Exercise 8.4

Question 1:
Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Answer 1:
(i). $\sin A$

$= \sqrt{\sin^2 A}$

$= \sqrt{\frac{1}{\text{cosec}^2 A}}$ $\quad \left[\because \sin A = \frac{1}{cosec\ A}\right]$

$= \sqrt{\frac{1}{1+\cot^2 A}}$ $\quad [\because \text{cosec}^2 A = 1 + \cot^2 A]$

$= \frac{1}{\sqrt{1+\cot^2 A}}$

(ii). $\cos A$

$= \sqrt{\cos^2 A}$

$= \sqrt{1 - \sin^2 A}$ $\quad [\because \cos^2 A = 1 - \sin^2 A]$

$= \sqrt{1 - \frac{1}{\text{cosec}^2 A}}$ $\quad \left[\because \sin A = \frac{1}{cosec\ A}\right]$

$= \sqrt{1 - \frac{1}{1+\cot^2 A}}$ $\quad [\because \text{cosec}^2 A = 1 + \cot^2 A]$

$$= \sqrt{\frac{1 + \cot^2 A - 1}{1 + \cot^2 A}} = \frac{\cot A}{\sqrt{1 + \cot^2 A}}$$

(iii). $\tan A = \frac{1}{\cot A}$ $\quad \left[\because \tan A = \frac{1}{cot\ A}\right]$

Question 2:
Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer 2:
(i). $\sin A$

$= \sqrt{\sin^2 A}$

$= \sqrt{1 - \cos^2 A}$ $\quad [\because \sin^2 A = 1 - \cos^2 A]$

$= \sqrt{1 - \frac{1}{\sec^2 A}}$ $\quad \left[\because \cos A = \frac{1}{sec\ A}\right]$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

(ii). $\cos A$

$\cos A = \frac{1}{sec\ A}$ $\quad \left[\because \cos A = \frac{1}{sec\ A}\right]$

(iii). $\tan A = \sqrt{\tan^2 A}$

$= \sqrt{\sec^2 A - 1}$ $\quad [\because \sec^2 A = 1 + \tan^2 A]$

(iv). $\text{cosec}\, A = \sqrt{\text{cosec}^2 A}$

$= \sqrt{1 + \cot^2 A}$ $\quad [\because \text{cosec}^2 A = 1 + \cot^2 A]$

$= \sqrt{1 + \frac{1}{\tan^2 A}}$ $\left[\because \cot A = \frac{1}{tan\,A}\right]$

$= \sqrt{1 + \frac{1}{\sec^2 A - 1}}$ $[\because \sec^2 A = 1 + \tan^2 A]$

$= \sqrt{\frac{\sec^2 A - 1 + 1}{\sec^2 A - 1}} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

(v). $\cot A = \sqrt{\cot^2 A}$

$= \sqrt{\frac{1}{\tan^2 A}}$ $\left[\because \cot A = \frac{1}{tan\,A}\right]$

$= \sqrt{\frac{1}{\sec^2 A - 1}}$ $[\because \sec^2 A = 1 + \tan^2 A]$

$= \frac{1}{\sqrt{\sec^2 A - 1}}$

Question 3:

Evaluate:

(i). $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$ **(ii).** $sin\, 25° \cos 65° + \cos 25° \sin 65°$

Answer 3:

(i). $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$= \frac{\sin^2 63° + \cos^2(90° - 27°)}{\cos^2 17° + \sin^2(90° - 73°)}$ $[\because \cos(90° - \theta) = \sin\theta \text{ and } \sin(90° - \theta) = \cos\theta]$

$= \frac{\sin^2 63° + \cos^2 63°}{\cos^2 17° + \sin^2 17°} = \frac{1}{1} = 1$ $[\because \sin^2\theta + \cos^2\theta = 1]$

(ii). $sin\, 25° \cos 65° + \cos 25° \sin 65°$

$= cos(90° - 25°) \cos 65° + \sin(90° - 25°) \sin 65° = cos\, 65° \cos 65° + \sin 65° \sin 65°$

$= \cos^2 65° + \sin^2 65° = 1$ $[\because \sin^2\theta + \cos^2\theta = 1]$

Question 4:

Choose the correct option. Justify your choice.

(i). $9sec^2 A - 9tan^2 \mathrm{B} =$

(A) 1 (B) 9 (C) 8 (D) 0

(ii). $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \mathrm{cosec}\,\theta) =$

(A) 0 (B) 1 (C) 2 (D) -1

(iii). $(\sec A + \tan A)(1 - \sin A) =$

(A) $\sec A$ (B) $\sin A$ (C) $cosec\, A$ (D) $\cos A$

(iv). $\frac{1 + tan^2 A}{1 + cot^2 A} =$

(A) $\sec^2 A$ (B) -1 (C) $cot^2 A$ (D) $tan^2 A$

Answer 4:

(i). $9sec^2 A - 9tan^2 \mathrm{B}$

Given that: $9sec^2 A - 9tan^2 \mathrm{B}$

$= 9(sec^2 A - tan^2 \mathrm{B}) = 9(1) = 9$ $[\because sec^2 A - tan^2 \mathrm{B} = 1]$

Hence, the option (B) is correct.

