Exercise 9.4

Question 1:

Multiply the binomials:

(i) $(2x+5)$ and $(4x-3)$

(ii) $(y-8)$ and $(3y-4)$

(iii) $(2.5l-0.5m)$ and $(2.5l+0.5m)$

(iv) $(a+3b)$ and $(x+5)$

(v) $(2pq+3q^2)$ and $(3pq-2q^2)$

(vi) $\left(\dfrac{3}{4}a^2+3b^2\right)$ and $4\left(a^2-\dfrac{2}{3}b^2\right)$

Answer 1:

(i)
$$(2x+5)\times(4x-3) = 2x(4x-3)+5(4x-3)$$
$$= 2x\times4x-2x\times3+5\times4x-5\times3$$
$$= 8x^2-6x+20x-15$$
$$= 8x^2+14x-15$$

(ii)
$$(y-8)\times(3y-4) = y(3y-4)-8(3y-4)$$
$$= y\times3y-y\times4-8\times3y-8\times-4$$
$$= 3y^2-4y-24y+12$$
$$= 3y^2-28y+12$$

(iii)
$$(2.5l-0.5m)\times(2.5l+0.5m) = 2.5l\times(2.5l+0.5m)-0.5m\times(2.5l+0.5m)$$
$$= 2.5l\times2.5l+0.5l\times0.5m-0.5m\times2.5l-0.5m\times0.5m$$
$$= 6.25l^2+1.25lm-1.25lm-0.25m^2$$
$$= 6.25l^2-0.25m^2$$

(iv)
$$(a+3b)\times(x+5) = a(x+5)+3b(x+5)$$
$$= a\times x+a\times5+3b\times x+3b\times5$$
$$= ax+5a+3bx+15b$$

(v)
$$(2pq+3q^2)(3pq-2q^2) = 2pq\times(3pq-2q^2)+3q^2(3pq-2q^2)$$
$$= 2pq\times3pq-2pq\times2q^2+3q^2\times3pq-3q^2\times2q^2$$
$$= 6p^2q^2-4pq^3+9pq^3-6q^4$$
$$= 6p^2q^2+5pq^3-6q^4$$

(vi)
$$\left(\frac{3}{4}a^2+3b^2\right)\times4\left(a^2-\frac{2}{3}b^2\right) = \left(\frac{3}{4}a^2+3b^2\right)\times\left(4a^2-\frac{8}{3}b^2\right)$$
$$= \frac{3}{4}a^2\times\left(4a^2-\frac{8}{3}b^2\right)+3b^2\times\left(4a^2-\frac{8}{3}b^2\right)$$
$$= \frac{3}{4}a^2\times4a^2-\frac{3}{4}a^2\times\frac{8}{3}b^2+3b^2\times4a^2-3b^2\times\frac{8}{3}b^2$$
$$= 3a^4-2a^2b^2+12a^2b^2-8b^4$$
$$= 3a^4+10a^2b^2-8b^4$$

Question 2:

Find the product:

(i) $(5-2x)(3+x)$

(ii) $(x+7y)(7x-y)$

(iii) $(a^2+b)(a+b^2)$

(iv) $(p^2-q^2)(2p+q)$

Answer 2:

(i) $(5-2x)(3+x) = 5\times(3+x)-2x(3+x) = 5\times3+5\times x-2x\times3-2x\times x$

$$= 15+5x-6x-2x^2 = 15-x-2x^2$$

(ii) $(x+7y)(7x-y) = x(7x-y)+7y\times(7x-y)$

$$= x\times7x-x\times y+7y\times7x-7y\times y$$

$$= 7x^2-xy+49xy-7y^2$$

$$= 7x^2+48xy-7y^2$$

(iii) $(a^2+b)(a+b^2) = a^2\times(a+b^2)+b\times(a+b^2)$

$$= a^2\times a+a^2\times b^2+b\times a+b\times b^2$$

$$= a^3+a^2b^2+ab+b^3$$

(iv) $(p^2-q^2)(2p+q) = p^2\times(2p+q)-q^2(2p+q)$

$$= p^2\times2p+p^2\times q-q^2\times2p-q^2\times q$$

$$= 2p^3+p^2q-2pq^2-q^3$$

Question 3:

