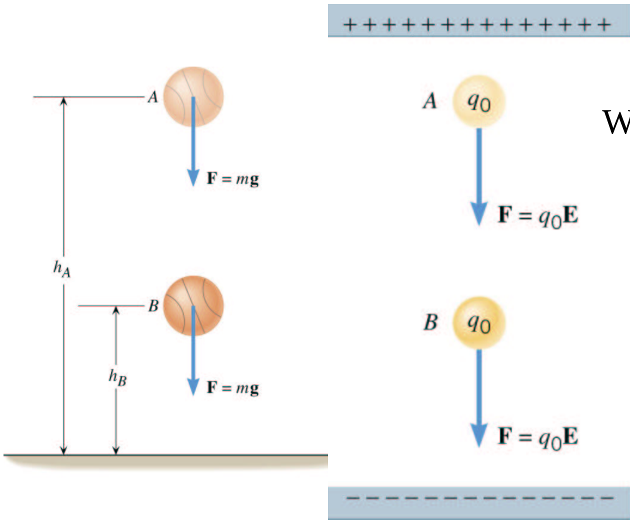
Electrical Potential Energy and Electric Potential

1 – Potential Energy and Electric Potential

The Coulomb force law is of the same form as the universal law of gravity $\rightarrow$ the electrostatic force is a **conservative** force.

Example: A small positive test charge, q_0, in a uniform electric field $\mathbf{E}$:



Work done by gravity:

$$W_{AB} = F \cdot s = mg(h_A - h_B)$$

Work done by constant elec. force:

$$W_{AB} = F \cdot s = |q_0| E \cdot s$$

Work=Change in PE:

$$W_{AB} = -\Delta GPE = GPE_A - GPE_B$$

$$W_{AB} = -\Delta EPE = EPE_A - EPE_B$$

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- A positive charge gains electric potential energy when it is moved in a direction opposite the electric field.
- A negative charge loses electric potential energy when it is moved in a direction opposite the electric field.

Energy Conservation: $W_{nc} = 0J \Rightarrow \Delta E = E_f - E_0 = 0J$

The total energy, E, of the universe does not change. Energy can only be transformed between different forms of energy but can never be destroyed or created out of nothing.

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh + \frac{1}{2}kx^2 + EPE \qquad (1)$$

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Electric Potential:

The potential difference between points **A** and **B**, $\Delta V = V_B - V_A$, is defined as the change in electric potential energy of a charge, q_0, moved from **A** to **B**, divided by the charge.

$$\Delta V = V_B - V_A = \frac{\Delta EPE}{q_0} = \frac{-W_{AB}}{q_0} \quad (2)$$

SI Unit: Volt: $1\ V = 1\ J/C$ ($1eV = 1.6 \times 10^{-19}$ J)

Note: Only potential differences can be measured, not the absolute values.

2 – Electric Potential due to a Point Charge

It is possible to define the potential of a point charge at a point in space.
Reference point: Point of zero potential for a point charge: ∞

Electric potential due to a point charge, q, at a distance r from the charge:

$$V = k\,\frac{q}{r} \quad (3)$$

- V only depends on q and r.
- The electric potential of two or more charges is obtained by applying the **superposition principle**: *The total potential at a point P is the sum of the potentials due to the individual point charges present*

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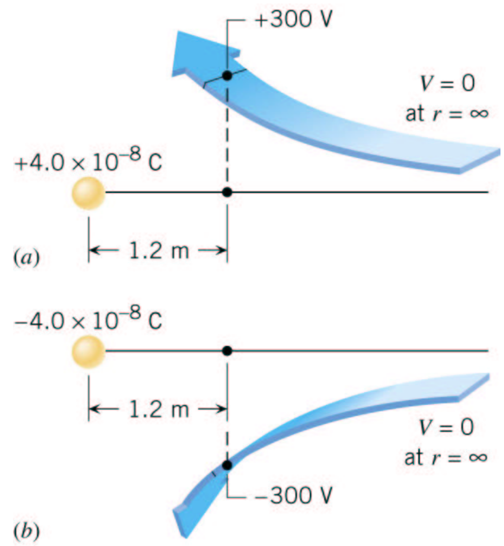

$q > 0 \rightarrow V > 0$, i.e. the potential has been raised with respect to the zero reference value.

$q < 0 \rightarrow V < 0$, i.e. the potential has decreased with respect to the zero reference value.

Electrical Potential Energy and Electric Potential

Electric Potential Energy of two point charges q_1 and q_2:

1. First, the charge q_1 is moved from ∞ to point **A**. Since there is no other charge present, no electric work is done and thus $EPE_{A1} = 0$.

