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Factors and Multiples

Factors:

An integer that divides into another integer exactly is called a **Factor**.

Example 1:

2 is a factor of 8, because 2 divide evenly into 8.

Example 2:

$15 \times 4 = 60$ implies that 15 and 4 are the factors of 60.

Example 3: What are the factors of 30?

Solution:

Step 1: The number 30 can be written as 5×6.
Step 2: $5 \times 6 = 5 \times 3 \times 2$
Step 3: So, the factors of 30 are 5, 3, and 2 or $30 = 5 \times 3 \times 2$.

Multiple:

The product of a number with any integer is called the **multiple** of that number. Or If a number p is multiplied with an integer q, then its multiple (product) n is given as $n = p \times q$

Example 1:

Find the first seven multiples of 3.

Solution:

Step 1: A multiple of a number is the product of the number with an integer.

Step 2:
$1 \times 3 = 3$
$2 \times 3 = 6$
$3 \times 3 = 9$
$4 \times 3 = 12$
$5 \times 3 = 15$
$6 \times 3 = 18$
$7 \times 3 = 21$

Step 3: So, the first seven multiples of 3 are 3, 6, 9, 12, 15, 18, and 21.

Example 2:

What are the multiples of 6 and 4?

Solution:

The multiples of 6 are 6, 12, 18, 24, 30, 36 . . .
The multiples of the number 4 are 4, 8, 12, 16, 20, 24...

Let us see what we conclude about factors and multiples:

1. Is there any number which occurs as a factor of every number? Yes. It is 1.

For example:

6 = 1 × 6,
18 = 1 × 18
We say **1 is a factor of every number.**

2. Can 7 be a factor of itself? Yes. You can write 7 as 7 = 7 × 1. You will find that every number can be expressed in this way. We say that **every number is a factor of itself.**

3. What are the factors of 16? They are 1, 2, 4, 8, and 16. Out of these factors do you find any factor which does not divide 16? NO. You will find that **every factor of a number is an exact divisor of that number.**

4. What are the factors of 34? They are 1, 2, 17 and 34. Out of these which are the greatest factor? It is 34. The other factors 1, 2 and 17 are less than 34.
We say that **every factor is less than or equal to the given number.**

5. The number 76 has 5 factors. How many factors do 136 or 96 have? You will find that you are able to count the number of factors of each of these. Even if the numbers are as large as 10576, 25642 etc. or larger, you can still count the number of factors of such numbers, (though you may find it difficult to factorize such numbers).
We say that **number of factors of a given number is finite.**

6. What are the multiples of 7? Obviously, 7, 14, 21, 28... You will find that each of these multiples is greater than or equal to 7. Will it happen with each number? Check this for the multiples of 6, 9 and 10. We find that **every multiple of a number is greater than or equal to that number.**

7. Write the multiples of 5. They are 5, 10, 15, 20 ... Do you think this list will end anywhere? No! The list is endless. Try it with multiples of 6, 7 etc.

We find that **the number of multiples of a given number is infinite.**

8. Can 7 be a multiple of itself? Yes, because 7 = 7×1. Will it be true for other numbers also? Yes. Try it with 3, 12 and 16. You will find that **every number is a multiple of itself.**

Perfect Number:

A number for which sum of all its factors is equal to twice the number is called a perfect number.

For example: Consider the number 6. The proper divisors of 6 are 1, 2, and 3. Sum of these divisors = 1 + 2 + 3 = 6. As the sum of the divisors is 6 and the number is also 6, so 6 is a perfect number.

Example: Is 28 a perfect number?

Solution: Yes.

Step 1: A perfect number is equal to the sum of all its factors.

Step 2: The factors of 28 are 1, 2, 4, 7, and 14.

Step 3: Sum of the factors = 1 + 2 + 4 + 7 + 14 = 28

Step 4: So, according to the definition, 28 is a perfect number.

Types of Number

Prime number:

A prime number is a positive integer that has exactly two factors, 1 and the number itself.

Example:

2, 3, 5, 7, 11, 13, 17, 19, etc. are all prime numbers. There are infinitely many prime numbers.

