Exercise 12.1

Question 1:

Evaluate:

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

Answer 1:

(i) $3^{-2} = \frac{1}{3^2}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= \frac{1}{9}$

(ii) $(-4)^{-2} = \frac{1}{(-4)^2}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= \frac{1}{16}$

(iii) $\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^5$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= (2)^5 = 32$

Question 2:

Simplify and express the result in power notation with positive exponent:

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$ (iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

(iv) $\left(3^{-7} \div 3^{-10}\right) \times 3^{-5}$ (v) $2^{-3} \times (-7)^{-3}$

Answer 2:

(i) $(-4)^5 \div (-4)^8 = (-4)^{5-8}$ $\left[\because\ a^m \div a^n = a^{m-n}\right]$

$= (-4)^{-3} = \frac{1}{(-4)^3}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

(ii) $\left(\frac{1}{2^3}\right)^2 = \frac{1^2}{\left(2^3\right)^2}$ $\left[\because\ \left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}\right]$

$= \frac{1}{2^{3\times 2}} = \frac{1}{2^6}$ $\left[\because\ \left(a^m\right)^n = a^{m\times n}\right]$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-3)^4 \times \frac{5^4}{3^4}$ $\left[\because\ \left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}\right]$

$= \left\{(-1)^4 \times 3^4\right\} \times \frac{5^4}{3^4}$ $\left[\because\ (ab)^m = a^m b^m\right]$

$= 3^{4-4} \times 5^4$ $\left[\because\ a^m \div a^n = a^{m-n}\right]$

$= 3^0 \times 5^4 = 5^4$ $\left[\because\ a^0 = 1\right]$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5} = 3^{-7-(-10)} \times 3^{-5}$ $\left[\because a^m \div a^n = a^{m-n}\right]$

$= 3^{-7+10} \times 3^{-5} = 3^3 \times 3^{-5} = 3^{3+(-5)}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= 3^{-2} = \dfrac{1}{3^2}$ $\left[\because a^{-m} = \dfrac{1}{a^m}\right]$

(v) $2^{-3} \times (-7)^{-3} = \dfrac{1}{2^3} \times \dfrac{1}{(-7)^3}$ $\left[\because a^{-m} = \dfrac{1}{a^m}\right]$

$= \dfrac{1}{\{2 \times (-7)\}^3} = \dfrac{1}{(-14)^3}$ $\left[\because (ab)^m = a^m b^m\right]$

Question 3:

Find the value of:

(i) $(3^0 + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$ (iii) $\left(\dfrac{1}{2}\right)^{-2} + \left(\dfrac{1}{3}\right)^{-2} + \left(\dfrac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$ (v) $\left\{\left(\dfrac{-2}{3}\right)^{-2}\right\}^2$

Answer 3:

(i) $(3^0 + 4^{-1}) \times 2^2 = \left(1 + \dfrac{1}{4}\right) \times 2^2$ $\left[\because a^{-m} = \dfrac{1}{a^m}\right]$

$= \left(\dfrac{4+1}{4}\right) \times 2^2 = \dfrac{5}{4} \times 2^2 = \dfrac{5}{2^2} \times 2^2$

$= 5 \times 2^{2-2}$ $\left[\because a^m \div a^n = a^{m-n}\right]$

$= 5 \times 2^0 = 5 \times 1 = 5$ $\left[\because a^0 = 1\right]$

(ii) $(2^{-1} \times 4^{-1}) \div 2^{-2} = \left(\dfrac{1}{2^1} \times \dfrac{1}{4^1}\right) \div 2^{-2}$ $\left[\because a^{-m} = \dfrac{1}{a^m}\right]$

$= \left(\dfrac{1}{2} \times \dfrac{1}{2^2}\right) \div 2^{-2} = \dfrac{1}{2^3} \div 2^{-2}$ $\left[\because a^m \times a^n = a^{m+n}\right]$

$= 2^{-3} \div 2^{-2} = 2^{-3-(-2)} = 2^{-3+2} = 2^{-1}$ $\left[\because a^m \div a^n = a^{m-n}\right]$

$= \dfrac{1}{2}$ $\left[\because a^{-m} = \dfrac{1}{a^m}\right]$

(iii) $\left(\dfrac{1}{2}\right)^{-2} + \left(\dfrac{1}{3}\right)^{-2} + \left(\dfrac{1}{4}\right)^{-2} = (2^{-1})^{-2} + (3^{-1})^{-2} + (4^{-1})^{-2}$ $\left[\because a^{-m} = \dfrac{1}{a^m}\right]$

$= 2^{-1\times(-2)} + 3^{-1\times(-2)} + 4^{-1\times(-2)}$ $\left[\because (a^m)^n = a^{m \times n}\right]$

