Exercise 9.5

Question 1:

Use a suitable identity to get each of the following products:

(i) $(x+3)(x+3)$ (ii) $(2y+5)(2y+5)$

(iii) $(2a-7)(2a-7)$ (iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right)$

(v) $(1.1m-0.4)(1.1m+0.4)$ (vi) $\left(a^2+b^2\right)\left(-a^2+b^2\right)$

(vii) $(6x-7)(6x+7)$ (viii) $(-a+c)(-a+c)$

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right)$ (x) $(7a-9b)(7a-9b)$

Answer 1:

(i) $(x+3)(x+3)=(x+3)^2$

$=(x)^2+2\times x\times 3+(3)^2$ [Using identity $(a+b)^2=a^2+2ab+b^2$]

$= x^2+6x+9$

(ii) $(2y+5)(2y+5)=(2y+5)^2$

$=(2y)^2+2\times 2y\times 5+(5)^2$ [Using identity $(a+b)^2=a^2+2ab+b^2$]

$= 4y^2+20y+25$

(iii) $(2a-7)(2a-7)=(2a-7)^2$

$=(2a)^2-2\times 2a\times 7+(7)^2$ [Using identity $(a-b)^2=a^2-2ab+b^2$]

$= 4a^2-28a+49$

(iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right)=\left(3a-\frac{1}{2}\right)^2$

$=(3a)^2-2\times 3a\times\frac{1}{2}+\left(\frac{1}{2}\right)^2$ [Using identity $(a-b)^2=a^2-2ab+b^2$]

$= 9a^2-3a+\frac{1}{4}$

(v) $(1.1m-0.4)(1.1m+0.4)=(1.1m)^2-(0.4)^2$

Using identity $(a-b)(a+b)=a^2-b^2$]

$= 1.21m^2-0.16$

(vi) $\left(a^2+b^2\right)\left(-a^2+b^2\right)=\left(b^2+a^2\right)\left(b^2-a^2\right)$

$=\left(b^2\right)^2-\left(a^2\right)^2$ [Using identity $(a-b)(a+b)=a^2-b^2$]

$= b^4-a^4$

(vii) $(6x-7)(6x+7)=(6x)^2-(7)^2$ [Using identity $(a-b)(a+b)=a^2-b^2$]

$= 36x^2-49$

(viii) $(-a+c)(-a+c)=(c-a)(c-a)=(c-a)^2$

$= (c)^2 - 2\times c\times a + (a)^2$ [Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= c^2 - 2ca + a^2$

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right)=\left(\frac{x}{2}+\frac{3y}{4}\right)^2$

$=\left(\frac{x}{2}\right)^2 + 2\times\frac{x}{2}\times\frac{3y}{4}+\left(\frac{3y}{4}\right)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= \frac{x^2}{4}+\frac{3}{4}xy+\frac{9}{16}y^2$

(x) $(7a-9b)(7a-9b)=(7a-9b)^2$

$=(7a)^2 - 2\times 7a\times 9b + (9b)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 49a^2 - 126ab + 81b^2$

Question 2:

Use the identity $(x+a)(x+b)=x^2+(a+b)x+ab$ to find the following products:

(i) $(x+3)(x+7)$ (ii) $(4x+5)(4x+1)$

(iii) $(4x-5)(4x-1)$ (iv) $(4x+5)(4x-1)$

(v) $(2x+5y)(2x+3y)$ (vi) $\left(2a^2+9\right)\left(2a^2+5\right)$

(vii) $(xyz-4)(xyz-2)$

Answer 2:

(i) $(x+3)(x+7)=(x)^2+(3+7)x+3\times 7$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= x^2 + 10x + 21$

(ii) $(4x+5)(4x+1)=(4x)^2+(5+1)4x+5\times 1$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 16x^2 + 6\times 4x + 5 = 16x^2 + 24x + 5$

(iii) $(4x-5)(4x-1) = (4x)^2+(-5-1)4x+(-5)\times(-1)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 16x^2 + (-6)\times 4x + 5 = 16x^2 - 24x + 5$

(iv) $(4x+5)(4x-1)=(4x)^2+\{5\times(-1)\}\times 4x+5\times(-1)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 16x^2 + (5-1)\times 4x - 5$

