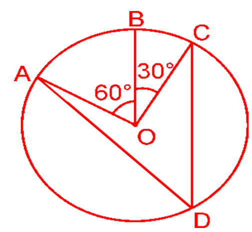
10 CIRCLES

EXERCISE 10.5

Q.1. *In the figure, A, B and C are three points on a circle with centre O such that ∠ BOC = 30° and ∠ AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ ADC.*



Sol. We have, $\angle BOC = 30°$ and $\angle AOB = 60°$

$\angle AOC = \angle AOB + \angle BOC = 60° + 30° = 90°$

We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.

$\therefore\ 2\angle ADC = \angle AOC$

$\Rightarrow\ \angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 90° \qquad \Rightarrow \angle ADC = 45°$ **Ans.**

Q.2. *A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.*

Sol. We have, OA = OB = AB

Therefore, ΔOAB is a equilateral triangle.

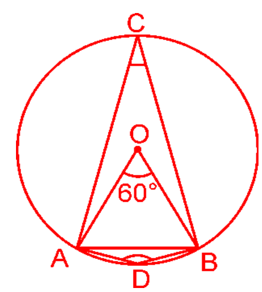
$\Rightarrow \quad \angle AOB = 60°$

We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.

$\therefore \quad \angle AOB = 2\angle ACB$

$\Rightarrow \quad \angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 60°$

$\Rightarrow \quad \angle ACB = 30°$



Also, $\angle ADB = \frac{1}{2}$ reflex $\angle AOB$

$= \frac{1}{2}(360° - 60°) = \frac{1}{2} \times 300° = 150°$

Hence, angle subtended by the chord at a point on the minor arc is 150° and at a point on the major arc is 30° **Ans.**

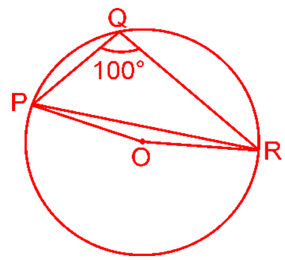
Q.3. *In the figure, ∠PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠OPR.*

Sol. Reflex angle POR $= 2\angle PQR$

$= 2 \times 100° = 200°$

Now, angle POR $= 360° - 200 = 160°$

Also,



$PO = OR$ [Radii of a circle]

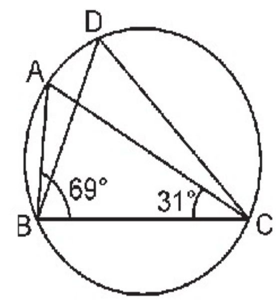
$\angle OPR = \angle ORP$ [Opposite angles of isosceles triangle]

In ΔOPR, $\angle POR = 160°$

$\therefore \angle OPR = \angle ORP = 10°$

[Angle sum property of a triangle]. **Ans.**

Q.4. *In the figure, $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$.*



Sol. In ΔABC, we have

$\angle ABC + \angle ACB + \angle BAC = 180°$

[Angle sum property of a triangle]

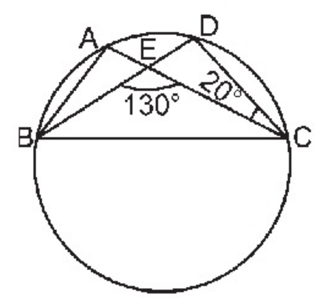
$\Rightarrow 69° + 31° + \angle BAC = 180°$

$\Rightarrow \angle BAC = 180° - 100° = 80°$

Also, $\angle BAC = \angle BDC$ [Angles in the same segment]

$\Rightarrow \angle BDC = 80°$ **Ans.**

Q.5. *In the figrue, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130°$ and $\angle ECD = 20°$. Find $\angle BAC$.*



Sol. $\angle BEC + \angle DEC = 180°$ [Linear pair]

$\Rightarrow 130° + \angle DEC = 180°$

$\Rightarrow \angle DEC = 180° - 130° = 50°$

Now, in ΔDEC,

$\Rightarrow \angle DEC + \angle DCE + \angle CDE = 180°$

[Angle sum property of a triangle]

