

प्रश्न 1:

त्रिकोणमितीय अनुपातों $\sin A$, $\cos A$ और $\tan A$ को के $\cot A$ पदों में व्यक्त कीजिए।

उत्तर 1:

(i). $\sin A$

$$\begin{aligned} &= \sqrt{\sin^2 A} \\ &= \sqrt{\frac{1}{\operatorname{cosec}^2 A}} \\ &= \sqrt{\frac{1}{1+\cot^2 A}} \\ &= \frac{1}{\sqrt{1+\cot^2 A}} \end{aligned}$$

$$\left[\text{क्योंकि } \sin A = \frac{1}{\operatorname{cosec} A} \right]$$

$$\left[\text{क्योंकि } \operatorname{cosec}^2 A = 1 + \cot^2 A \right]$$

(ii). $\cos A$

$$\begin{aligned} &= \sqrt{\cos^2 A} \\ &= \sqrt{1 - \sin^2 A} \\ &= \sqrt{1 - \frac{1}{\operatorname{cosec}^2 A}} \\ &= \sqrt{1 - \frac{1}{1+\cot^2 A}} \\ &= \sqrt{\frac{1+\cot^2 A - 1}{1+\cot^2 A}} \\ &= \frac{\cot A}{\sqrt{1+\cot^2 A}} \end{aligned}$$

$$\left[\text{क्योंकि } \cos^2 A = 1 - \sin^2 A \right]$$

$$\left[\text{क्योंकि } \sin A = \frac{1}{\operatorname{cosec} A} \right]$$

$$\left[\text{क्योंकि } \operatorname{cosec}^2 A = 1 + \cot^2 A \right]$$

(iii). $\tan A = \frac{1}{\cot A}$

$$\left[\text{क्योंकि } \tan A = \frac{1}{\cot A} \right]$$

प्रश्न 2:

$\angle A$ के अन्य सभी त्रिकोणमितीय अनुपातों को $\sec A$ के पदों में लिखिए।

उत्तर 2:

(i). $\sin A$

$$\begin{aligned} &= \sqrt{\sin^2 A} \\ &= \sqrt{1 - \cos^2 A} \\ &= \sqrt{1 - \frac{1}{\sec^2 A}} \\ &= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} \\ &= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \end{aligned}$$

$$\left[\text{क्योंकि } \sin^2 A = 1 - \cos^2 A \right]$$

$$\left[\text{क्योंकि } \cos A = \frac{1}{\sec A} \right]$$

(ii). $\cos A$

$$\cos A = \frac{1}{\sec A}$$

$$\left[\text{क्योंकि } \cos A = \frac{1}{\sec A} \right]$$

गणित

(www.tiwariacademy.com)
(पाठ - 8) (त्रिकोणमिति का परिचय)
(कक्षा 10)

$$\text{(iii). } \tan A = \sqrt{\tan^2 A} \\ = \sqrt{\sec^2 A - 1}$$

$$[\text{क्योंकि } \sec^2 A = 1 + \tan^2 A]$$

$$\text{(iv). } \operatorname{cosec} A = \sqrt{\operatorname{cosec}^2 A} \\ = \sqrt{1 + \cot^2 A} \\ = \sqrt{1 + \frac{1}{\tan^2 A}} \\ = \sqrt{1 + \frac{1}{\sec^2 A - 1}} \\ = \sqrt{\frac{\sec^2 A - 1 + 1}{\sec^2 A - 1}} \\ = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

$$[\text{क्योंकि } \operatorname{cosec}^2 A = 1 + \cot^2 A]$$

$$[\text{क्योंकि } \cot A = \frac{1}{\tan A}]$$

$$[\text{क्योंकि } \sec^2 A = 1 + \tan^2 A]$$

$$\text{(v). } \cot A = \sqrt{\cot^2 A} \\ = \sqrt{\frac{1}{\tan^2 A}} \\ = \sqrt{\frac{1}{\sec^2 A - 1}} \\ = \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$[\text{क्योंकि } \cot A = \frac{1}{\tan A}]$$

$$[\text{क्योंकि } \sec^2 A = 1 + \tan^2 A]$$

प्रश्न 3:

मान निकालिए:

$$\text{(i). } \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$\text{(ii). } \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