$= 16x^2 + 4\times 4x - 5$

$= 16x^2 + 16x - 5$

(v) $(2x+5y)(2x+3y)=(2x)^2+(5y+3y)\times 2x+5y\times 3y$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 4x^2+8y\times 2x+15y^2 = 4x^2+16xy+15y^2$

(vi) $(2a^2+9)(2a^2+5)=(2a^2)^2+(9+5)\times 2a^2+9\times 5$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= 4a^4+14\times 2a^2+45 = 4a^4+28a^2+45$

(vii) $(xyz-4)(xyz-2)=(xyz)^2+(-4-2)\times xyz+(-4)\times(-2)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= x^2y^2z^2-6xyz+8$

Question 3:

Find the following squares by using identities:

(i) $(b-7)^2$ (ii) $(xy+3z)^2$ (iii) $(6x^2-5y)^2$

(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2$ (v) $(0.4p-0.5q)^2$ (vi) $(2xy+5y)^2$

Answer 3:

(i) $(b-7)^2=(b)^2-2\times b\times 7+(7)^2$ [Using identity $(a-b)^2=a^2-2ab+b^2$]

$= b^2-14b+49$

(ii) $(xy+3z)^2=(xy)^2+2\times xy\times 3z+(3z)^2$ [Using identity $(a+b)^2=a^2+2ab+b^2$]

$= x^2y^2+6xyz+9z^2$

(iii) $(6x^2-5y)^2=(6x^2)^2-2\times 6x^2\times 5y+(5y)^2$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$= 36x^4-60x^2y+25y^2$

(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2=\left(\frac{2}{3}m\right)^2+2\times\frac{2}{3}m\times\frac{3}{2}n+\left(\frac{3}{2}n\right)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

$= \frac{4}{9}m^2+2mn+\frac{9}{4}n^2$

(v) $(0.4p-0.5q)^2=(0.4p)^2-2\times 0.4p\times 0.5q+(0.5q)^2$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$= 0.16p^2-0.40pq+0.25q^2$

(vi) $(2xy+5y)^2=(2xy)^2+2\times 2xy\times 5y+(5y)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

$= 4x^2y^2+20xy^2+25y^2$

Question 4:

Simplify:

(i) $\left(a^2-b^2\right)^2$

(ii) $(2x+5)^2-(2x-5)^2$

(iii) $(7m-8n)^2+(7m+8n)^2$

(iv) $(4m+5n)^2+(5m+4n)^2$

(v) $(2.5p-1.5q)^2-(1.5p-2.5q)^2$

(vi) $(ab+bc)^2-2ab^2c$

(vii) $\left(m^2-n^2m\right)^2+2m^3n^2$

Answer 4:

(i) $\left(a^2-b^2\right)^2=\left(a^2\right)^2-2\times a^2\times b^2+\left(b^2\right)^2$ [Using identity $(a-b)^2=a^2-2ab+b^2$]

$= a^4-2a^2b^2+b^4$

(ii) $(2x+5)^2-(2x-5)^2=(2x)^2+2\times 2x\times 5+(5)^2-\left[(2x)^2-2\times 2x\times 5+(5)^2\right]$

[Using identities $(a+b)^2=a^2+2ab+b^2$ and $(a-b)^2=a^2-2ab+b^2$]

$= 4x^2+20x+25-\left[4x^2-20x+25\right]$

$= 4x^2+20x+25-4x^2+20x-25$

$= 40x$

(iii)

$(7m-8n)^2+(7m+8n)^2=(7m)^2-2\times 7m\times 8n+(8n)^2+\left[(7m)^2+2\times 7m\times 8n+(8n)^2\right]$

[Using identities $(a+b)^2=a^2+2ab+b^2$ and $(a-b)^2=a^2-2ab+b^2$]

$= 49m^2-112mn+64n^2+\left[49m^2+112mn+64n^2\right]$

$= 49m^2-112mn+64n^2+49m^2+112mn+64n^2$

$= 98m^2+128n^2$

(iv) $(4m+5n)^2+(5m+4n)^2=(4m)^2+2\times 4m\times 5n+(5n)^2+(5m)^2+2\times 5m\times 4n+(4n)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