2.Then, the charge q_2 is moved from ∞ to point **B** which is a distance r apart from **A.** Now electric work must be done due to the electric potential created by charge q_1: $W_{\infty B} = -q_2 V_1$ and thus $EPE_{B2} = q_2 V_1$.

$$EPE = EPE_{A1} + EPE_{B2} = 0 + q_2 V_1 = q_1 V_2 + 0 = k\,\frac{q_1 q_2}{r} \qquad (4)$$

- If the two charges have the same sign, EPE is positive (like charges repel, so positive work must be done to bring the two charges near one another)

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3 – Potentials and Charged Conductors

Relation between electric work and electric potential when a charge q_0 is moved from point **A** to **B**:

$W_{AB} = -\Delta EPE$, $\Delta EPE = q_0(V_B - V_A) \rightarrow$

$$W_{AB} = -q_0(V_B - V_A) \tag{5}$$

- **No work ($W = 0$) is required to move a charge between two points that are at the same potential ($V_B = V_A$)**

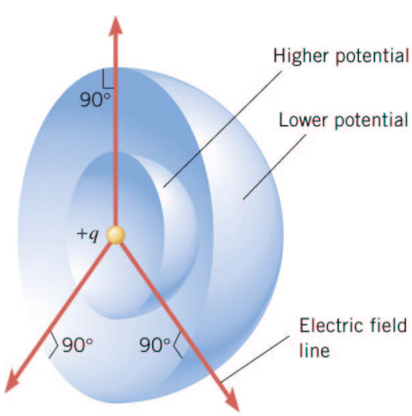
  On a charged conductor, **E** is perpendicular to its surface, and **ZERO** inside $\rightarrow$ no work is done if a charge is moved $\rightarrow$

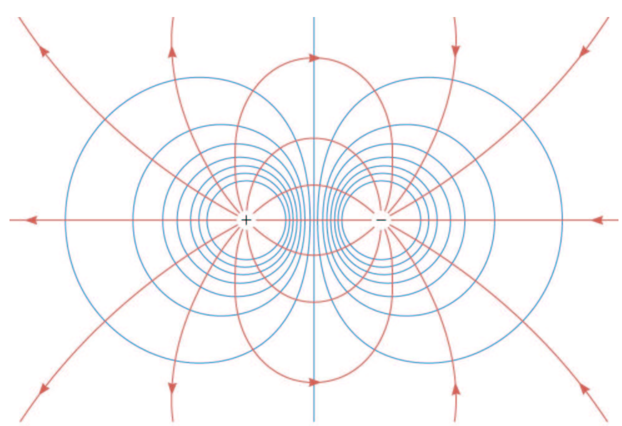
- **The electric potential is a constant everywhere on the surface of a charged conductor**

  Furthermore, it is constant everywhere inside a conductor, and equal to its value at the surface.

Electrical Potential Energy and Electric Potential

- An area on which all points are at the same potential is called an **equipotential surface**.
- The electric field **E** at every point of an equipotential surface is perpendicular to the surface.

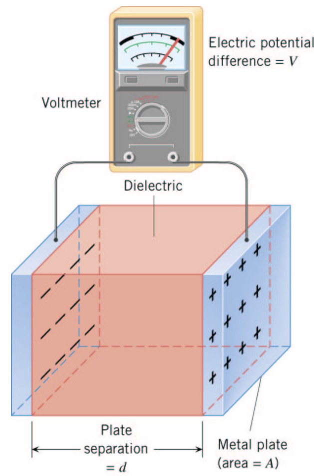




4 – Capacitors and Dielectrics

A capacitor is a device to store electric charge (electric energy) and is used in a variety of electric circuits.

A capacitor consists of two conductors placed near to each other without touching:



Definition of **capacitance**:

$$C = \frac{q}{V} \tag{6}$$

SI unit: 1 F $= 1\ C/V$ (Farad)

5 – The Parallel Plate Capacitor

The capacitance of a device depends on the geometric arrangement of the conductors. For a parallel plate capacitor whose plates are separated by a vacuum:

$$C = \epsilon_0 \frac{A}{d} \tag{7}$$

ϵ_0 is the **permittivity of free space**: $\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ C^2/N \cdot m^2$
If an insulator (**dielectric**) is placed between the two conductors, the electric field between the conductors decreases by (κ=dielectric constant, see Tab.19.1)

$$E = \frac{E_0}{\kappa} \tag{8}$$

and the capacitance increases, i.e. more charge can be stored per volt.
For a parallel plate capacitor whose plates are separated by a dielectric:

$$C = \kappa \epsilon_0 \frac{A}{d} \tag{9}$$

6 – Energy stored in a Charged Capacitor

1. Start with an uncharged capacitor and move a small charge, Δq on capacitor. Voltage: $\delta V = \delta q / C$
2. As the charge accumulates and the voltage increases: to move a charge δq on capacitor which is at a voltage V, work, $\delta W = V \delta q$, needs to be done.
3. The total work done is
$$W = \frac{1}{2} q V \tag{10}$$
4. The work done is stored as energy:
$$\text{Energy stored} = \frac{1}{2} q V = \frac{1}{2} C V^2 = \frac{q^2}{2C} \tag{11}$$
Parallel plate capacitor (with $V = Ed$):
$$\text{Energy stored} = \frac{1}{2} C V^2 = \frac{1}{2} C (Ed)^2 \tag{12}$$