

Mathematics

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(Chapter - 8) (Introduction to Trigonometry)

(Class 10)

Exercise 8.1

Question 1:

In $\triangle ABC$, right angled at B, $AB = 24\text{cm}$, $BC = 7\text{cm}$. Determine:

(i) $\sin A$, $\cos A$

(ii) $\sin C$, $\cos C$

Answer 1:

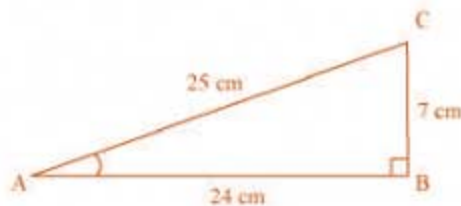
In $\triangle ABC$, by Pythagoras theorem, we have

$$\begin{aligned}AC^2 &= AB^2 + BC^2 \\&= (24\text{ cm})^2 + (7\text{ cm})^2 \\&= (576 + 49)\text{ cm}^2 \\&= 625\text{ cm}^2\end{aligned}$$

$$\Rightarrow AC = \sqrt{625} = 25\text{ cm}$$

$$(i) \sin A = \frac{BC}{AC} = \frac{7}{25} \text{ and } \cos A = \frac{AB}{AC} = \frac{24}{25}$$

$$(ii) \sin C = \frac{AB}{AC} = \frac{24}{25} \text{ and } \cos C = \frac{BC}{AC} = \frac{7}{25}$$



Question 2:

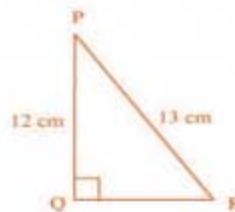
In figure, find $\tan P - \cot R$.

Answer 2:

In $\triangle PQR$, by Pythagoras theorem, we have

$$\begin{aligned}QR^2 &= PR^2 - PQ^2 \\&= (13)^2 - (12)^2 \\&= 169 - 144 = 25 \\&\Rightarrow QR = \sqrt{25} = 5\end{aligned}$$

$$\text{Hence, } \tan P - \cot R = \frac{QR}{PQ} - \frac{QR}{PQ} = \frac{5}{12} - \frac{5}{12} = 0$$



Question 3:

If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Answer 3:

$$\text{Given that: } \sin A = \frac{3}{4}$$

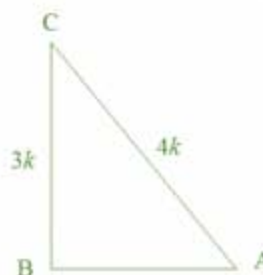
$$\text{Let } \sin A = \frac{3k}{4k}, \text{ where } k \text{ is a real number.}$$

In $\triangle ABC$, by Pythagoras theorem, we have

$$\begin{aligned}AB^2 &= AC^2 - BC^2 \\&= (4k)^2 - (3k)^2 \\&= 16k^2 - 9k^2 \\&= 7k^2\end{aligned}$$

$$\Rightarrow AB = \sqrt{7k^2} = \sqrt{7}k$$

$$\text{Hence, } \cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4} \text{ and } \tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$



Question 4:

Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.

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Answer 4:

Given that: $15 \cot A = 8$

$$\Rightarrow \cot A = \frac{8}{15}$$

Let $\cot A = \frac{8k}{15k}$, where k is a real number.

In $\triangle ABC$, by Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

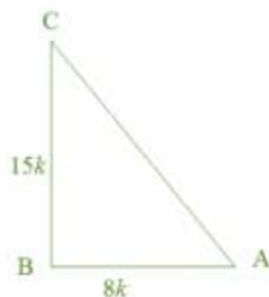
$$= (8k)^2 + (15k)^2$$

$$= 64k^2 + 225k^2$$

$$= 289k^2$$

$$\Rightarrow AC = \sqrt{289k^2} = 17k$$

$$\text{Hence, } \sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17} \text{ and } \sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$$



Question 5:

Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Answer 5:

Given that: $\sec \theta = \frac{13}{12}$

Let $\sec \theta = \frac{13k}{12k}$, where k is a real number.