$= 16m^2+40mn+25n^2+25m^2+40mn+16n^2$

$= 16m^2+25m^2+40mn+40mn+25n^2+16n^2$

$= 41m^2+80mn+41n^2$

(v) $(2.5p-1.5q)^2-(1.5p-2.5q)^2$

$=(2.5p)^2-2\times 2.5p\times 1.5q+(1.5q)^2-\left[(1.5p)^2-2\times 1.5p\times 2.5q+(2.5q)^2\right]$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$= 6.25p^2-7.50pq+2.25q^2-\left[2.25p^2-7.50pq+6.25q^2\right]$

$= 6.25p^2-7.50pq+2.25q^2-2.25p^2+7.50pq-6.25q^2$

$= 4p^2 - 4q^2$

(vi) $(ab+bc)^2 - 2ab^2c = (ab)^2 + 2\times ab\times bc + (bc)^2 - 2ab^2c$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c$

$= a^2b^2 + b^2c^2$

(vii) $(m^2 - n^2m)^2 + 2m^3n^2 = (m^2)^2 - 2\times m^2\times n^2m + (n^2m)^2 + 2m^3n^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= m^4 - 2m^3n^2 + n^4m^2 + 2m^3n^2$

$= m^4 + n^4m^2$

Question 5:

Show that:

(i) $(3x+7)^2 - 84x = (3x-7)^2$

(ii) $(9p-5q)^2 + 180pq = (9p+5q)^2$

(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

(iv) $(4pq+3q)^2 - (4pq-3q)^2 = 48pq^2$

(v) $(a-b)(a+b) + (b-c)(b+c) + (c-a)(c+a) = 0$

Answer 5:

(i) L.H.S. $= (3x+7)^2 - 84x = (3x)^2 + 2\times 3x\times 7 + (7)^2 - 84x$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= 9x^2 + 42x + 49 - 84x$

$= 9x^2 - 42x + 49$

$= (3x-7)^2$ [$\because\ (a-b)^2 = a^2 - 2ab + b^2$]

= R.H.S.

(ii) L.H.S. $= (9p-5q)^2 + 180pq = (9p)^2 - 2\times 9p\times 5q + (5q)^2 + 180pq$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 81p^2 - 90pq + 25q^2 + 180pq$

$= 81p^2 + 90pq + 25q^2$

$= (9p+5q)^2$ [$\because\ (a+b)^2 = a^2 + 2ab + b^2$]

(iii) L.H.S. $= \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \left(\frac{4}{3}m\right)^2 - 2\times\frac{4}{3}m\times\frac{3}{4}n + \left(\frac{3}{4}n\right)^2 + 2mn$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= \frac{16}{9}m^2 - 2mn + \frac{9}{16}n^2 + 2mn$

$= \frac{16}{9}m^2 + \frac{9}{16}n^2$

= R.H.S.

(iv) L.H.S. $= (4pq+3q)^2 - (4pq-3q)^2$

$= (4pq)^2 + 2\times 4pq \times 3q + (3q)^2 - \left[(4pq)^2 - 2\times 4pq\times 3q + (3q)^2\right]$

[Using identities $(a+b)^2 = a^2+2ab+b^2$ and $(a-b)^2 = a^2-2ab+b^2$]

$= 16p^2q^2 + 24pq^2 + 9q^2 - \left[16p^2q^2 - 24pq^2 + 9q^2\right]$

$= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2 = 48pq^2$ = R.H.S.

(v) L.H.S. $= (a-b)(a+b) + (b-c)(b+c) + (c-a)(c+a)$

$= a^2 - b^2 + b^2 - c^2 + c^2 - a^2$ [Using identity $(a-b)(a+b) = a^2-b^2$]

= 0 = R.H.S.