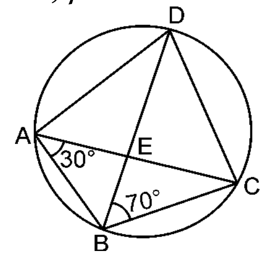
$\Rightarrow 50° + 20° + \angle CDE = 180°$

$\Rightarrow \angle CDE = 180° - 70° = 110°$

Also, $\angle CDE = \angle BAC$ [Angles in same segment]

$\Rightarrow \angle BAC = 110°$ **Ans.**

Q.6. *ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC = 30°$, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.*



Sol. $\angle CAD = \angle DBC = 70°$ [Angles in the same segment]

Therefore, $\angle DAB = \angle CAD + \angle BAC$

$= 70° + 30° = 100°$

But, $\angle DAB + \angle BCD = 180°$

[Opposite angles of a cyclic quadrilateral]

So, $\angle BCD = 180° - 100° = 80°$

Now, we have AB = BC

Therefore, $\angle BCA = 30°$ [Opposite angles of an isosceles triangle]

Again, $\angle DAB + \angle BCD = 180°$

[Opposite angles of a cyclic quadrilateral]

$\Rightarrow 100° + \angle BCA + \angle ECD = 180°$ [$\because \angle BCD = \angle BCA + \angle ECD$]

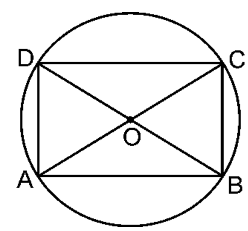
$\Rightarrow 100° + 30° + \angle ECD = 180°$

$\Rightarrow 130° + \angle ECD = 180°$

$\Rightarrow \angle ECD = 180° - 130° = 50°$

Hence, $\angle BCD = 80°$ and $\angle ECD = 50°$ **Ans.**

Q.7. *If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.*



Sol. Given : ABCD is a cyclic quadrilateral, whose diagonals AC and BD are diameter of the circle passing through A, B, C and D.

To Prove : ABCD is a rectangle.

Proof : In ΔAOD and ΔCOB

AO = CO [Radii of a circle]

OD = OB [Radii of a circle]

$\angle AOD = \angle COB$ [Vertically opposite angles]

$\therefore$ $\Delta AOD \cong \Delta COB$ [SAS axiom]

$\therefore$ $\angle OAD = \angle OCB$ [CPCT]

But these are alternate interior angles made by the transversal AC, intersecting AD and BC.

$\therefore$ AD || BC

Similarly, AB || CD.

Hence, quadrilateral ABCD is a parallelogram.

Also, $\angle ABC = \angle ADC$ ..(i) [Opposite angles of a ||gm are equal]

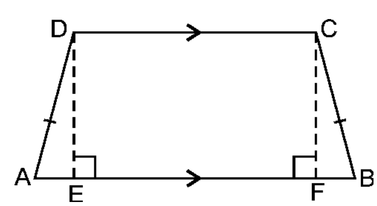
And, $\angle ABC + \angle ADC = 180°$ ...(ii)

[Sum of opposite angles of a cyclic quadrilateral is 180°]

$\Rightarrow \angle ABC = \angle ADC = 90°$ [From (i) and (ii)]

$\therefore$ ABCD is a rectangle. [A ||gm one of whose angles is 90° is a rectangle] **Proved.**

Q.8. *If the non-parallel sides of a trapezium are equal, prove that it is cyclic.*



Sol. Given : A trapezium ABCD in which AB || CD and AD = BC.

To Prove : ABCD is a cyclic trapezium.

Construction : Draw DE $\perp$ AB and CF $\perp$ AB.

Proof : In ΔDEA and ΔCFB, we have

AD = BC [Given]

$\angle DEA = \angle CFB = 90°$ [DE $\perp$ AB and CF $\perp$ AB]

$DE = CF$

[Distance between parallel lines remains constant]

$\therefore \quad \Delta DEA \cong \Delta CFB$ [RHS axiom]

$\Rightarrow \quad \angle A = \angle B$ ...(i) [CPCT]

and, $\angle ADE = \angle BCF$ ..(ii) [CPCT]

Since, $\angle ADE = \angle BCF$ [From (ii)]

$\Rightarrow \quad \angle ADE + 90° = \angle BCF + 90°$

$\Rightarrow \quad \angle ADE + \angle CDE = \angle BCF + \angle DCF$

$\Rightarrow \quad \angle D = \angle C$ ..(iii)