उत्तर 3:

$$\text{(i). } \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$= \frac{\sin^2 63^\circ + \cos^2 (90^\circ - 27^\circ)}{\cos^2 17^\circ + \sin^2 (90^\circ - 73^\circ)}$$

$$= \frac{\sin^2 63^\circ + \cos^2 63^\circ}{\cos^2 17^\circ + \sin^2 17^\circ}$$

$$= \frac{1}{1} = 1$$

$$[\text{क्योंकि } \cos(90^\circ - \theta) = \sin \theta \text{ और } \sin(90^\circ - \theta) = \cos \theta]$$

$$[\text{क्योंकि } \sin^2 \theta + \cos^2 \theta = 1]$$

$$\text{(ii). } \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

$$= \cos(90^\circ - 25^\circ) \cos 65^\circ + \sin(90^\circ - 25^\circ) \sin 65^\circ$$

$$= \cos 65^\circ \cos 65^\circ + \sin 65^\circ \sin 65^\circ$$

$$= \cos^2 65^\circ + \sin^2 65^\circ = 1 \quad [\text{क्योंकि } \sin^2 \theta + \cos^2 \theta = 1]$$

प्रश्न 4:

सही विकल्प चुनिए और अपने विकल्प की पुष्टि कीजिए:

$$\text{(i). } 9\sec^2 A - 9\tan^2 B \text{ बराबर है:}$$

$$(A) 1$$

$$(B) 9$$

$$(C) 8$$

$$(D) 0$$

(कक्षा 10)

(ii). $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$ बराबर है:

- (A) 0 (B) 1 (C) 2 (D) -1

(iii). $(\sec A + \tan A)(1 - \sin A)$ बराबर है:

- (A) $\sec A$ (B) $\sin A$ (C) $\operatorname{cosec} A$ (D) $\cos A$

(iv). $\frac{1 + \tan^2 A}{1 + \cot^2 A}$ बराबर है:

- (A) $\sec^2 A$ (B) -1 (C) $\cot^2 A$ (D) $\tan^2 A$

उत्तर 4:

(i). $9\sec^2 A - 9\tan^2 B$

दिया है $9\sec^2 A - 9\tan^2 B$

$$= 9(\sec^2 A - \tan^2 B)$$

$$= 9(1) \quad [\text{क्योंकि } \sec^2 A - \tan^2 B = 1]$$

$$= 9$$

इसलिए, विकल्प (B) सही है।

(ii). $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$

दिया है $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$

$$= 1 + \cot \theta - \operatorname{cosec} \theta + \tan \theta + \tan \theta \cot \theta - \tan \theta \operatorname{cosec} \theta + \sec \theta + \sec \theta \cot \theta - \sec \theta \operatorname{cosec} \theta$$

$$= 1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} + \frac{\sin \theta}{\cos \theta} + 1 - \frac{\sin \theta}{\cos \theta} \times \frac{1}{\sin \theta} + \frac{1}{\cos \theta} + \frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} - \frac{1}{\cos \theta} \times \frac{1}{\sin \theta}$$

$$[\text{क्योंकि } \tan \theta \cot \theta = 1]$$

$$= 1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} + \frac{\sin \theta}{\cos \theta} + 1 - \frac{1}{\cos \theta} + \frac{1}{\cos \theta} + \frac{1}{\sin \theta} - \frac{1}{\sin \theta \cos \theta}$$

$$= 1 + \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} + 1 - \frac{1}{\sin \theta \cos \theta}$$

$$= 2 + \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} - \frac{1}{\sin \theta \cos \theta}$$

$$= 2 + \frac{\cos^2 \theta + \sin^2 \theta - 1}{\sin \theta \cos \theta}$$

$$= 2 + \frac{1-1}{\sin \theta \cos \theta} \quad [\text{क्योंकि } \cos^2 \theta + \sin^2 \theta = 1]$$

$$= 2 + 0 = 2$$

इसलिए, विकल्प (C) सही है।

(iii). $(\sec A + \tan A)(1 - \sin A)$

दिया है $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) (1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A} \right) (1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A} = \cos A$$

इसलिए, विकल्प (D) सही है।

(iv). $\frac{1+\tan^2 A}{1+\cot^2 A}$

दिया है $\frac{1+\tan^2 A}{1+\cot^2 A}$

$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$

[क्योंकि $\operatorname{cosec}^2 A = 1 + \cot^2 A$, $\sec^2 A = 1 + \tan^2 A$]