Question 6:

Using identities, evaluate:

(i)	71^2	(ii)	99^2	(iii)	102^2
(iv)	998^2	(v)	5.2^2	(vi)	297×303
(vii)	78×82	(viii)	8.9^2	(ix)	1.05×9.5

Answer 6:

(i) $71^2 = (70+1)^2 = (70)^2 + 2\times 70\times 1 + (1)^2$

[Using identity $(a+b)^2 = a^2-2ab+b^2$]

= 4900 + 140 + 1 = 5041

(ii) $99^2 = (100-1)^2 = (100)^2 - 2\times 100\times 1 + (1)^2$

[Using identity $(a-b)^2 = a^2-2ab+b^2$]

= 10000 – 200 + 1 = 9801

(iii) $102^2 = (100+2)^2 = (100)^2 + 2\times 100\times 2 + (2)^2$

[Using identity $(a+b)^2 = a^2+2ab+b^2$]

= 10000 + 400 + 4 = 10404

(iv) $998^2 = (1000-2)^2 = (1000)^2 - 2\times 1000\times 2 + (2)^2$

[Using identity $(a-b)^2 = a^2-2ab+b^2$]

= 1000000 – 4000 + 4 = 996004

(v) $5.2^2 = (5+0.2)^2 = (5)^2 + 2\times 5\times 0.2 + (0.2)^2$

[Using identity $(a+b)^2 = a^2+2ab+b^2$]

= 25 + 2.0 + 0.04 = 27.04

(vi) 297 x 303 = (300 – 3) x (300 + 3) = $(300)^2 - (3)^2$

[Using identity $(a-b)(a+b) = a^2-b^2$]

= 90000 – 9 = 89991

(vii) 78 x 82 = (80 – 2) x (80 + 2) = $(80)^2 - (2)^2$

[Using identity $(a-b)(a+b)=a^2-b^2$]

= 6400 – 4 = 6396

(viii) $8.9^2=(8+0.9)^2=(8)^2+2\times8\times0.9+(0.9)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

= 64 + 14.4 + 0.81 = 79.21

(ix) 1.05 x 9.5 = (10 + 0.5) x (10 – 0.5) = $(10)^2-(0.5)^2$

[Using identity $(a-b)(a+b)=a^2-b^2$]

Question 7:

Using $a^2-b^2=(a+b)(a-b)$, find

(i) 51^2-49^2 (ii) $(1.02)^2-(0.98)^2$ (iii) 153^2-147^2 (iv) $12.1^2-7.9^2$

Answer 7:

(i) $51^2-49^2=(51+49)(51-49)$ [Using identity $(a-b)(a+b)=a^2-b^2$]

= 100 x 2 = 200

(ii) $(1.02)^2-(0.98)^2=(1.02+0.98)(1.02-0.98)$

[Using identity $(a-b)(a+b)=a^2-b^2$]

= 2.00 x 0.04 = 0.08

(iii) $153^2-147^2=(153+147)(153-147)$ [Using identity $(a-b)(a+b)=a^2-b^2$]

= 300 x 6 = 1800

(iv) $12.1^2-7.9^2=(12.1+7.9)(12.1-7.9)$ [Using identity $(a-b)(a+b)=a^2-b^2$]

= 20.0 x 4.2 = 84.0 = 84

Question 8:

Using $(x+a)(x+b)=x^2+(a+b)x+ab$, find

(i) 103 x 104 (ii) 5.1 x 5.2 (iii) 103 x 98 (iv) 9.7 x 9.8

Answer 8:

(i) 103 x 104 = (100 + 3) x (100 + 4) = $(100)^2+(3+4)\times100+3\times4$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

= 10000 + 7 x 100 + 12 = 10000 + 700 + 12 = 10712

(ii) 5.1 x 5.2 = (5 + 0.1) x (5 + 0.2) = $(5)^2+(0.1+0.2)\times5+0.1\times0.2$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

= 25 + 0.3 x 5 + 0.02 = 25 + 1.5 + 0.02 = 26.52

(iii) 103 x 98 = (100 + 3) x (100 – 2) = $(100)^2+(3-2)\times100+3\times(-2)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

= 10000 + (3 – 2) x 100 – 6 = 10000 + 100 – 6 = 10094

(iv) 9.7 x 9.8 = (10 – 0.3) x (10 – 0.2)

= $(10)^2+\{(-0.3)+(-0.2)\}\times10+(-0.3)\times(-0.2)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

= $100+\{-0.3-0.2\}\times10+0.06$ = 100 – 0.5 x 10 + 0.06 = 100 – 5 + 0.06 = 95.06