10 CIRCLES

EXERCISE 10.4

Q.1. *Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.*

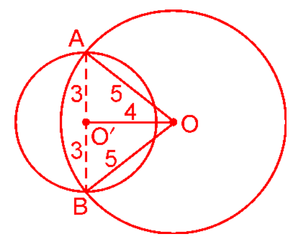
Sol. In $\Delta AOO'$,

$AO^2 = 5^2 = 25$

$AO'^2 = 3^2 = 9$

$OO'^2 = 4^2 = 16$

$AO'^2 + OO'^2 = 9 + 16 = 25 = AO^2$



$\Rightarrow \angle AO'O = 90°$

[By converse of pythagoras theorem]

Similarly, $\angle BO'O = 90°$.

$\Rightarrow \angle AO'B = 90° + 90° = 180°$

$\Rightarrow$ AO′B is a straight line. whose mid-point is O.

$\Rightarrow AB = (3 + 3)$ cm = 6 cm **Ans.**

Q.2. *If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.*

Sol. Given : AB and CD are two equal chords of a circle which meet at E.

To prove : AE = CE and BE = DE

Construction : Draw $OM \perp AB$ and $ON \perp CD$ and join OE. **Proof :**

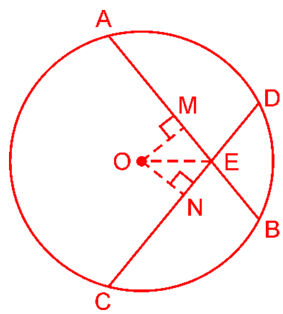
In ΔOME and ΔONE

OM = ON [Equal chords are equidistant]

OE = OE [Common]

$\angle OME = \angle ONE$ [Each equal to 90°]

$\therefore \Delta OME \cong \Delta ONE$ [RHS axiom]



$\Rightarrow$ EM = EN ...(i) [CPCT]

Now AB = CD [Given]

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow$ AM = CN ..(ii) [Perpendicular from centre bisects the chord]

Adding (i) and (ii), we get

EM + AM = EN + CN

$\Rightarrow$ AE = CE ..(iii)

Now, AB = CD ..(iv)

$\Rightarrow$ AB – AE = CD – AE [From (iii)]

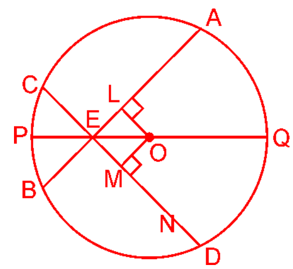
$\Rightarrow$ BE = CD – CE **Proved.**

Q.3. *If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.*

Sol. Given : AB and CD are two equal chords of a circle which meet at E within the circle and a line PQ joining the point of intersection to the centre.

To Prove : $\angle AEQ = \angle DEQ$

Construction : Draw OL ⊥ AB and OM ⊥ CD.



Proof : In ΔOLE and ΔOME, we have

OL = OM [Equal chords are equidistant]

OE = OE [Common]

∠OLE = ∠OME [Each = 90°]

∴ ΔOLE ≅ ΔOME [RHS congruence]

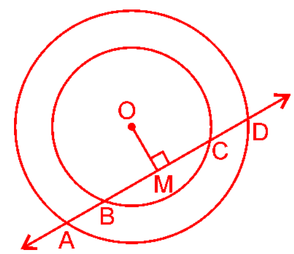
⇒ ∠LEO = ∠MEO [CPCT]

Q.4. *If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig.)*

Sol. **Given :** A line AD intersects two concentric circles at A, B, C and D, where O is the centre of these circles.

To prove : AB = CD

Construction : Draw OM ⊥ AD.



Proof : AD is the chord of larger circle.

∴ AM = DM ..(i) [OM bisects the chord]

BC is the chord of smaller circle

∴ BM = CM ..(ii) [OM bisects the chord]

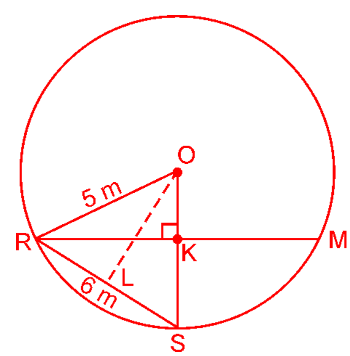
Subtracting (ii) from (i), we get

AM – BM = DM – CM

⇒ AB = CD **Proved.**

Q.5. *Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?*

Sol. Let Reshma, Salma and Mandip be represented by R, S and M respectively.



Draw OL ⊥ RS,

$OL^2 = OR^2 - RL^2$

$OL^2 = 5^2 - 3^2$ [RL = 3 m, because OL ⊥ RS]

$= 25 - 9 = 16$

$OL = \sqrt{16} = 4$

Now, area of triangle ORS $= \frac{1}{2} \times KR \times 05$

$= \frac{1}{2} \times KR \times 05$

Also, area of ΔORS $= \frac{1}{2} \times RS \times OL = \frac{1}{2} \times 6 \times 4 = 12 \text{ m}^2$

$\Rightarrow \frac{1}{2} \times KR \times 5 = 12$

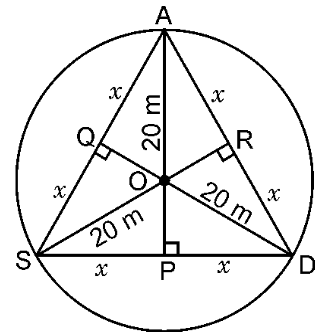
$\Rightarrow KR = \frac{12 \times 2}{5} = \frac{24}{5} = 4.8$ m

⇒ RM = 2KR

⇒ RM = 2 × 4.8 = 9.6 m

Hence, distance between Reshma and Mandip is 9.6 m **Ans.**

Q.6. *A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are siting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.*



Sol. Let Ankur, Syed and David be represented by A, S and D respectively.

Let PD = SP = SQ = QA = AR = RD = x

In ΔOPD,

$OP^2 = 400 - x^2$

$\Rightarrow \quad OP = \sqrt{400 - x^2}$

$\Rightarrow \quad AP = 2\sqrt{400 - x^2} + \sqrt{400 - x^2}$

[$\because$ centroid divides the median in the ratio 2 : 1]

$= 3\sqrt{400 - x^2}$

Now, in ΔAPD,

$PD^2 = AD^2 - DP^2$

$\Rightarrow \quad x^2 = (2x)^2 - \left(3\sqrt{400 - x^2}\right)^2$

$\Rightarrow \quad x^2 = 4x^2 - 9(400 - x^2)$

$\Rightarrow \quad x^2 = 4x^2 - 3600 + 9x^2$

$\Rightarrow \quad 12x^2 = 3600$

$\Rightarrow \quad x^2 = \dfrac{3600}{12} = 300$

$\Rightarrow \quad x = 10\sqrt{3}$

Now, SD = $2x = 2 \times 10\sqrt{3} = 20\sqrt{3}$

$\therefore$ ASD is an equilateral triangle.

$\Rightarrow$ SD = AS = AD = $20\sqrt{3}$

Hence, length of the string of each phone is $20\sqrt{3}$ m **Ans.**