In $\triangle ABC$, by Pythagoras theorem, we have

$$BC^2 = AC^2 - AB^2$$

$$= (13k)^2 - (12k)^2$$

$$= 169k^2 - 144k^2$$

$$= 25k^2$$

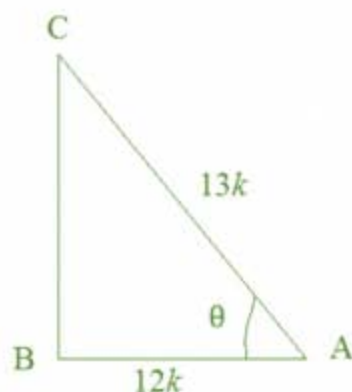
$$\Rightarrow BC = \sqrt{25k^2} = 5k$$

$$\text{Hence, } \sin \theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos \theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan \theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\operatorname{cosec} \theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5} \text{ and } \cot \theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$



Question 6:

If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

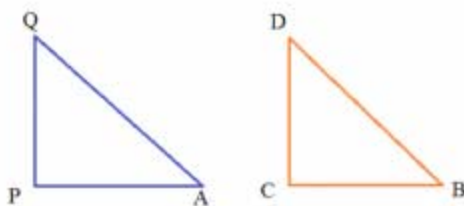
Answer 6:

Given that: $\cos A = \cos B$

$$\cos A = \cos B$$

$$\Rightarrow \frac{AP}{AQ} = \frac{BC}{BD}$$

$$\Rightarrow \frac{AP}{BC} = \frac{AQ}{BD}$$



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$$\text{Let } \frac{AP}{BC} = \frac{AQ}{BD} = k \quad \dots (i)$$

Therefore, $AP = k(BC)$ and $AQ = k(BD)$

Now, in $\triangle APQ$ and $\triangle BCD$

$$\frac{PQ}{CD} = \frac{\sqrt{AQ^2 - AP^2}}{\sqrt{BD^2 - BC^2}} = \frac{\sqrt{(k \cdot BD)^2 - (k \cdot BC)^2}}{\sqrt{BD^2 - BC^2}} = \frac{k\sqrt{BD^2 - BC^2}}{\sqrt{BD^2 - BC^2}} = k \quad \dots (ii)$$

From the equation (i) and (ii), we get

$$\frac{AP}{BC} = \frac{AQ}{BD} = \frac{PQ}{CD}$$

So, $\triangle APQ \sim \triangle BCD$

[SSS similarity criteria]

Hence, $\angle A = \angle B$

Question 7:

If $\cot \theta = \frac{7}{8}$, evaluate: (i) $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$ (ii) $\cot^2 \theta$

Answer 7:

Given that: $\cot \theta = \frac{7}{8}$

Let $\cot \theta = \frac{7k}{8k}$, where k is a real number.

In $\triangle ABC$, by Pythagoras theorem, we have

$$AC^2 = BC^2 + AB^2$$

$$= (8k)^2 + (7k)^2$$

$$= 64k^2 + 49k^2$$

$$= 113k^2$$

$$\Rightarrow AC = \sqrt{113k^2} = \sqrt{113}k$$

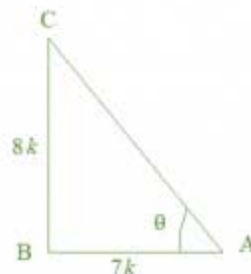
$$(i) \frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$$

$$= \frac{\left(1 + \frac{8}{\sqrt{113}}\right)\left(1 - \frac{8}{\sqrt{113}}\right)}{\left(1 + \frac{7}{\sqrt{113}}\right)\left(1 - \frac{7}{\sqrt{113}}\right)}$$

$$= \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}} = \frac{\frac{113 - 64}{113}}{\frac{113 - 49}{113}} = \frac{49}{64}$$

$$(ii) \cot^2 \theta$$

$$= (\cot \theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$



Question 8:

If $3 \cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer 8:

Given that: $3 \cot A = 4$

$$\Rightarrow \cot A = \frac{4}{3}$$

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Let $\cot A = \frac{4k}{3k}$, where k is a real number.

In $\triangle ABC$, by Pythagoras theorem, we have

$$AC^2 = BC^2 + AB^2$$

$$= (3k)^2 + (4k)^2$$

$$= 9k^2 + 16k^2$$

$$= 25k^2$$

$$\Rightarrow AC = \sqrt{25k^2} = 5k$$

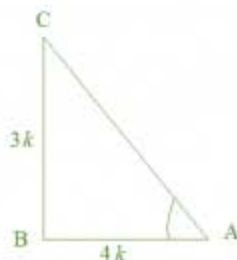
Therefore,

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{7}{25}$$

and

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{16-9}{25} = \frac{7}{25}$$

$$\text{Hence, } \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$



Question 9:

In triangle ABC, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Answer 9:

Given that: $\tan A = \frac{1}{\sqrt{3}}$

Let $\tan A = \frac{1k}{\sqrt{3}k}$, where k is a real number.