(ii). $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$
Given that: $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$
$= 1+\cot\theta - cosec\,\theta + \tan\theta + \tan\theta\cot\theta - \tan\theta\, cosec\,\theta + \sec\theta + \sec\theta\cot\theta - \sec\theta\, cosec\,\theta$

$$= 1+\frac{\cos\theta}{\sin\theta}-\frac{1}{sin\,\theta}+\frac{sin\,\theta}{cos\,\theta}+1-\frac{sin\,\theta}{cos\,\theta}\times\frac{1}{sin\,\theta}+\frac{1}{cos\,\theta}+\frac{1}{cos\,\theta}\times\frac{cos\,\theta}{sin\,\theta}-\frac{1}{cos\,\theta}\times\frac{1}{sin\,\theta}\quad[\because\ \tan\theta\cot\theta=1]$$

$$= 1+\frac{\cos\theta}{\sin\theta}-\frac{1}{sin\,\theta}+\frac{sin\,\theta}{cos\,\theta}+1-\frac{1}{cos\,\theta}+\frac{1}{cos\,\theta}+\frac{1}{sin\,\theta}-\frac{1}{\sin\theta\, cos\,\theta}$$

$$= 1+\frac{\cos\theta}{\sin\theta}+\frac{sin\,\theta}{cos\,\theta}+1-\frac{1}{\sin\theta\, cos\,\theta}$$

$$= 2+\frac{\cos\theta}{\sin\theta}+\frac{sin\,\theta}{cos\,\theta}-\frac{1}{\sin\theta cos\,\theta}\ = 2+\frac{\cos^2\theta+\sin^2\theta-1}{\sin\theta cos\,\theta}$$

$$= 2+\frac{1-1}{\sin\theta cos\,\theta}\qquad[\because\ \cos^2\theta+\sin^2\theta=1]$$

$= 2+0=2$
Hence, the option (C) is correct.

(iii). $(\sec A+\tan A)(1-\sin A)$
Given that: $(\sec A+\tan A)(1-\sin A)$

$$=\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)=\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)=\frac{1-\sin^2 A}{\cos A}=\frac{\cos^2 A}{\cos A}=\cos A$$

Hence, the option (D) is correct.

(iv). $\frac{1+tan^2 A}{1+cot^2 A}$

Given that: $\frac{1+tan^2 A}{1+cot^2 A}$

$$=\frac{\sec^2 A}{\operatorname{cosec}^2 A}\qquad[\because\ \operatorname{cosec}^2 A=1+\cot^2 A,\ \sec^2 A=1+\tan^2 A\ ]$$

$$=\frac{\left(\frac{1}{\cos^2 A}\right)}{\left(\frac{1}{\sin^2 A}\right)}=\frac{1}{cos^2\,A}\times\frac{sin^2\,A}{1}=tan^2\,A$$

Hence, the option (D) is correct.

Question 5:

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i). $(coscec\ \theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

(ii). $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

(iii). $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

(iv). $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$

(v). $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec} A+\cot A$, using the identity $\operatorname{cosec}^2 A=1+\cot^2 A$

(vi). $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii). $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(viii). $(\sin A+cosec\ A)^2+(\cos A+sec\ A)^2=7+\tan^2 A+\cot^2 A$

(ix). $(cosec\ A-\sin A)(sec\ A-\cos A)=\frac{1}{\tan A+\cot A}$

(x). $\frac{1+\tan^2 A}{1+\cot^2 A}=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Answer 5:

(i). $(cosec\,\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

LHS $= (cosec\,\theta - \cot\theta)^2$

$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$ $\quad [\because \sin^2\theta = 1-\cos^2\theta]$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = \text{RHS}$

(ii). $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

LHS $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A} = \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)\cos A}$

$= \frac{1+1+2\sin A}{(1+\sin A)\cos A}$ $\quad [\because \sin^2 A + \cos^2 A = 1]$

$= \frac{2+2sin\,A}{(1+sin\,A)\,cos\,A}$

$= \frac{2(1+sin\,A)}{(1+sin\,A)\,cos\,A} = \frac{2}{cos\,A} = 2\sec A = \text{RHS}$

(iii). $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\,\theta$

LHS $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \left[\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}\right] + \left[\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}\right]$ $\quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta},\ \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

$= \left[\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}\right] + \left[\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}\right] = \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$ $\quad [\because (\cos\theta-\sin\theta) = -(\sin\theta-\cos\theta)]$

$= \frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\cos\theta\sin\theta)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$ $\quad [\because a^3-b^3 = (a-b)(a^2+b^2+ab)]$

