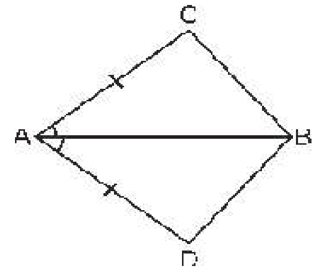
7 TRIANGLES

EXERCISE 7.1

Q.1. *In quadrilateral ACBD, AC = AD and AB bisects ∠A (see Fig.). Show that ΔABC ≅ ΔABD. What can you say about BC and BD?*



Sol. In ΔABC and ΔABD, we have

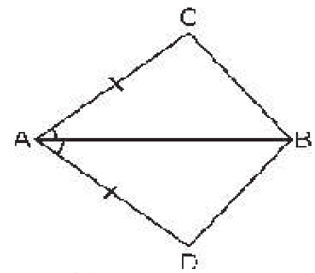
AC = AD [Given]

∠CAB = ∠DAB [Q AB bisects ∠A]

AB = AB [Common]

∴ ΔABC ≅ ΔABD. [By SAS congruence] **Proved.**

Therefore, BC = BD. (CPCT). **Ans.**

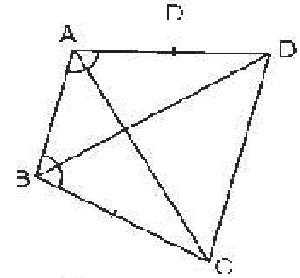


Q.2. *ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (see Fig.). Prove that*

(i) Δ ABD ≅ Δ BAC

(ii) BD = AC

(iii) ∠ ABD = ∠ BAC



Sol. In the given figure, ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA.

In ΔABD and ΔBAC, we have

AD = BC [Given]

∠DAB = ∠CBA [Given]

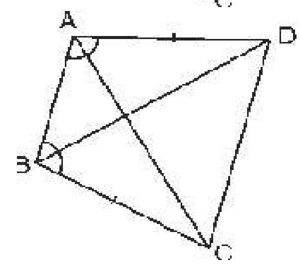
AB = AB [Common]

∴ ΔABD ≅ ΔBAC [By SAS congruence]

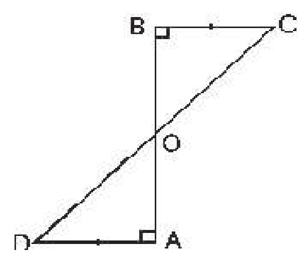
∴ BD = AC [CPCT]

and ∠ABD = ∠BAC [CPCT]

Proved



Q.3. *AD and BC are equal perpendiculars to a line segment AB (see Fig.). Show that CD bisects AB.*



Sol. In ΔAOD and ΔBOC, we have,

∠AOD = ∠BOC [Vertically opposite angles)

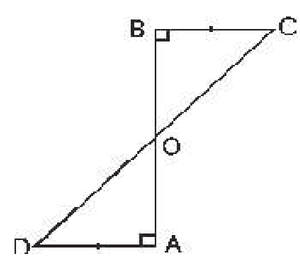
∠CBO = ∠DAO [Each = 90°]

and AD = BC [Given]

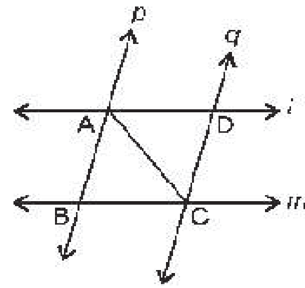
∴ ΔAOD ≅ ΔBOC [By AAS congruence]

Also, AO = BO [CPCT]

Hence, CD bisects AB **Proved.**



Q.4. *l and m are two parallel lines intersected by another pair of parallel lines p and q (see Fig.). Show that Δ ABC ≅ ΔCDA.*



Sol. In the given figure, ABCD is a parallelogram in which AC is a diagonal, i.e., AB || DC and BC || AD.

In Δ ABC and Δ CDA, we have,

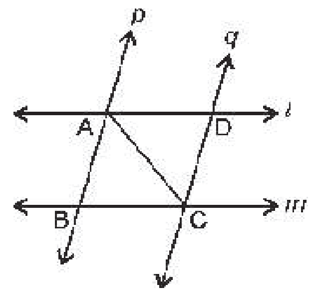
∠BAC = ∠DCA [Alternate angles]

∠BCA = ∠DAC [Alternate angles]

AC = AC [Common]

∴ Δ ABC ≅ Δ CDA [By ASA congruence]

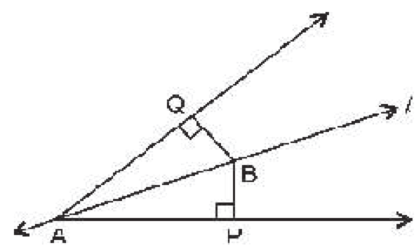
Proved.



Q.5. *Line l is the bisector of an angle A and B is any point on l. BP and BQ are perpendiculars from B to the arms of ∠A (see Fig.). Show that :*

(i) Δ APB ≅ Δ AQB

(ii) BP = BQ or B is equidistant from the arms of ∠A.