Example: Is 41 a prime number?

Solution: Yes.
Because the only divisors of 41 are 1 and 41 so, 41 is a prime number.

Composite Number:

A whole number that has factors other than 1 and the number itself is a Composite Number

For Example:

4, 6, 9, 15, 32, 45 are some examples of composite numbers.

Example:

Is 121 a composite number?

Solution:

Step 1: Here, only 121 has factors other than 1 and itself.
Step 2: The factors of 121 are 1, 11, and 121. So, 121 is a composite number.

Note: 1 is neither a prime nor a composite number.

Even and Odd numbers

Even number:

An Even Number is a number that is divisible by 2. For example: {0, 2, 4, 6, 8 . . .}

A number with 0, 2, 4, 6, and 8 at the ones place is an even number.
So, 350, 4862, 59246 are even numbers.

Odd Number:

An odd number is a whole number that is not divisible by 2. For example: {1, 3, 5, 7, 9 . . .} is the set of odd numbers.

An odd number is a whole number that has 1, 3, 5, 7, and 9 in the ones place. The following are few examples of odd numbers. 27, 55, 89, 45, 99, 221, 999, 100, 375

Let us try to find some interesting facts:

(a) Which is the smallest even number? It is 2. Which is the smallest prime number? It is again 2.
Thus, **2 is the smallest prime number which is even.**

(b) The other prime numbers are 3, 5, 7, 11, 13 . . . Do you find any even number in this list? Of course not, they are all odd.
Thus, we can say that **every prime number except 2 is odd.**

Tests for Divisibility of Numbers

Divisibility by 10:

If a number has 0 in the ones place then it is divisible by 10.

Example:

1,470 is divisible by 10 since the last digit is 0.

Divisibility by 5:

A number which has either 0 or 5 in its ones place is divisible by 5.

Example:

195 is divisible by 5 since the last digit is 5.

Divisibility by 2:

A number is divisible by 2 if it has any of the digits 0, 2, 4, 6 or 8 in its ones place.

Example:

168 is divisible by 2 since the last digit is 8.

Divisibility by 3:

If the sum of the digits is a multiple of 3, then the number is divisible by 3.

Example:

168 is divisible by 3 since the sum of the digits is 15 (1+6+8=15), and 15 is divisible by 3.

Divisibility by 6:

If a number is divisible by 2 and 3 both then it is divisible by 6 also.

Example:

168 is divisible by 6 since it is divisible by 2 and 3 both.

Divisibility by 4:

A number with 3 or more digits is divisible by 4 if the number formed by its last two digits (i.e. ones and tens) is divisible by 4.

Example:

316 is divisible by 4 since 16 is divisible by 4.

Divisibility by 8:

A number with 4 or more digits is divisible by 8, if the number formed by the last three digits is divisible by 8.

Example:

7120 is divisible by 8 since 120 is divisible by 8.

Divisibility by 9:

If the sum of the digits of a number is divisible by 9, then the number itself is divisible by 9.

Example:

549 is divisible by 9 since the sum of the digits is 18 (5+4+9=18), and 18 is divisible by 9.

Divisibility by 11:

First find the difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) of the number.
If the difference is either 0 or divisible by 11, then the number is divisible by 11.

Example:

Is 60258 divisible by 11?

Solution:
Sum of odd places, 6 + 2 + 8 = 16
Sum of even places, 0 + 5 = 5
Difference of 16-5 = 11
So, 60258 is divisible by 11.

<u>**Common Factors and Common Multiples**</u>

Common Factors:

A Common Factor is a number that divides two or more numbers exactly.

Example:

Find the common factor of 6 and 8?

The factors of 6 are 1, 2, 3, and 6.

The factors of 8 are 1, 2, 4, and 8.

So, the common factors of 6 and 8 are 1, 2.

Example:

Find all the common factors of 16, 28, and 32.

Solution: We have,

$16 = 1 \times 16$,
$16 = 2 \times 8$,
$16 = 4 \times 4$
The factors of 16 are 1, 2, 4, 8, and 16.

