Mathematics

(www.tiwariacademy.in)
(Chapter – 7) (Coordinate Geometry)
(Class 10)
Exercise 7.2

Question 1:

Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2:3.

Answer 1:

Let the point P divides the line joining the points $A(-1, 7)$ and $B(4, -3)$ into $2 : 3$.

$A(-1,7)$ — 2 — P — 3 — $B(4,-3)$

Using section formula $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$, the coordinates of P are given by

$$= \left(\frac{2\times 4+3\times(-1)}{2+3}, \frac{2\times(-3)+3\times 7}{2+3}\right) = \left(\frac{5}{5}, \frac{15}{5}\right) = (1, 3)$$

Question 2:

Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

Answer 2:

Let the points P and Q be the points of trisection of the line joining $A(4, -1)$ and $B(-2, -3)$.

$A(4,-1)$ — 1 — P — 1 — Q — 1 — $B(-2,-3)$

Hence, $AP: PB = 1: 2$

Using section formula:

$$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

The coordinates of P are given by

$$= \left(\frac{1\times(-2)+2\times 4}{1+2}, \frac{1\times(-3)+2\times(-1)}{1+2}\right) = \left(\frac{6}{3}, \frac{-5}{3}\right) = \left(2, -\frac{5}{3}\right)$$

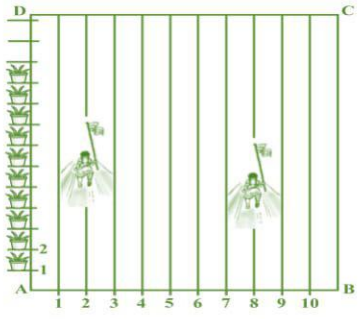
and, $AQ: QB = 2: 1$

Using section formula $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$, the coordinates of Q are given by

$$= \left(\frac{2\times(-2)+1\times 4}{2+1}, \frac{2\times(-3)+1\times(-1)}{2+1}\right) = \left(\frac{0}{3}, \frac{-7}{3}\right) = \left(0, -\frac{7}{3}\right)$$

Question 3:

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Figure. Niharika runs $\frac{1}{4}$ th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$ th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Answer 3:

Niharika runs $\frac{1}{4}$ th the distance AD on the 2nd line and posts a green flag.

Therefore, the coordinates of the flag posted by Niharika = $N\left(2, \frac{1}{4} \times 100\right) = N(2, 25)$

Preet runs $\frac{1}{5}$ th the distance AD on the eighth line and posts a red flag.

Therefore, the coordinates of flag posted by Preet = $P\left(8, \frac{1}{5} \times 100\right) = P(8, 20)$

Distance between the two flags = $NP = \sqrt{(8-2)^2 + (20-25)^2} = \sqrt{36+25} = \sqrt{61}$

If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, then the coordinates of the flag posted by Rashmi = coordinates of mid points of NP

$$= R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = R\left(\frac{2+8}{2}, \frac{25+20}{2}\right) = R\left(5, \frac{45}{2}\right) = R(5, 22.5)$$

Hence, Rashmi should post her flag in fifth line at a distance of 22.5 m.

Question 4:

Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

Answer 4:

Let the point $P(-1, 6)$, divides the line segment joining $A(-3, 10)$ and $B(6, -8)$ into k: 1.

$A(-3, 10)$ — k — $P(-1, 6)$ — 1 — $B(6, -8)$

Using the section formula $\left(\frac{k.x_2 + 1.x_1}{k+1}, \frac{k.y_2 + 1.y_1}{k+1}\right)$, the coordinates of point P

$$P(-1, 6) = \left(\frac{k \times 6 + 1 \times (-3)}{k+1}, \frac{k \times (-8) + 1 \times 10}{k+1}\right)$$

$$\Rightarrow P(-1, 6) = \left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$$

On comparing

$$\frac{6k-3}{k+1} = -1 \quad \text{and} \quad \frac{-8k+10}{k+1} = 6$$

$$\Rightarrow 6k - 3 = -k - 1$$

$$\Rightarrow 7k = 2 \quad \Rightarrow k = \frac{2}{7}$$

Hence, the point $P(-1, 6)$, divides the line segment joining the points $A(-3, 10)$ and $B(6, -8)$ into 2: 7.

