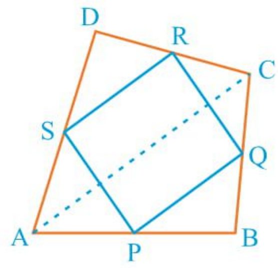
Exercise 8.2

Question 1:

ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Figure). AC is a diagonal. Show that :

(i) SR || AC and SR = $\frac{1}{2}$AC **(ii)** PQ = SR

(iii) PQRS is a parallelogram.



Answer 1:

(i) In ΔACD,

S is mid-point of DA. [∵ Given]

R is mid-point of DC [∵ Given]

Hence, SR || AC and SR = $\frac{1}{2}$AC ... (1) [∵ Mid Point Theorem]

(ii) In ΔACD,

P is mid-point of AB [∵ Given]

Q is mid-point of BC [∵ Given]

Hence, PQ || AC and PQ = $\frac{1}{2}$AC ... (2) [∵ Mid Point Theorem]

From (1) and (2), we have

PQ || SR ... (3) [∵ PQ || AC and SR || AC]

and PQ = SR ... (4) [∵ SR = $\frac{1}{2}$AC and PQ = $\frac{1}{2}$AC]

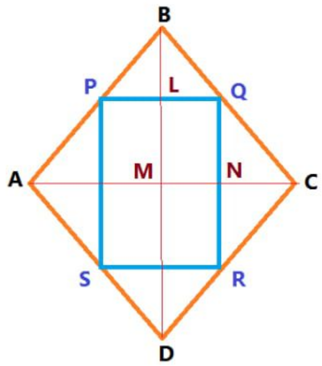
(iii) In PQRS,

PQ || SR and PQ = SR [∵ From (3) and (4)]

Hence, PQRS is a parallelogram.

Question 2:

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.



Answer 2:

In ΔABC,

P is mid-point of AB [∵ Given]

Q is mid-point of BC [∵ Given]

Hence, PQ || AC and PQ = $\frac{1}{2}$AC ... (1) [∵ Mid Point Theorem]

Similarly, in ΔACD,

S is mid-point of AD [∵ Given]

R is mid-point of CD [∵ Given]

Hence, SR || AC and SR = $\frac{1}{2}$AC ... (2) [∵ Mid Point Theorem]

From (1) and (2), we have

PQ || SR ... (3) [∵ PQ || AC and SR || AC]

and PQ = SR ... (4) [∵ SR = $\frac{1}{2}$AC and PQ = $\frac{1}{2}$AC]

Hence, PQRS is a parallelogram.

Similarly, in ΔBCD,

Q is mid-point of BC [∵ Given]

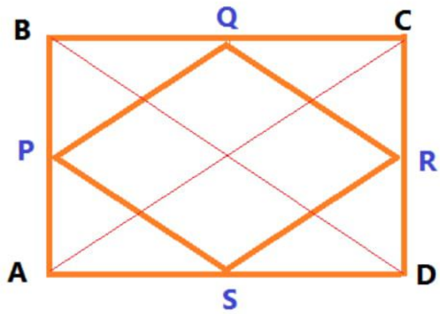
R is mid-point of CD [∵ Given]

Hence, QR || BD [∵ Mid Point Theorem]
⇒ QN || LM ... (5)
and, LQ || MN ... (6) [∵ PQ || AC]
From (5) and (6), we have
LMNQ is a parallelogram.
Hence, ∠LMN = ∠LQN [∵ Opposite angles of a parallelogram]
But, ∠LMN = 90° [∵ Digonals of a rhombus are perpendicular to each other]
Hence, ∠LQN = 90°
A parallelogram whose one angle is right angle, is a rectangle. Hence, PQRS is a rectangle.

Question 3:

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Answer 3:



In ΔABC,
P is mid-point of AB [∵ Given]
Q is mid-point of BC [∵ Given]
Hence, PQ || AC and $PQ = \frac{1}{2}AC$ ... (1) [∵ Mid Point Theorem]

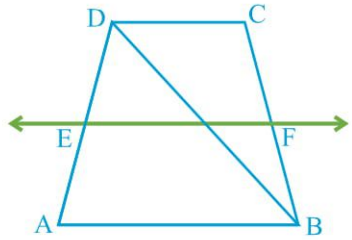
Similarly, in ΔACD,
S is mid-point of AD [∵ Given]
R is mid-point of CD [∵ Given]
Hence, SR || AC and $SR = \frac{1}{2}AC$ ... (2) [∵ Mid Point Theorem]

From (1) and (2), we have
PQ || SR ... (3) [∵ PQ || AC and SR || AC]
and PQ = SR ... (4) [∵ $SR = \frac{1}{2}AC$ and $PQ = \frac{1}{2}AC$]
Hence, PQRS is a parallelogram.