$= \left(\frac{1}{\cos^2 A} \right)$

$= \left(\frac{1}{\sin^2 A} \right)$

$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A$

इसलिए, विकल्प (D) सही है।

प्रश्न 5:

निम्नलिखित सर्वसमिकाएँ सिद्ध कीजिए, जहाँ वे कोण, जिनके लिए व्यंजक परिभाषित है, न्यून कोण है:

(i). $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1-\cos \theta}{1+\cos \theta}$

(ii). $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2 \sec A$

(iii). $\frac{\tan \theta}{1-\cot \theta} + \frac{\cot \theta}{1-\tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$

(iv). $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

(v). सर्वसमिका $\operatorname{cosec}^2 A = 1 + \cot^2 A$ को लागू करके

$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

(vi). $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii). $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$

(viii). $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix). $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$ (x). $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A} \right)^2 = \tan^2 A$

उत्तर 5:

(i). $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1-\cos \theta}{1+\cos \theta}$

वाम पक्ष $= (\operatorname{cosec} \theta - \cot \theta)^2$

$= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2$

$= \left(\frac{1-\cos \theta}{\sin \theta} \right)^2$

$= \frac{(1-\cos \theta)^2}{\sin^2 \theta}$

$= \frac{(1-\cos \theta)^2}{1-\cos^2 \theta}$

[क्योंकि $\sin^2 \theta = 1 - \cos^2 \theta$]

$= \frac{(1-\cos \theta)(1-\cos \theta)}{(1-\cos \theta)(1+\cos \theta)}$

$= \frac{1-\cos \theta}{1+\cos \theta} = \text{दाया पक्ष}$

$$(ii). \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2 \sec A$$

$$\text{वाम पक्ष} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) \cos A}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A) \cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A) \cos A}$$

$$= \frac{2 + 2\sin A}{(1 + \sin A) \cos A}$$

$$= \frac{2(1 + \sin A)}{(1 + \sin A) \cos A}$$

$$= \frac{2}{\cos A} = 2 \sec A = \text{दाया पक्ष}$$

$$[\text{क्योंकि } \sin^2 A + \cos^2 A = 1]$$

$$(iii). \frac{\tan \theta}{1-\cot \theta} + \frac{\cot \theta}{1-\tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$\text{वाम पक्ष} = \frac{\tan \theta}{1-\cot \theta} + \frac{\cot \theta}{1-\tan \theta}$$

$$= \left[\frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} \right] + \left[\frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \right]$$

$$[\text{क्योंकि } \tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}]$$

$$= \left[\frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} \right] + \left[\frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \right]$$

$$= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta (\cos \theta - \sin \theta)}$$

$$= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)} \quad [\text{क्योंकि } (\cos \theta - \sin \theta) = -(\sin \theta - \cos \theta)]$$

$$= \frac{\sin^3 \theta - \cos^3 \theta}{\cos \theta \sin \theta (\sin \theta - \cos \theta)}$$

$$= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \cos \theta \sin \theta)}{\cos \theta \sin \theta (\sin \theta - \cos \theta)} \quad [\text{क्योंकि } a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]$$

$$= \frac{(1 + \cos \theta \sin \theta)}{\cos \theta \sin \theta} \quad [\text{क्योंकि } \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{1}{\cos \theta \sin \theta} + \frac{\cos \theta \sin \theta}{\cos \theta \sin \theta}$$

$$= \sec \theta \operatorname{cosec} \theta + 1$$

$$= \text{दाया पक्ष}$$

(iv). $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

वाम पक्ष = $\frac{1+\sec A}{\sec A}$

= $\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$

= $\frac{\cos A + 1}{\cos A} \times \frac{\cos A}{\cos A}$

= $\frac{1 + \cos A}{1}$

= $\frac{1 + \cos A}{1} \times \frac{1 - \cos A}{1 - \cos A}$

= $\frac{1 - \cos^2 A}{1 - \cos A}$

= $\frac{\sin^2 A}{1 - \cos A}$

= दाया पक्ष

[क्योंकि $\sec A = \frac{1}{\cos A}$]

[क्योंकि $1 - \cos^2 A = \sin^2 A$]

(v). $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

वाम पक्ष = $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

= $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

= $\frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$

= $\frac{\cot A + \operatorname{cosec} A - (1)}{\cot A + 1 - \operatorname{cosec} A}$

= $\frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A}$ [क्योंकि $\operatorname{cosec}^2 A - \cot^2 A = 1$]

