9 AREAS OF PARALLELOGRAMS AND TRIANGLES

EXERCISE 9.2

Q.1. *In the figure, ABCD is a paralle-logram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.*

Sol. Area of parallelogram ABCD

= AB × AE

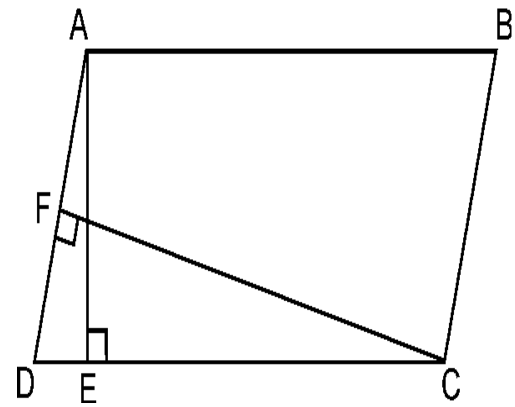
= 16 × 8 cm^2 = 128 cm^2

Also, area of parallelogram ABCD

= AD × FC = (AD × 10) cm^2

∴ AD × 10 = 128

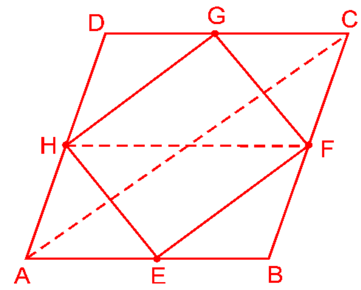
⇒ $AD = \frac{128}{10}$ = **12.8 cm Ans.**



Q.2. *If E, F, G, and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD).*

Sol. **Given :** A parallelogram ABCD · E, F, G, H are mid-points of sides AB, BC, CD, DA respectively

To Porve : ar (EFGH) = $\frac{1}{2}$ ar (ABCD)



Construction : Join AC and HF.

Proof : In ΔABC,
E is the mid-point of AB.
F is the mid-point of BC.

$\Rightarrow$ EF is parallel to AC and $EF = \frac{1}{2} AC$ ... (i)

Similarly, in ΔADC, we can show that

$HG \parallel AC$ and $HG = \frac{1}{2} AC$ ... (ii)

From (i) and (ii)
$EF \parallel HG$ and $EF = HG$
$\therefore$ EFGH is a parallelogram. [One pour of opposite sides is equal and parallel]

In quadrilateral ABFH, we have

$HA = FB$ and $HA \parallel FB$ $\quad [AD = BC \Rightarrow \frac{1}{2} AD = \frac{1}{2} BC \Rightarrow HA = FB]$

$\therefore$ ABFH is a parallelogram. [One pair of opposite sides is equal and parallel]

Now, triangle HEF and parallelogram HABF are on the same base HF and between the same parallels HF and AB.

$\therefore$ Area of $\Delta HEF = \frac{1}{2}$ area of HABF ... (iii)

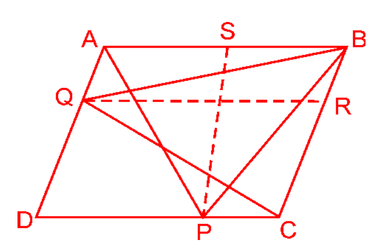
Similarly, area of $\Delta HGF = \frac{1}{2}$ area of HFCD ... (iv)

Adding (iii) and (iv),
Area of ΔHEF + area of ΔHGF

$= \frac{1}{2}$ (area of HABF + area of HFCD)

$\Rightarrow$ ar (EFGH) $= \frac{1}{2}$ ar (ABCD) **Proved.**

Q.3. *P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).*



Sol. **Given :** A parallelogram ABCD. P and Q are any points on DC and AD respectively.

To prove : ar (APB) = ar (BQC)

Construction : Draw $PS \parallel AD$ and $QR \parallel AB$.

Proof : In parallelogram ABRQ, BQ is the diagonal.

$\therefore$ area of $\Delta BQR = \frac{1}{2}$ area of ABRQ ... (i)

In parallelogram CDQR, CQ is a diagonal.

$\therefore$ area of $\Delta RQC = \frac{1}{2}$ area of CDQR ... (ii)

Adding (i) and (ii), we have

area of ΔBQR + area of ΔRQC

$= \frac{1}{2}$ [area of ABRQ + area of CDQR]

$\Rightarrow$ area of $\Delta BQC = \frac{1}{2}$ area of ABCD ... (iii)

Again, in parallelogram DPSA, AP is a diagonal.

$\therefore$ area of $\Delta ASP = \frac{1}{2}$ area of DPSA ... (iv)

In parallelogram BCPS, PB is a diagonal.