Simplify:

(i) $(x^2-5)(x+5)+25$

(ii) $(a^2+5)(b^2+3)+5$

(iii) $(t+s^2)(t^2-s)$

(iv) $(a+b)(c-d)+(a-b)(c+d)+2(ac+bd)$

(v) $(x+y)(2x+y)+(x+2y)(x-y)$

(vi) $(x+y)(x^2-xy+y^2)$

(vii) $(1.5x-4y)(1.5x+4y+3)-4.5x+12y$

(viii) $(a+b+c)(a+b-c)$

Answer 3:

(i) $(x^2-5)(x+5)+25 = x^2(x+5)-5(x+5)+25$

$$= x^2\times x+x^2\times5-5\times x-5\times5+25$$

$$= x^3+5x^2-5x-25+25$$

$$= x^3+5x^2-5x$$

(ii) $\left(a^2+5\right)\left(b^3+3\right)+5 = a^2\left(b^3+3\right)+5\left(b^3+3\right)+5$

$$= a^2 \times b^3 + a^2 \times 3 + 5 \times b^3 + 5 \times 3 + 5$$
$$= a^2 b^3 + 3a^2 + 5b^3 + 15 + 5$$
$$= a^2 b^3 + 3a^2 + 5b^3 + 20$$

(iii) $\left(t+s^2\right)\left(t^2-s\right) = t\left(t^2-s\right)+s^2\left(t^2-s\right)$

$$= t \times t^2 - t \times s + s^2 \times t^2 - s^2 \times s$$
$$= t^3 - st + s^2 t^2 - s^3$$

(iv) $(a+b)(c-d)+(a-b)(c+d)+2(ac+bd)$

$$= a(c-d)+b(c-d)+a(c+d)-b(c+d)+2ac+2bd$$
$$= ac-ad+bc-bd+ac+ad-bc-bd+2ac+2bd$$
$$= ac+ac-ad+ad+bc-bc-bd-bd+2ac+2bd$$
$$= 2ac-2bd+2ac+2bd$$
$$= 4ac$$

(v) $(x+y)(2x+y)+(x+2y)(x-y) = x(2x+y)+y(2x+y)+x(x-y)+2y(x-y)$

$$= 2x^2+xy+2xy+y^2+x^2-xy+2xy-2y^2$$
$$= 2x^2+x^2+xy+2xy-xy+2xy+y^2-2y^2$$
$$= 3x^2+4xy-y^2$$

(vi) $(x+y)\left(x^2-xy+y^2\right) = x\left(x^2-xy+y^2\right)+y\left(x^2-xy+y^2\right)$

$$= x^3-x^2y+xy^2+x^2y-xy^2+y^3$$
$$= x^3-x^2y+x^2y+xy^2-xy^2+y^3$$
$$= x^3+y^3$$

(vii) $(1.5x-4y)(1.5x+4y+3)-4.5x+12y$

$$= 1.5x(1.5x+4y+3)-4y(1.5x+4y+3)-4.5x+12y$$
$$= 2.25x^2+6.0xy+4.5x-6.0xy-16y^2-12y-4.5x+12y$$
$$= 2.25x^2+6.0xy-6.0xy+4.5x-4.5x-16y^2-12y+12y$$
$$= 2.25x^2-16y^2$$

(viii) $(a+b+c)(a+b-c) = a(a+b-c)+b(a+b-c)+c(a+b-c)$

$$= a^2+ab-ac+ab+b^2-bc+ac+bc-c^2$$
$$= a^2+ab+ab-ac+ac-bc+bc+b^2-c^2$$
$$= a^2+b^2-c^2+2ab$$