$28 = 1 \times 28$,
$28 = 2 \times 14$,
$28 = 4 \times 7$
The factors of 28 are 1, 2, 4, 7, 14, and 28.

$32 = 1 \times 32$,
$32 = 2 \times 16$,
$32 = 4 \times 8$
The factors of 32 are 1, 2, 4, 8, 16, and 32.
So, the common factors of 16, 28, and 32 are 1, 2, and 4.

Common Multiple:

A common multiple is a number that is a multiple of two or more other numbers.

For Example:

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48…

Multiples of 8 are 8, 16, 24, 32, 40, 48…
So, the common multiples of 6 and 8 are 24, 48.

Example:

Find the common multiples of 5 and 10.

Solution:

The multiples of 5 are 5, 10, 15, 20, 25…

The multiples of 10 are 10, 20, 30, 40, 50…

We observe that all the multiples of 10 are also the multiples of 5.Therefore, 10, 20, 30, 40. . . are the common multiples of 5 and 10.

Co-prime number:

Two numbers having only 1 as a common factor are called co-prime numbers.

For Example:

4 and 15 are co-prime numbers.

Some More Divisibility Rules

Let us observe a few more rules about the divisibility of numbers:

(i) Can you give a factor of 18? It is 9. Name a factor of 9? It is 3. Is 3 a factor of 18? Yes it is. Take any other factor of 18, say 6. Now, 2 is a factor of 6 and it also divides 18. Check this for the other factors of 18. Consider 24. It is divisible by 8 and the factors of 8 i.e. 1, 2, 4 and 8 also divide 24. So, we may say that **if a number is divisible by another number then it is divisible by each of the factors of that number.**

(ii) The number 80 is divisible by 4 and 5. It is also divisible by 4 × 5 = 20, and 4 and 5 are co-primes. Similarly, 60 is divisible by 3 and 5 which are co-primes. 60 is also divisible by 3 × 5 = 15. **If a number is divisible by two co-prime numbers then it is divisible by their product also.**

(iii) The numbers 16 and 20 are both divisible by 4. The number 16 + 20 = 36 is also divisible by 4. Check this for other pairs of numbers.
If two given numbers are divisible by a number, then their sum is also divisible by that number.

(iv) The numbers 35 and 20 are both divisible by 5. Is their difference 35 – 20 = 15 also divisible by 5? Try this for other pairs of numbers also. **If two given numbers are divisible by a number, then their difference is also divisible by that number.**

Prime Factorization:

Prime factorization is to write a composite number as a product of its prime factors.

Examples:

Find the prime factorization of 48.

Solution:

The prime factorization of 48 is $2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$.

Example:

Find the prime factorization of 108.

Solution:

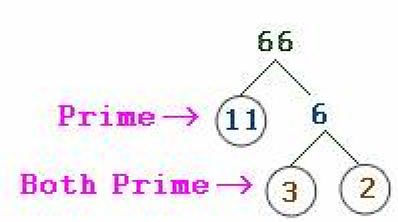
The prime factorization of 108 = $3 \times 3 \times 3 \times 2 \times 2$

Factor Tree:

Factor Tree is a hierarchical structure used to represent the prime factors of a number. All the composite numbers can be written as the product of the prime numbers using the factor tree.

For example:

To find the prime factors of 66, we use here a factor tree.



The circled numbers in the factor tree are the prime factors of 66.

$66 = 11 \times 3 \times 2$.

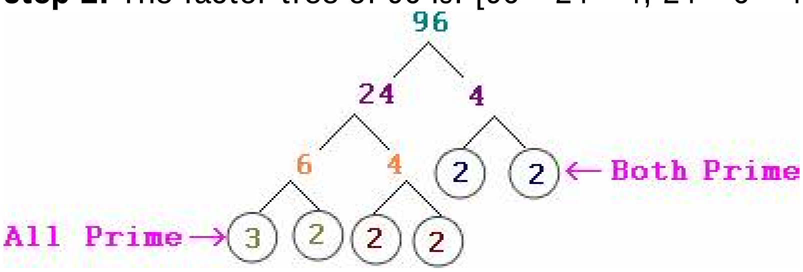
The prime factorization of 66 is $11 \times 3 \times 2$.