In $\triangle ABC$, by Pythagoras theorem, we have

$$AC^2 = BC^2 + AB^2$$

$$= (1k)^2 + (\sqrt{3}k)^2$$

$$= k^2 + 3k^2$$

$$= 4k^2$$

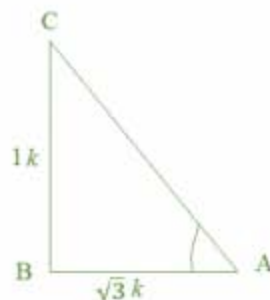
$$\Rightarrow AC = \sqrt{4k^2} = 2k$$

(i) $\sin A \cos C + \cos A \sin C$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

(ii) $\cos A \cos C - \sin A \sin C$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$



Question 10:

In $\triangle PQR$, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

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Answer 10:

Given that: in ΔPQR , angle Q is right angled.

Let $QR = x$, therefore, $PR = 25 - x$

In ΔPQR , by Pythagoras theorem, we have

$$PR^2 = PQ^2 + QR^2$$

$$\Rightarrow (25 - x)^2 = (5)^2 + (x)^2$$

$$\Rightarrow 625 + x^2 - 50x = 25 + x^2$$

$$\Rightarrow 625 - 50x = 25$$

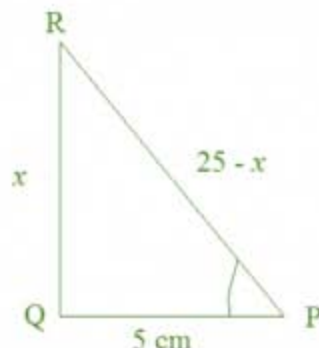
$$\Rightarrow 50x = 600$$

$$\Rightarrow x = 12$$

$$\Rightarrow QR = 12$$

$$\text{Therefore, } PR = 25 - 12 = 13$$

$$\text{Now, } \sin P = \frac{QR}{PR} = \frac{12}{13}, \cos P = \frac{PQ}{PR} = \frac{5}{13} \text{ and } \tan P = \frac{QR}{PQ} = \frac{12}{5}$$



Question 11:

State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of \cot and A.

(v) $\sin \theta = \frac{4}{3}$ for some angle θ .

Answer 11:

(i) False,

Because, $\tan A = \frac{\text{Opposite side of angle A}}{\text{Adjacent side of angle A}}$, if opposite side > adjacent side, then the value of $\tan A$ is greater than 1.

(ii) True,

Because, $\sec A = \frac{\text{Hypotenuse}}{\text{Adjacent side of angle A}}$ and we know that hypotenuse is always greater than adjacent side.

(iii) False,

Because, $\cos A$ is used for cosine of angle A.

(iv) False,

Because, $\cot A$ is used for cotangent of angle A.

(v) False,

Because, $\sin \theta = \frac{\text{Opposite side of angle A}}{\text{Hypotenuse}}$, we know that hypotenuse is always greater than opposite side.

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उत्तर 11:

(i) असत्य,

क्योंकि $\tan A = \frac{\text{कोण } A \text{ की सम्मुख भुजा}}{\text{कोण } A \text{ की संलग्न भुजा}}$, यदि कोण की सम्मुख भुजा > संलग्न भुजा, तो $\tan A$ का मान 1 से अधिक होगा।

(ii) सत्य,

क्योंकि $\sec A = \frac{\text{कर्ण}}{\text{कोण } A \text{ की संलग्न भुजा}}$, हम जानते हैं कि कर्ण, संलग्न भुजा से सदैव बड़ी होती है।

(iii) असत्य,

क्योंकि $\cos A$, कोण A के cosine के लिए प्रयुक्त एक संक्षिप्त रूप है।

(iv) असत्य,

क्योंकि $\cot A$, कोण A का *cotangent* होता है।

(v) असत्य,

क्योंकि $\sin \theta = \frac{\text{कोण } A \text{ की सम्मुख भुजा}}{\text{कर्ण}}$, हम जानते हैं कि कर्ण, सम्मुख भुजा से सदैव बड़ी होती है।

