Exercise 2.3

Question 1:

Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by:

(i) $x + 1$ **(ii)** $x - \frac{1}{2}$ **(iii)** x **(iv)** $x + \pi$ **(v)** $5 + 2x$

Answer 1:

Let $p(x) = x^3 + 3x^2 + 3x + 1$

(i) $x + 1$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $x + 1$, remainder is given by $p(-1)$

$= (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$= -1 + 3 - 3 + 1$

$= 0$

(ii) $x - \frac{1}{2}$

Putting $x - \frac{1}{2} = 0$, we get, $x = \frac{1}{2}$

Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $x - \frac{1}{2}$, remainder is given by $p\left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$

$= \frac{1}{8} + 3 \times \frac{1}{4} + 3 \times \frac{1}{2} + 1$

$= \frac{1 + 6 + 12 + 8}{8}$

$= \frac{27}{8}$

(iii) x

Putting $x = 0$, we get

Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by x, remainder is given by $p(0)$

$= (0)^3 + 3(0)^2 + 3(0) + 1$

$= 0 + 1$

$= 0$

(iv) $x + \pi$

Putting $x + \pi = 0$, we get, $x = -\pi$

Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $x + \pi$, remainder is given by $p(-\pi)$

$= (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$

(v) $5 + 2x$

Putting $5 + 2x = 0$, we get, $x = -\frac{5}{2}$

Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $5 + 2x$, remainder is given by $p\left(-\frac{5}{2}\right)$

$$= \left(-\frac{5}{2}\right)^3 + 3\left(-\frac{5}{2}\right)^2 + 3\left(-\frac{5}{2}\right) + 1$$

$$= -\frac{125}{8} + 3 \times \frac{25}{4} - 3 \times \frac{5}{2} + 1$$

$$= \frac{-125 + 150 - 60 + 8}{8}$$

$$= -\frac{27}{8}$$

Question 2:

Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Answer 2:

Let $p(x) = x^3 - ax^2 + 6x - a$

Putting $x - a = 0$, we get, $x = a$

Using remainder theorem, when $p(x) = x^3 - ax^2 + 6x - a$ is divided by $x - a$, remainder is given by $p(a)$

$= (a)^3 - a(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a$

$= 5a$

Question 3:

Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Answer 3:

Let $p(x) = 3x^3 + 7x$

Putting $7 + 3x = 0$, we get, $x = -\frac{7}{3}$

Using remainder theorem, when $p(x) = 3x^3 + 7x$ is divided by $7 + 3x$, remainder is given by $p\left(-\frac{7}{3}\right)$

$$= 3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right)$$

$$= -\frac{343}{27} - \frac{49}{3}$$

$$= \frac{-343 - 147}{9} = -\frac{490}{9}$$

Since, remainder $p\left(-\frac{7}{3}\right) \neq 0$, hence, $7 + 3x$ is not a factor of $3x^3 + 7x$.