Mathematics

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(Chapter – 8) (Introduction to Trigonometry)
(Class 10)

Exercise 8.2

Question 1:

Evaluate the following:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\frac{\cos 45°}{\sec 30° + cosec\ 30°}$

(iv) $\frac{\sin 30° + \tan 45° - cosec\ 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Answer 1:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

Putting the value of each trigonometric ratios, we get

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

Putting the value of each trigonometric ratios, we get

$$2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\frac{\cos 45°}{\sec 30° + cosec\ 30°}$

Putting the value of each trigonometric ratios, we get

$$\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{2}(2 + 2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}} \times \frac{2\sqrt{2} - 2\sqrt{6}}{2\sqrt{2} - 2\sqrt{6}} = \frac{2\sqrt{6} - 2\sqrt{18}}{(2\sqrt{2})^2 - (2\sqrt{6})^2} = \frac{2\sqrt{6} - 6\sqrt{2}}{8 - 24} = \frac{-2(3\sqrt{2} - \sqrt{6})}{-16}$$

$$= \frac{3\sqrt{2} - \sqrt{6}}{8}$$

(iv) $\frac{\sin 30° + \tan 45° - cosec\ 60°}{\sec 30° + \cos 60° + \cot 45°}$

Putting the value of each trigonometric ratios, we get

$$\frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}$$

$$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4} = \frac{27 - 12\sqrt{3} - 12\sqrt{3} + 16}{(3\sqrt{3})^2 - 4^2}$$

$$= \frac{43 - 24\sqrt{3}}{27 - 16} = \frac{43 - 24\sqrt{3}}{11}$$

(v) $\frac{5\cos^2 60°+4\sec^2 30°-\tan^2 45°}{\sin^2 30°+\cos^2 30°}$

Putting the value of each trigonometric ratios, we get

$$\frac{5\left(\frac{1}{2}\right)^2+4\left(\frac{2}{\sqrt{3}}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\frac{\frac{5}{4}+\frac{16}{3}-1}{\frac{1}{4}+\frac{3}{4}}=\frac{\frac{15+64-12}{12}}{\frac{4}{4}}=\frac{67}{12}$$

Question 2:

Choose the correct option and justify your choice:

(i) $\frac{2\tan 30°}{1+\tan^2 30°}=$

(A) $sin\ 60°$ (B) $cos\ 60°$ (C) $tan\ 60°$ (D) $sin\ 30°$

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°}=$

(A) $tan\ 90°$ (B) 1 (C) $sin\ 45°$ (D) 0

(iii) $\sin 2A = 2\sin A$ is true when $A =$

(A) 0 (B) 30° (C) 45° (D) 60°

(iv) $\frac{2\tan 30°}{1-\tan^2 30°}=$

(A) $cos\ 60°$ (B) $sin\ 60°$ (C) $tan\ 60°$ (D) $sin\ 30°$

Answer 2:

(i) $\frac{2\tan 30°}{1+\tan^2 30°}$

Putting the value of each trigonometric ratios, we get

$$\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}=\frac{6}{4\sqrt{3}}=\frac{\sqrt{3}}{2}$$

We know that $sin\ 60°=\frac{\sqrt{3}}{2}$, hence the option (A) is correct.

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°}$

Putting the value of each trigonometric ratios, we get

$\frac{1-(1)^2}{1+(1)^2}=\frac{1-1}{1+1}=\frac{0}{2}=0$

Hence, the option (D) is correct.

(iii) $\sin 2A = 2\sin A$

We know that $sin\ 0 = 0$, hence, the option (A) is correct.

(iv) $\frac{2\tan 30°}{1-\tan^2 30°}$

Putting the value of each trigonometric ratios, we get

$$\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}=\frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}=\frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}=\frac{3}{\sqrt{3}}=\sqrt{3}$$

We know that $tan\ 60°=\sqrt{3}$, hence, the option (C) is correct.

Question 3:

If $tan\ (A + B) = \sqrt{3}$ and $tan\ (A - B) = \frac{1}{\sqrt{3}}$, $0° < A + B \leq 90°, A > B$, find A and B.

Answer 3:

Given that: $tan\ (A + B) = \sqrt{3}$

$\Rightarrow\ tan\ (A + B) = \tan 60°$ $\quad [\because\ \tan 60° = \sqrt{3}]$

$\Rightarrow A + B = 60°$ ... (i)

Given that: $tan\ (A - B) = \frac{1}{\sqrt{3}}$

$\Rightarrow\ tan\ (A - B) = \tan 30°$ $\quad \left[\because\ \tan 30° = \frac{1}{\sqrt{3}}\right]$

$\Rightarrow A - B = 30°$ ... (ii)

Solving the equations (i) and (ii), we get

$2A = 90° \quad \Rightarrow A = 45°$

From equation (1), we get

$45° + B = 60° \quad \Rightarrow B = 15°$

Hence, $A = 45°$ and $B = 15°$

Question 4:

State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as θ increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = 0°$.

Answer 4:

(i) False,

Let $A = 30°$ and $B = 60°$

Therefore, $LHS = \sin(A + B) = \sin(30° + 60°) = \sin 90° = 1$ and

$$\text{RHS} = \sin A + \sin B = \sin 30° + \sin 60° = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2} \neq 1$$

Hence, $\sin(A + B) \neq \sin A + \sin B$

(ii) True,

As we know that $\sin 0° = 0, \sin 30° = \frac{1}{2}, \sin 45° = \frac{1}{\sqrt{2}}, \sin 60° = \frac{\sqrt{3}}{2}$ and $\sin 90° = 1$

Hence, for the increasing values of θ, $\sin\theta$ is also increasing.

(iii) False,

As we know that $\cos 0° = 1, \cos 30° = \frac{\sqrt{3}}{2}, \cos 45° = \frac{1}{\sqrt{2}}, \cos 60° = \frac{1}{2}$ and $\cos 90° = 0$

Hence, for the increasing values of θ, $\cos\theta$ is decreasing.

(iv) False,

$\because \cos 30° = \frac{\sqrt{3}}{2}$, but $\sin 30° = \frac{1}{2}$.

(v) True,

$\because \tan 0° = 0$, we know that $\cot 0° = \frac{1}{\tan 0°} = \frac{1}{0}$, which is not defined.