Sol. In Δ APB and Δ AQB, we have

∠PAB = ∠QAB [*l* is the bisector of ∠ A]

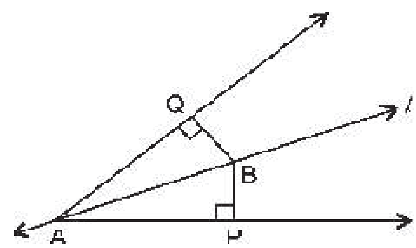
∠APB = ∠AQB [Each = 90°]

AB = AB [Common]

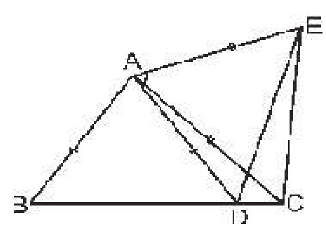
∴ ΔAPB ≅ ΔAQB [By AAS congruence]

Also, BP = BQ [By CPCT]

i.e., B is equidistant from the arms of ∠A. **Proved**



Q.6. *In the figure, AC = AE, AB = AD and ∠ BAD = ∠ EAC. Show that BC = DE.*



Sol. ∠BAD = ∠EAC [Given]

⇒ ∠BAD + ∠DAC = ∠EAC + ∠DAC [Adding ∠DAC to both sides]

⇒ ∠BAC = ∠EAC ... (i)

Now, in ΔABC and ΔADE, we have

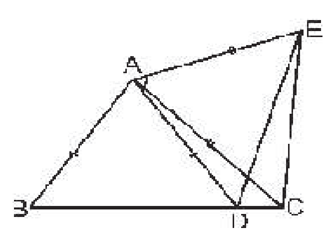
AB = AD [Given]

AC = AE [Given)

⇒ ∠BAC = ∠DAE [From (i)]

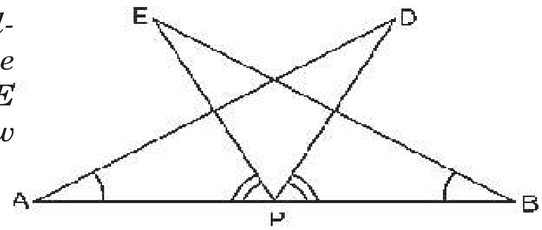
∴ ΔABC ≅ ΔADE [By SAS congruence]

⇒ BC = DE. [CPCT] **Proved.**



Q.7. *AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB (see Fig.). Show that*

(i) *ΔDAP ≅ ΔEBP* (ii) *AD = BE*



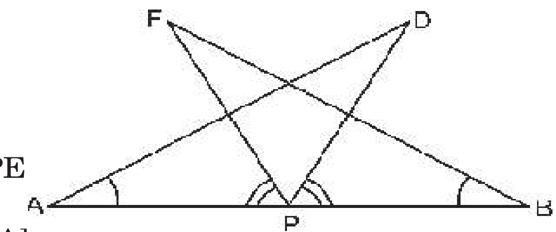
Sol. In ΔDAP and ΔEBP, we have

AP = BP [Q P is the mid-point of line segment AB]

∠BAD = ∠ABE [Given]

∠EPB = ∠DPA

[Q ∠EPA = ∠DPB ⇒ ∠EPA + ∠DPE = ∠DPB + ∠DPE]



∴ Δ DPA ≅ Δ EPB [ASA]

⇒ AD = BE [By CPCT] **Proved.**

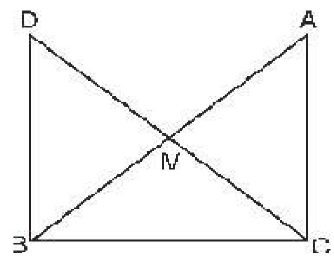
Q.8. *In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig.). Show that :*

(i) *ΔAMC ≅ ΔBMD*

(ii) *∠DBC is a right angle.*

(iii) *ΔDBC ≅ ΔACB*

(iv) *CM* $= \frac{1}{2}$ *AB*



Sol. In ΔBMB and ΔDMC, we have

(i) DM = CM [Given]

BM = AM

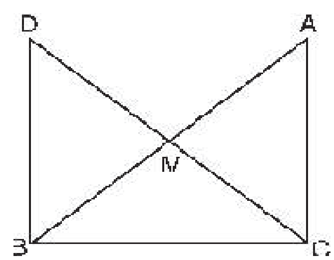
[Q M is the mid-point of AB]

∠DMB = ∠AMC

[Vertically opposite angles]

∴ ΔAMC ≅ ΔBMD [By SAS]

Proved.



(ii) AC || BD [Q ∠DBM and ∠CAM are alternate angles]

⇒ ∠DBC + ∠ACB = 180° [Sum of co-interior angles]

[Q ∠ACB = 90°] **Proved.**

⇒ ∠DBC = 90° **Proved.**

(iii) In ΔDBC and ΔACB, we have

DB = AC [CPCT]

BC = BC [Common]

∠DBC = ∠ACB [Each = 90°]

∴ ΔDBC ≅ ΔACB [By SAS] **Proved.**

(iv) ∴ AB = CD [CPCT]

⇒ $\frac{1}{2}$AB = $\frac{1}{2}$CD

Hence, $\frac{1}{2}$AB = CM [CM = $\frac{1}{2}$CD] **Proved.**