Exercise 8.3

Question 1:
Evaluate:

(i). $\frac{\sin 18°}{\cos 72°}$ **(ii).** $\frac{\tan 26°}{\cot 64°}$ **(iii).** $\cos 48° - \sin 42°$ **(iv).** $cosec\ 31° - \sec 59°$

Answer 1:
(i) $\frac{\sin 18°}{\cos 72°}$

$= \frac{\sin 18°}{\cos 72°} = \frac{\cos(90°-18°)}{\cos 72°}$ $[\because \cos(90° - \theta) = \sin\theta]$

$= \frac{\cos 72°}{\cos 72°} = 1$

(ii) $\frac{\tan 26°}{\cot 64°}$

$\frac{\tan 26°}{\cot 64°} = \frac{\cot(90°-26°)}{\cot 64°}$ $[\because \cot(90° - \theta) = \tan\theta]$

$= \frac{\cot 64°}{\cot 64°} = 1$

(iii) $\cos 48° - \sin 42°$

$\cos 48° - \sin 42° = \cos 48° - \cos(90° - 42°)$ $[\because \cos(90° - \theta) = \sin\theta]$

$= \cos 48° - \cos 48° = 0$

(iv) $cosec\ 31° - \sec 59°$

$\text{cosec}\, 31° - \sec 59° = \text{cosec}\, 31° - \text{cosec}(90° - 59°)$ $[\because \text{cosec}(90° - \theta) = \sec\theta]$

$= \text{cosec}\, 31° - \text{cosec}\, 31° = 0$

Question 2:
Show that:

(i). $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

(ii). $\cos 38° \cos 52° - \sin 38° \sin 52° = 0$

Answer 2:
(i). $\tan 48° \tan 23° \tan 42° \tan 67° = 1$

$\text{LHS} = \tan 48° \tan 23° \tan 42° \tan 67°$

$= \tan 48° \tan 23° \cot(90° - 42°) \cot(90° - 67°)$ $[\because \cot(90° - \theta) = \tan\theta]$

$= \tan 48° \tan 23° \cot 48° \cot 23°$

$= \tan 48° \tan 23° \times \frac{1}{\tan 48°} \times \frac{1}{\tan 23°}$

$= 1 = \text{RHS}$

(ii). $\cos 38° \cos 52° - \sin 38° \sin 52° = 0$

$\text{LHS} = \cos 38° \cos 52° - \sin 38° \sin 52°$

$= \cos 38° \cos 52° - \cos(90° - 38°) \cos(90° - 52°)$ $[\because \cos(90° - \theta) = \sin\theta]$

$= \cos 38° \cos 52° - \cos 52° \cos 38°$

$= 0 = \text{RHS}$

Question 3:

If $tan\ 2A = cot(A - 18°)$, where $2A$ is an acute angle, find the value of A.

Answer 3:

Given that: $tan\ 2A = cot(A - 18°)$

$\Rightarrow cot(90° - 2A) = cot(A - 18°)$ $[\because\ \cot(90° - \theta) = \tan\theta]$

$\Rightarrow 90° - 2A = A - 18°$ $\Rightarrow 90° + 18° = 3A$

$\Rightarrow 3A = 108°$ $\Rightarrow A = 36°$

Hence, $A = 36°$

Question 4:

If $tan\ A = cot$B, prove that $A + B = 90°$.

Answer 4:

Given that: $tan\ A = cot$B

$\Rightarrow cot(90° - A) = cot$B $[\because\ \cot(90° - \theta) = \tan\theta]$

$\Rightarrow 90° - A = B$ $\Rightarrow 90° = A + B$

Hence, $A + B = 90°$

Question 5:

If $sec\ 4A = cosec(A - 20°)$, where $4A$ is an acute angle, find the value of A.

Answer 5:

Given that: $sec\ 4A = cosec(A - 20°)$

$\Rightarrow cosec(90° - 4A) = cosec(A - 20°)$ $[\because\ \text{cosec}(90° - \theta) = \sec\theta]$

$\Rightarrow 90° - 4A = A - 20°$

$\Rightarrow 90° + 20° = 5A$ $\Rightarrow 5A = 110°$ $\Rightarrow A = 22°$

Hence, $A = 22°$

Question 6:

If A, B and C are interior angles of a triangle ABC, then show that

$$\sin\left(\frac{B + C}{2}\right) = \cos\frac{A}{2}$$

Answer 6:

LHS $= \sin\left(\frac{B+C}{2}\right)$

$= \sin\left(\frac{180° - A}{2}\right)$ $[\because\ A + B + C = 180°]$

$= \sin\left(90° - \frac{A}{2}\right)$

$= \cos\frac{A}{2} =$ RHS $[\because\ \sin(90° - \theta) = \cos\theta]$

Question 7:

Express sin 67°+ cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Answer 7:

We know that $\sin\theta = \cos(90° - \theta)$ and $\cos\theta = \sin(90° - \theta)$

Therefore, $\sin 67° + cos75° = cos(90° - 67°) + sin(90° - 75°)$

$= \cos 23° + \sin 15°$