$\therefore$ area of $\Delta BPS = \frac{1}{2}$ area of BCPS ... (v)

Adding (iv) and (v)

area of ΔASP + area of $\Delta BPS = \frac{1}{2}$(area of DPSA + area of BCPS)

$\Rightarrow$ area of $\Delta APB = \frac{1}{2}$ (area of ABCD) ... (vi)

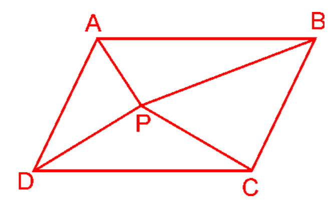
From (iii) and (vi), we have

area of ΔAPB = area of ΔBQC. **Proved.**

Q.4. *In the figure, P is a point in the interior of a parallelogram ABCD. Show that*

(i) ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC) = ar(APB) + ar (PCD)



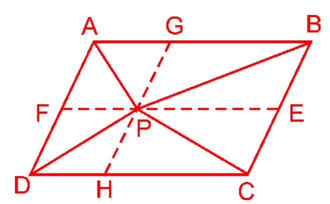
Sol. Given : A parallelogram ABCD. P is a point inside it.

To prove : (i) ar (APB) + ar(PCD)

$= \frac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC)

= ar (APB) + ar (PCD)



Construction : Draw EF through P parallel to AB, and GH through P parallel to AD.

Proof : In parallelogram FPGA, AP is a diagonal,

$\therefore$ area of ΔAPG = area of ΔAPF ... (i)

In parallelogram BGPE, PB is a diagonal,

$\therefore$ area of ΔBPG = area of ΔEPB ... (ii)

In parallelogram DHPF, DP is a diagonal,

∴ area of ΔDPH = area of ΔDPF ... (iii)
In parallelogram HCEP, CP is a diagonal,
∴ area of ΔCPH = area of ΔCPE ... (iv)
Adding (i), (ii), (iii) and (iv)
area of ΔAPG + area of ΔBPG + area of ΔDPH + area of ΔCPH
= area of ΔAPF + area of ΔEPB + area of ΔDPF + area ΔCPE
⇒ [area of ΔAPG + area of ΔBPG] + [area of ΔDPH + area of ΔCPH]
= [area of ΔAPF + area of ΔDPF] + [area of ΔEPB + area of ΔCPE]
⇒ area of ΔAPB + area of ΔCPD = area of ΔAPD + area of ΔBPC
... (v)
But area of parallelogram ABCD
= area of ΔAPB + area of ΔCPD + area of ΔAPD + area of ΔBPC
... (vi)
From (v) and (vi)

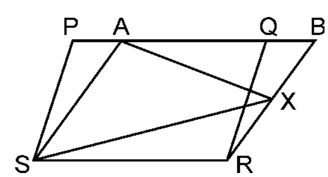
area of ΔAPB + area of ΔPCD = $\frac{1}{2}$ area of ABCD

or, ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD) **Proved.**

(ii) From (v),
⇒ ar (APD) + ar (PBC) = ar (APB) + ar (CPD) **Proved.**

Q.5. *In the figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that*
(i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ *ar (PQRS)*



Sol. Given : PQRS and ABRS are parallelograms and X is any point on side BR.
To prove : (i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ ar (PQRS)

Proof : (i) In ΔASP and BRQ, we have
∠SPA = ∠RQB [Corresponding angles] ...(1)
∠PAS = ∠QBR [Corresponding angles] ...(2)
∴ ∠PSA = ∠QRB [Angle sum property of a triangle] ...(3)
Also, PS = QR [Opposite sides of the parallelogram PQRS] ...(4)
So, ΔASP ≅ ΔBRQ [ASA axiom, using (1), (3) and (4)]
Therefore, area of ΔPSA = area of ΔQRB
[Congruent figures have equal areas] ...(5)
Now, ar (PQRS) = ar (PSA) + ar (ASRQ]
= ar (QRB) + ar (ASRQ]
= ar (ABRS)
So, ar (PQRS) = ar (ABRS) **Proved.**

(ii) Now, ΔAXS and ||gm ABRS are on the same base AS and between same parallels AS and BR

$\therefore$ area of $\Delta AXS = \frac{1}{2}$ area of ABRS

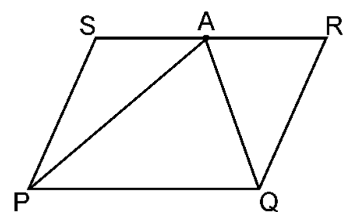
$\Rightarrow$ area of $\Delta AXS = \frac{1}{2}$ area of PQRS [$\because$ ar (PQRS) = ar (ABRS]

$\Rightarrow$ ar of (AXS) = $\frac{1}{2}$ ar of (PQRS) **Proved.**

Q.6. *A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?*

Sol. The field is divided in three triangles.

Since triangle APQ and parallelogram PQRS are on the same base PQ and between the same parallels PQ and RS.



$\therefore$ ar (APQ) = $\frac{1}{2}$ ar (PQRS)

$\Rightarrow$ 2ar (APQ) = ar(PQRS)

But ar (PQRS) = ar(APQ) + ar (PSA) + ar (ARQ)

$\Rightarrow$ 2 ar (APQ) = ar(APQ) + ar(PSA) + ar (ARQ)

$\Rightarrow$ ar (APQ) = ar(PSA) + ar(ARQ)

Hence, area of ΔAPQ = area of ΔPSA + area of ΔARQ.

To sow wheat and pulses in equal portions of the field separately, farmer sow wheat in ΔAPQ and pulses in other two triangles or pulses in ΔAPQ and wheat in other two triangles. **Ans.**