Similarly, in ΔBCD,
Q is mid-point of BC [∵ Given]
R is mid-point of CD [∵ Given]
Hence, $QR = \frac{1}{2}BD$ ... (5) [∵ Mid Point Theorem]
Given that: AC = BD ... (6) [∵ Diagonals of a rectangle are equal]
From (1), (5) and (6), we have
PQ = QR
A parallelogram whose adjacent sides are equal, is a rhombus. Hence PQRS is a rhombus.

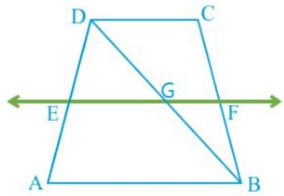
Question 4:

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Figure). Show that F is the mid-point of BC.



Answer 4:

In ΔABD,
E is mid-point of AD [∵ Given]
and EG || AB [∵ Given]
Hence, G is mid-point of BD [∵ Converse of Mid-Point Theorem]



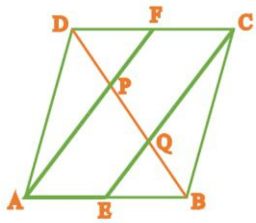
Similarly,
In ΔBCD,
G is mid-point of BD [∵ Proved above]
and FG || DC [∵ Given]
Hence, F is mid-point of BC [∵ Converse of Mid-Point Theorem]

Question 5:

In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Figure). Show that the line segments AF and EC trisect the diagonal BD.

Answer 5:

In quadrilateral ABCD,
AB = CD [∵ Given]
$\frac{1}{2}$AB = $\frac{1}{2}$ CD
⇒AE = CF [∵ E and F are the mid-points of AB and CD respectively]
In quadrilateral AECF,
AE = CF [∵ Proved above]
AE || CF [∵ Opposite sides of a parallelogram]
Hence, AECF is a parallelogram.



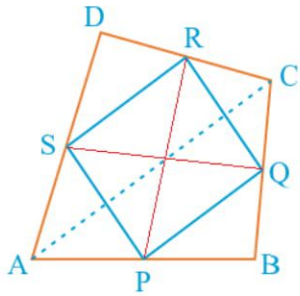
In ΔDCQ,
F is mid-point of DC [∵ Given]
and FP || CQ [∵ AECF is a parallelogram]
Hence, P is mid-point of DQ [∵ Converse of Mid-Point Theorem]
Hence, DP = PQ ... (1)
Similarly,
In ΔABP,
E is mid-point of AB [∵ Given]
and EQ || AP [∵ AECF is a parallelogram]
Hence, Q is mid-point of PB [∵ Converse of Mid-Point Theorem]
Hence, PQ = QB ... (2)
From (1) and (2), we have
DP = PQ = QB
Hence, line segment AF and EC trisect BD.

Question 6:

Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Answer 6:

Given: ABCD is a quadrilateral in which P, Q, R and S are the mid-points of AB, BC, CD and DA respectively.



In ΔACD,
S is mid-point of DA [∵ Given]
R is mid-point of DC [∵ Given]
Hence, SR || AC and SR = $\frac{1}{2}$AC ... (1) [∵ Mid-Point Theorem]

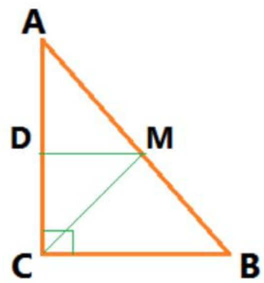
In ΔACD,
P is mid-point of AB [∵ Given]
Q is mid-point of BC [∵ Given]
Hence, PQ || AC and PQ = $\frac{1}{2}$AC ... (2) [∵Mid-Point Theorem]

From (1) and (2), we have
PQ || SR ... (3) [∵ PQ || AC and SR || AC]
and PQ = SR ... (4) [∵ SR = $\frac{1}{2}$AC and PQ = $\frac{1}{2}$AC]
In quadrilateral PQRS,
PQ || SR and PQ = SR [∵ From (3) and (4)]
Hence, PQRS is a parallelogram and diagonals of parallelogram bisects each other.
Therefore, SQ and PR bisects each other.

Question 7:

ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that
(i) D is the mid-point of AC
(ii) MD ⊥ AC
(iii) CM = MA = $\frac{1}{2}$AB

Answer 7:



(i) In ΔABC,
M is mid-point of AB [∵ Given]
and DM || BC [∵ Given]
Hence, D is mid-point of AC [∵ Converse of Mid-Point Theorem]

(ii) ∠ADM = ∠ACB [∵ Corresponding Angles]
⇒∠ADM = 90° [∵ ∠ACB 90°]
Hence, MD ⊥ AC

(iii) In ΔAMD and ΔCMD,
AD = D C [∵ Proved above]
∠ADM = ∠CDM [∵ Each 90°]
DM = DM [∵ Common]
Hence, ΔAMD ≅ ΔCMD [∵ SAS Congruency rule]
AM = CM [∵ CPCT]
But AM = $\frac{1}{2}$AB [∵ Given]
Therefore, CM = AM = $\frac{1}{2}$AB