$= 2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0 = \left(\dfrac{1}{3} + \dfrac{1}{4} + \dfrac{1}{5}\right)^0$ $\left[\because a^{-m} = \dfrac{1}{a^m}\right]$

$$= \left(\frac{20+15+12}{60}\right)^0 = \left(\frac{47}{60}\right)^0 = 1 \qquad \left[\because\ a^0 = 1\right]$$

(v) $$\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2 = \left(\frac{-2}{3}\right)^{-2\times 2} \qquad \left[\because\ \left(a^m\right)^n = a^{m\times n}\right]$$

$$= \left(\frac{-2}{3}\right)^{-4} = \left(\frac{-3}{2}\right)^4 \qquad \left[\because\ a^{-m} = \frac{1}{a^m}\right]$$

$$= \frac{81}{16}$$

Question 4:

Evaluate:

(i) $\frac{8^{-1}\times 5^3}{2^{-4}}$ (ii) $\left(5^{-1}\times 2^{-1}\right)\times 6^{-1}$

Answer 4:

(i) $$\frac{8^{-1}\times 5^3}{2^{-4}} = \frac{\left(2^3\right)^{-1}\times 5^3}{2^{-4}} = \frac{2^{-3}\times 5^3}{2^{-4}} \qquad \left[\because\ \left(a^m\right)^n = a^{m\times n}\right]$$

$$= 2^{-3-(-4)}\times 5^3 = 2^{-3+4}\times 5^3 \qquad \left[\because\ a^m \div a^n = a^{m-n}\right]$$

$$= 2\times 125 = 250$$

(ii) $$\left(5^{-1}\times 2^{-1}\right)\times 6^{-1} = \left(\frac{1}{5}\times\frac{1}{2}\right)\times\frac{1}{6} \qquad \left[\because\ a^{-m} = \frac{1}{a^m}\right]$$

$$= \frac{1}{10}\times\frac{1}{6} = \frac{1}{60}$$

Question 5:

Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Answer 5:

$5^m \div 5^{-3} = 5^5$

$\Rightarrow \quad 5^{m-(-3)} = 5^5 \qquad \left[\because\ a^m \div a^n = a^{m-n}\right]$

$\Rightarrow \quad 5^{m+3} = 5^5$

Comparing exponents both sides, we get

$\Rightarrow \quad m+3 = 5$

$\Rightarrow \quad m = 5-3$

$\Rightarrow \quad m = 2$

Question 6:

Evaluate:

(i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$

Answer 6:

(i) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}=\left\{\left(\frac{3}{1}\right)^{1}-\left(\frac{4}{1}\right)^{1}\right\}$ $\left[\because\ a^{-m}=\frac{1}{a^m}\right]$

$=\{3-4\}=-1$

(ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}=\frac{5^{-7}}{8^{-7}}\times\frac{8^{-4}}{5^{-4}}$ $\left[\because\ \left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}\right]$

$=5^{-7-(-4)}\times 8^{-4-(-7)}$ $\left[\because\ a^m\div a^n=a^{m-n}\right]$

$=5^{-7+4}\times 8^{-4+7}=5^{-3}\times 8^{3}=\frac{8^3}{5^3}$ $\left[\because\ a^{-m}=\frac{1}{a^m}\right]$

$=\frac{512}{125}$

Question 7:

Simplify:

(i) $\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}\quad (t\neq 0)$ (ii) $\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}$

Answer 7:

(i) $\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}=\frac{5^2\times t^{-4}}{5^{-3}\times 5\times 2\times t^{-8}}=\frac{5^{2-(-3)-1}\times t^{-4-(-8)}}{2}$ $\left[\because\ a^m\div a^n=a^{m-n}\right]$

$=\frac{5^{2+3-1}\times t^{-4+8}}{2}=\frac{5^4\times t^4}{2}=\frac{625}{2}t^4$

(ii) $\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}=\frac{3^{-5}\times(2\times 5)^{-5}\times 5^3}{5^{-7}\times(2\times 3)^{-5}}=\frac{3^{-5}\times 2^{-5}\times 5^{-5}\times 5^3}{5^{-7}\times 2^{-5}\times 3^{-5}}$ $\left[\because\ (ab)^m=a^m b^m\right]$

$=\frac{3^{-5}\times 2^{-5}\times 5^{-5+3}}{5^{-7}\times 2^{-5}\times 3^{-5}}=\frac{3^{-5}\times 2^{-5}\times 5^{-2}}{5^{-7}\times 2^{-5}\times 3^{-5}}$ $\left[\because\ a^m\times a^n=a^{m+n}\right]$

$=3^{-5-(-5)}\times 2^{-5-(-5)}\times 5^{-2-(-7)}$ $\left[\because\ a^m\div a^n=a^{m-n}\right]$

$=3^{-5+5}\times 2^{-5+5}\times 5^{-2+7}=3^0\times 2^0\times 5^5$

= 1 x 1 x 3125 $\left[\because\ a^0=1\right]$

= 3125