[$\angle ADE + \angle CDE = \angle D, \angle BCF + \angle DCF = \angle C$]

$\therefore \angle A = \angle B$ and $\angle C = \angle D$ [From (i) and (iii)] (iv)

$\angle A + \angle B + \angle C + \angle D = 360°$ [Sum of the angles of a quadrilateral is 360°]

$\Rightarrow 2(\angle B + \angle D) = 360°$ [Using (iv)]

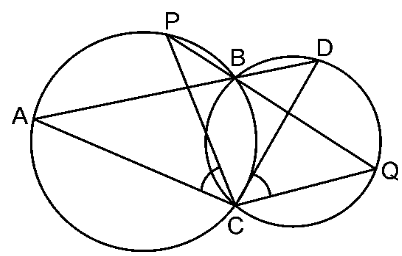
$\Rightarrow \angle B + \angle D = 180°$

$\Rightarrow$ Sum of a pair of opposite angles of quadrilateral ABCD is 180°.

$\Rightarrow$ ABCD is a cyclic trapezium **Proved.**

Q.9. *Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig.). Prove that $\angle ACP = \angle QCD$.*

Sol. Given : Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.



To Prove : $\angle ACP = \angle QCD$.

Proof : $\angle ACP = \angle ABP$ ...(i)

[Angles in the same segment]

$\angle QCD = \angle QBD$ ..(ii)

[Angles in the same segment]

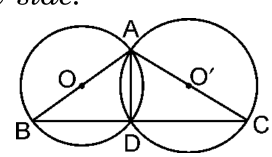
But, $\angle ABP = \angle QBD$ ..(iii) [Vertically opposite angles]

By (i), (ii) and (ii) we get

$\angle ACP = \angle QCD$ **Proved.**

Q.10. *If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.*

Sol. Given : Sides AB and AC of a triangle ABC are diameters of two circles which intersect at D.



To Prove : D lies on BC.

Proof : Join AD

$\angle ADB = 90°$ ...(i) [Angle in a semicircle]

Also, $\angle ADC = 90°$ ..(ii)

Adding (i) and (ii), we get

$\angle ADB + \angle ADC = 90° + 90°$

$\Rightarrow \quad \angle ADB + \angle ADC = 180°$

$\Rightarrow$ BDC is a straight line.

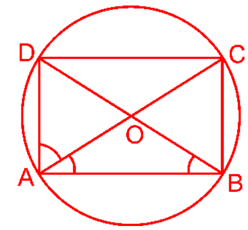
$\therefore$ D lies on BC

Hence, point of intersection of circles lie on the third side BC. **Proved.**

Q.11. *ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.*

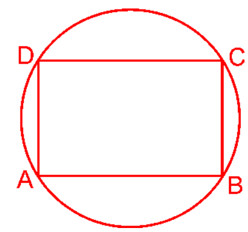
Sol. Given : ABC and ADC are two right triangles with common hypotenuse AC.

To Prove : $\angle CAD = \angle CBD$



Proof : Let O be the mid-point of AC.
Then OA = OB = OC = OD
Mid point of the hypotenuse of a right triangle is equidistant from its vertices with O as centre and radius equal to OA, draw a circle to pass through A, B, C and D.
We know that angles in the same segment of a circle are equal.
Since, $\angle CAD$ and $\angle CBD$ are angles of the same segment.
Therefore, $\angle CAD = \angle CBD$. **Proved.**

Q.12. *Prove that a cyclic parallelogram is a rectangle.*



Sol. **Given :** ABCD is a cyclic parallelogram.
To prove : ABCD is a rectangle.
Proof : $\angle ABC = \angle ADC$ ...(i)
[Opposite angles of a ||gm are equal]
But, $\angle ABC + \angle ADC = 180°$ ... (ii)
[Sum of opposite angles of a cyclic quadrilateral is 180°]
$\Rightarrow \angle ABC = \angle ADC = 90°$ [From (i) and (ii)]
$\therefore$ ABCD is a rectangle
[A ||gm one of whose angles is 90° is a rectangle]
Hence, a cyclic parallelogram is a rectangle. **Proved.**