Example:

Find the prime factorization of 96.

Solution:

Step 1: The factor tree of 96 is: [96 = 24 × 4, 24 = 6 × 4, 4 = 2 × 2, 6 = 3 × 2]



Step 2: The circled numbers in the factor tree are the prime factors of 96.

Step 3: 96 = 3 × 2 × 2 × 2 × 2 × 2 = 31 × 25.

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Highest Common Factor (HCF):

The Highest Common Factor (HCF) of two or more numbers is the highest number that divides the numbers exactly.

Example:

To find the HCF of 12, 24, and 36, first we list out the factors of the three numbers.

12 – 1, 2, 3, 4, 6, and **12**

24 – 1, 2, 3, 4, 6, 8, **12**, and 24

36 - 1, 2, 3, 4, 6, 8, **12**, 18, and 36

So, the HCF of the numbers 12, 24, and 36 is 12.

In another method, we need to write all the prime factors of the three numbers 12, 24, and 36.

12 – **2** × **2** × **3**
24 – **2** × **2** × 2 × **3**
36 - **2** × **2** × **3** × 3

Then we list out all the common prime factors. The common prime factors of the three numbers are 2 × 2 × 3. Then we have to multiply the common prime factors.
2 × 2 × 3 = 12
so, the HCF of the three numbers 12, 24, and 36 is 12.

Example:
Find the HCF of 18 and 35.

Solution: First we list the factors of 18 and 35.

18 = 1, 2, 3, 6, 9, and 18

35 = 1, 5, 7, and 35

The common factor of 18 and 35 is 1.
So, the HCF of 18 and 35 is 1.

Least Common Multiple (LCM):

Least common multiple is the smallest nonzero number that is a common multiple of two or more numbers considered.

Example:

The LCM of 6, 9, and 15 is 90.

6 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90 . . .

9 = 9, 18, 27, 36, 45, 54, 63, 72, 81, 90 . . .

15 = 15, 30, 45, 60, 75, 90 . . .

Example:

Find the least common multiple of 13 and 11.

Solution:

As 13 and 11 are co-primes; the least common multiple is nothing but their product. So, least common multiple of 13 and 11 is 143.

Some Problems on HCF and LCM

Example:

Two tankers contain 850 litres and 680 litres of kerosene oil respectively. Find the maximum capacity of a container which can measure the kerosene oil of both the tankers when used an exact number of times.

Solution:

The required container has to measure both the tankers in a way that the count is an exact number of times. So its capacity must be an exact divisor of the capacities of both the tankers. Moreover, this capacity should be maximum Thus; the maximum capacity of such a container will be the HCF of 850 and 680.

$850 = 2 \times 5 \times 5 \times 17 = 2 \times 5 \times 17 \times 5$ and

$680 = 2 \times 2 \times 2 \times 5 \times 17 = 2 \times 5 \times 17 \times 2 \times 2$

The common factors of 850 and 680 are 2, 5 and 17.

Thus, the HCF of 850 and 680 is $2 \times 5 \times 17 = 170$.

Therefore, maximum capacity of the required container is 170 litres. It will fill the first container in 5 and the second in 4 refills.

Example:

In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

Solution:

The distance covered by each one of them is required to be the same as well as minimum. The required minimum distance each should walk would be the lowest common multiple of the measures of their steps. Can you describe why? Thus, we find the LCM of 80, 85 and 90. The LCM of 80, 85 and 90 is 12240.
The required minimum distance is 12240 cm.

Example:

Find the least number which when divided by 12, 16, 24 and 36 leaves a remainder 7 in each case.

Solution:

We first find the LCM of 12, 16, 24 and 36 as follows:
LCM of 12, 16, 24, 36 = $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$
144 is the least number which when divided by the given numbers will leave remainder 0 in each case. But we need the least number that leaves remainder 7 in each case.
Therefore, the required number is 7 more than 144. The required least number = $144 + 7 = 151$.