$= \frac{(1+\cos\theta\sin\theta)}{\cos\theta\sin\theta}$ $\quad [\because \sin^2\theta+\cos^2\theta = 1]$

$= \frac{1}{\cos\theta\sin\theta} + \frac{\cos\theta\sin\theta}{\cos\theta\sin\theta}$

$= \sec\theta\,\text{cosec}\,\theta + 1 = \text{RHS}$

(iv). $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

LHS $= \frac{1+\sec A}{\sec A}$

$= \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$ $\left[\because \sec A = \frac{1}{\cos A}\right]$

$= \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}} = \frac{1+\cos A}{1} = \frac{1+\cos A}{1} \times \frac{1-\cos A}{1-\cos A} = \frac{1-\cos^2 A}{1-\cos A}$

$= \frac{\sin^2 A}{1-\cos A}$ $[\because 1-\cos^2 A = \sin^2 A]$

$=$ RHS

(v). $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

LHS $= \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$ [Dividing Numerator and Denominator by $\sin A$]

$= \frac{\cot A + \operatorname{cosec} A - (1)}{\cot A + 1 - \operatorname{cosec} A}$

$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A}$ $[\because \operatorname{cosec}^2 A - \cot^2 A = 1]$

$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{\cot A + 1 - \operatorname{cosec} A} = \frac{(\cot A + \operatorname{cosec} A)(1 - \operatorname{cosec} A + \cot A)}{1 - \operatorname{cosec} A + \cot A}$

$= \cot A + \operatorname{cosec} A$ $=$ RHS

(vi). $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

LHS $= \sqrt{\frac{1+\sin A}{1-\sin A}}$

$= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$

$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$ $[\because 1-\sin^2 A = \cos^2 A]$

$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

$= \sec A + \tan A =$ RHS

(vii). $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

LHS $= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta\,(1-2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta[2(1-\sin^2\theta)-1]}$ $\quad [\because\ \cos^2\theta = 1-\sin^2\theta]$

$= \frac{\sin\theta\,(1-2\sin^2\theta)}{\cos\theta\,(2-2\cos^2\theta-1)} = \frac{\sin\theta\,(1-2\sin^2\theta)}{\cos\theta\,(1-2\sin^2\theta)}$

$= \frac{\sin\theta}{\cos\theta} = \tan\theta \quad = \text{RHS}$

(viii). $(\sin A + cosec\ A)^2 + (\cos A + sec\ A)^2 = 7 + \tan^2 A + \cot^2 A$

LHS $= (\sin A + cosec\ A)^2 + (\cos A + sec\ A)^2$

$= \sin^2 A + \text{cosec}^2 A + 2\sin A\, cosec\ A + \cos^2 A + \sec^2 A + 2\cos A\, sec\ A$

$= (\sin^2 A + \cos^2 A) + \text{cosec}^2 A + 2 + \sec^2 A + 2$

$[\because\ \cos A\, sec\ A = 1, \quad \sin A\, cosec\ A = 1\,]$

$= 1 + (1+\cot^2 A) + 2 + (1+\tan^2 A) + 2$ $\quad [\because\ \text{cosec}^2 A = 1+\cot^2 A, \quad \sec^2 A = 1+\tan^2 A\,]$

$= 7 + \tan^2 A + \cot^2 A = \text{RHS}$

(ix). $(cosec\ A - \sin A)(sec\ A - \cos A) = \frac{1}{\tan A + \cot A}$

LHS $= (cosec\ A - \sin A)(sec\ A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right) = \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right) = \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$

$= \sin A\, cos\ A$ $\quad \dots \text{(i)}$

$= \text{RHS} = \frac{1}{\tan A + \cot A}$

$= \frac{1}{\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)} = \frac{1}{\left(\frac{\sin^2 A + \cos^2 A}{\cos A \sin A}\right)} = \frac{1}{\left(\frac{1}{\cos A \sin A}\right)}$

$= \cos A \sin A$ $\quad \dots \text{(ii)}$

From the equation (i) and (ii), we get, LHS = RHS

(x). $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

LHS $= \frac{1+\tan^2 A}{1+\cot^2 A}$

$= \frac{\sec^2 A}{\text{cosec}^2 A}$ $\quad [\because\ \text{cosec}^2 A = 1+\cot^2 A,\ \sec^2 A = 1+\tan^2 A\,]$

$= \frac{\left(\frac{1}{\cos^2 A}\right)}{\left(\frac{1}{\sin^2 A}\right)} = \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A = \text{RHS}$

Now, $\left(\frac{1-\tan A}{1-\cot A}\right)^2$

$= \left(\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right)^2 = \left(\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}}\right)^2 = \left(\frac{\cos A - \sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A}\right)^2$

$= \left(-\frac{\sin A - \cos A}{\cos A} \times \frac{\sin A}{\sin A - \cos A}\right)^2 = \left(-\frac{\sin A}{\cos A}\right)^2 = \tan^2 A = \text{RHS}$