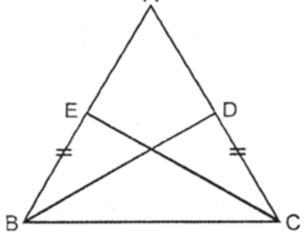
Reason : $8 + 7 > 4$

$7 + 4 > 8$

and $8 + 4 > 7$

because in each case the sum of two sides is greater than the third side.

Q.6. ABC is an isosceles triangle with AB = AC and BD and CE are its two medians. Show that BD = CE.



Ans. Given : ABC is an isoceles triangle with AB = AC and BD and CE are its two medians.

To prove : BD = CE

Proof : AB = AC (Given)

$\therefore \quad \angle ABC = \angle ACB$ ...(i)

[Angles opposite to equal sides are equal]

$$AB = AC$$

$$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$$

$\Rightarrow$ BE = CD ... (iii)

[E and D are mid-points]

and BC = BC (Common)... (iii)

From (i), (ii) and (iii)

Thus, $\Delta EBC = \Delta DCB$

(Using SSS congruence rule)

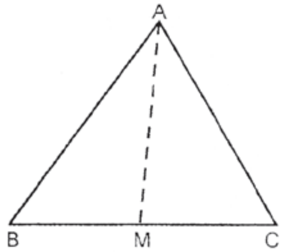
Hence, BD = CE (CPCT)

Q.7. M is a point on side BC of a triangle ABC such that AM is the bisector of $\angle$BAC. Is it true to say that perimeter of the triangle is greater than 2 AM? Give reason for your answer.

Ans. Yes,

Given : ABC is a triangle and AM is the bisector of $\angle$BAC.

To prove : $AB + BC + AC > 2AM$



Reason : In ΔABM, we have

$AB + BM > AM$ ... (i)

[the sum of any two sides is greater than the 3rd side]

Similarly, $AC + CM > AM$ ...(ii)

On adding (i) and (ii), we get

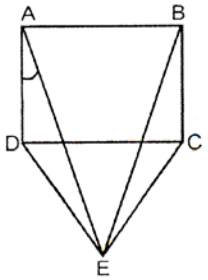
$$AB + BM + AC + CM > 2AM$$

$$\Rightarrow AB + (BM + CM) + AC > 2AM$$

$$\Rightarrow AB + BC + AC > 2AM$$

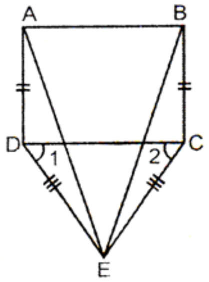
i.e., perimeter of $\Delta ABC > 2AM$ **Proved.**

Q.8. CDE is an equilateral triangle formed on a side CD of a square ABCD (Fig.). Show that $\Delta ADE = \Delta BCE$.



Ans. Given : CDE is an equlateral triangle on a side CD of square ABCD.

To Prove : $\Delta ADE = \Delta BCE$



Proof : In ΔADE and ΔBCE,

$AD = BC$ ...(i)

(Sides of a square)

$DE = CF$ ... (ii)

[equal sides of an equilateral Δs)

$\angle D = \angle C$ (each 90°)

$\angle 1 = \angle 2$ (each 60°)

Hence, $\angle D + \angle 1 = \angle C + \angle 2$

$\Rightarrow \angle ADE = \angle BCE$ ... (iii)

From (i), (ii) and (iii)

Hence, $\Delta ADE = \Delta BCE$ (by SAS rule)

Q.9. If $\Delta PQR = \Delta EDF$, then is it true to say that PR = EF? Give reason for your answer.

Ans. Yes, $PR \leftrightarrow EF$

Reason : PR = EF

This is a pair of corrsponding sides.

[$\Delta PQR = \Delta EDF$ (given)]

Q.10. In ΔPQR, $\angle P = 70?$ and $\angle R = 30?$, Which side of this triangle is the longest? Give reason for your answer.

Ans. $\angle P = 70°$ (Given)

$\angle R = 30°$ (Given)

$\therefore \angle Q = 180° - (\angle P + \angle R)$

$= 180° - (70° + 30°) = 80°$

Hence, PR > PQ and PR > QR.

[Side opposite to greater angle is longer]

i.e., PR is the longest side.

Multiple Choice Questions

Q.1. Which of the following is not a criterion for congruency of triangles?

(a) SAS (b) ASA
(c) SSA (d) SSS

Ans. (c)

Q.2. In triangle ABC and DEF, $\angle A = \angle D$, $\angle B = \angle E$ and AB = EF, then are the two triangles congruent ? If yes, by which congruency criterion?

(a) Yes, by AAS (b) No
(c) Yes, by ASA (d) Yes, by RHS

Ans. (b)

Q.3. Two sides of a triangle are of lengths 7 cm and 3.5 cm. The length of the third side of the triangle cannot be

(a) 3.6 cm (b) 4.1 cm
(c) 3.4 cm (d) 3.8 cm

Ans. (c)

Q.4. If $\Delta ABC \cong \Delta DEF$ by SSS congruence rule then:

(a) AB = EF, BC = FD, CA = DE
(b) AB = FD, BC = DE, CA = EF
(c) AB = DE, BC = EF, CA = FD
(d) AB = DE, BC = EF, DC = DE

Ans. (c)

Q.5. If $\Delta ABC = \Delta PQR$ and ΔABC is not congruent to ΔRPQ, then which of the following is not true :

(a) BC = PQ
(b) AC = PR
(c) QR = BC
(d) AB = PQ

Ans. (A)

Q.6. Among the following which is not a criterion for congruency of two triangles?

(a) SSA
(b) SAS
(c) ASA
(d) SSS

Ans. (a)

Q.7. In Triangle ABC and DEF, AB = FD and $\angle A = \angle D$. The two triangles will be congruent by SAS axiom if:

(a) BC = EF
(b) AC = DE
(c) AC = EF
(d) BC = DE

Ans. (b)

Q.8. If the sides of a triangle are produced in order, then the sum of the exterior angles so formed is equal to :
(a) 90°
(b) 180°
(c) 270°
(d) 360°

Ans. 360°

Q.9. In right triangle DEF if ∠E = 90°, then:
(a) DF is the shortest side
(b) DF is the longest side
(c) EF is the longest side
(d) DE is the longest side

Ans. (b)

Q.10. In ΔABC, ∠C = ∠A and BC = 6 cm and AC = 5 cm. Then the length of AB is :
(a) 6 cm
(b) 5 cm
(c) 3 cm
(d) 2.5 cm

Ans. (a)

Q.11. If E is a point of SQ of an isosceles ΔPQR such that PQ = PR and PE bisects ∠QPR, then :
(a) QE = ER
(b) QP > QE
(c) QE > QP
(d) ER > RP

Ans. (a)

Q.12. In ΔPQR, ∠P = 60° and ∠Q = 50°. Which side of the triangle is the longest ?
(a) PQ (b) QR
(c) PR (d) None

Ans. (a)