= $\frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{\cot A + 1 - \operatorname{cosec} A}$

= $\frac{(\cot A + \operatorname{cosec} A)(1 - \operatorname{cosec} A + \cot A)}{1 - \operatorname{cosec} A + \cot A}$

= $\cot A + \operatorname{cosec} A$

= दाया पक्ष

[अंश और हर को $\sin A$ से भाग करने पर]

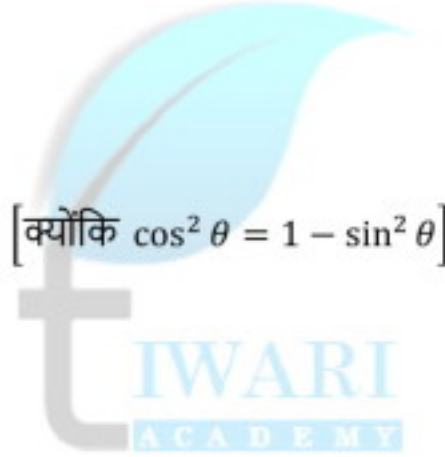
(vi). $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

वाम पक्ष = $\sqrt{\frac{1+\sin A}{1-\sin A}}$

$$\begin{aligned}
 &= \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 + \sin A}{1 + \sin A}} \\
 &= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} \\
 &= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} \quad \left[\text{क्योंकि } 1 - \sin^2 A = \cos^2 A \right] \\
 &= \frac{1 + \sin A}{\cos A} \\
 &= \frac{1}{\cos A} + \frac{\sin A}{\cos A} \\
 &= \sec A + \tan A \\
 &= \text{दाँया पक्ष}
 \end{aligned}$$

(vii). $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$

$$\begin{aligned}
 \text{वाम पक्ष} &= \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} \\
 &= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)} \\
 &= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta [2(1 - \sin^2 \theta) - 1]} \quad \left[\text{क्योंकि } \cos^2 \theta = 1 - \sin^2 \theta \right] \\
 &= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 - 2 \cos^2 \theta - 1)} \\
 &= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (1 - 2 \sin^2 \theta)} \\
 &= \frac{\sin \theta}{\cos \theta} = \tan \theta \\
 &= \text{दाँया पक्ष}
 \end{aligned}$$



(viii). $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$$\begin{aligned}
 \text{वाम पक्ष} &= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\
 &= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A \sec A \\
 &= (\sin^2 A + \cos^2 A) + \operatorname{cosec}^2 A + 2 + \sec^2 A + 2 \\
 &\quad \left[\text{क्योंकि } \cos A \sec A = 1, \quad \sin A \operatorname{cosec} A = 1 \right] \\
 &= 1 + (1 + \cot^2 A) + 2 + (1 + \tan^2 A) + 2 \\
 &\quad \left[\text{क्योंकि } \operatorname{cosec}^2 A = 1 + \cot^2 A, \quad \sec^2 A = 1 + \tan^2 A \right] \\
 &= 7 + \tan^2 A + \cot^2 A \\
 &= \text{दाँया पक्ष}
 \end{aligned}$$

(कक्षा 10)

(ix). $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

वाम पक्ष = $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right)$$

$$= \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right)$$

$$= \sin A \cos A \quad \dots (i)$$

= दाया पक्ष = $\frac{1}{\tan A + \cot A}$

$$= \frac{1}{\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right)} = \frac{1}{\left(\frac{\sin^2 A + \cos^2 A}{\cos A \sin A} \right)} = \frac{1}{\left(\frac{1}{\cos A \sin A} \right)}$$

$$= \cos A \sin A \quad \dots (ii)$$

समीकरण (i) और (ii) से

वाम पक्ष = दाया पक्ष

(x). $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$

वाम पक्ष = $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \left(\frac{1}{\cos^2 A} \right) \left(\frac{1}{\sin^2 A} \right)$$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A$$

= दाया पक्ष

अब, $\left(\frac{1 - \tan A}{1 - \cot A} \right)^2$

$$= \left(\frac{1 - \frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}} \right)^2 = \left(\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}} \right)^2$$

$$= \left(\frac{\cos A - \sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} \right)^2$$

$$= \left(-\frac{\sin A - \cos A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} \right)^2$$

$$= \left(-\frac{\sin A}{\cos A} \right)^2 = \tan^2 A = \text{दाया पक्ष}$$