Question 5:

Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Answer 5:

Let the line segment joining the points $A(1, -5)$ and $B(-4, 5)$ is divided by x −axis at point $P(x, 0)$ into k: 1.

$A(1, -5)$ — k — $P(x, 0)$ — 1 — $B(-4, 5)$

Using the section formula $\left(\frac{k.x_2 + 1.x_1}{k+1}, \frac{k.y_2 + 1.y_1}{k+1}\right)$, the coordinates of point P

$$P(x,0)=\left(\frac{k\times(-4)+1\times 1}{k+1},\frac{k\times 5+1\times(-5)}{k+1}\right)\Rightarrow P(x,0)=\left(\frac{-4k+1}{k+1},\frac{5k-5}{k+1}\right)$$

On comparing

$\frac{-4k+1}{k+1}=x$ and $\frac{5k-5}{k+1}=0$

$\Rightarrow 5k-5=0 \quad \Rightarrow k=1$

Putting the value of k in $\frac{-4k+1}{k+1}=x$, we have

$\frac{-4(1)+1}{1+1}=x \quad \Rightarrow x=-\frac{3}{2}$

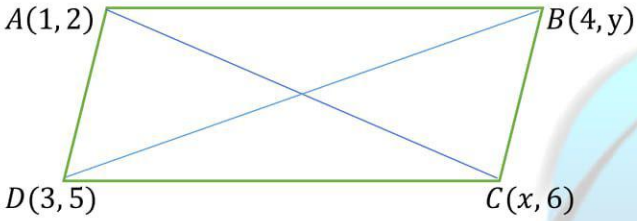
Hence, the line segment joining the points $A(1,-5)$and$B(-4,5)$ is divided by x −axis at point $P\left(-\frac{3}{2},0\right)$ into 1: 1.

Question 6:

If (1, 2), $(4, y)$, $(x, 6)$ and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Answer 6:

Given that: Points $A(1,2), B(4,y), C(x,6)$ and $D(3,5)$ are vertices of a parallelogram.



The diagonals of parallelogram bisect each other. Therefore,

Coordinates of mid points of AC = Coordinates of mid points of BD

$\Rightarrow\left(\frac{1+x}{2},\frac{2+6}{2}\right)=\left(\frac{3+4}{2},\frac{5+y}{2}\right)$

On comparing

$\frac{1+x}{2}=\frac{7}{2}$ and $\frac{8}{2}=\frac{5+y}{2}$

$\Rightarrow 1+x=7$ and $8=5+y$

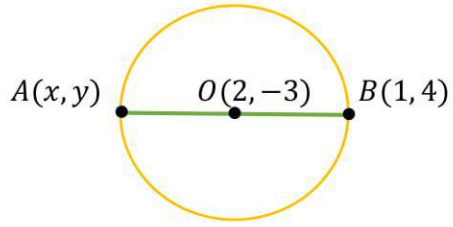
$\Rightarrow x=6$ and $y=3$

Question 7:

Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).

Answer 7:

Here, AB is the diameter of circle with centre $O(2,-3)$ and the coordinates of B $(1,4)$. Let the coordinates of point A is (x,y).



The centre $O(2,-3)$ is the mid-point of the diameter AB, therefore
Coordinates of mid-point of AB = Coordinates of point O

$\Rightarrow \left(\frac{x+1}{2}, \frac{y+4}{2}\right) = (2,-3)$

On comparing,

$\frac{x+1}{2} = 2$ and $\frac{y+4}{2} = -3$

$\Rightarrow x + 1 = 4$ and $y + 4 = -6$

$\Rightarrow x = 3$ and $y = -10$

Hence, the coordinates of A is $(3,-10)$.

Question 8:

If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB.

Answer 8:

Given that: $AP = \frac{3}{7}AB$, therefore, $BP = \frac{4}{7}AB$

$\Rightarrow AP: PB = 3:4$

The point P divides the line segment joining the points $A(-2,-2)$ and $B(2,-4)$ into 3: 4.

$A(-2,-2)$ 3 P 4 $B(2,-4)$

Using section formula

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

The coordinates of P is given by

$$= \left(\frac{3\times 2 + 4\times(-2)}{3+4}, \frac{3\times(-4) + 4\times(-2)}{3+4}\right) = \left(-\frac{2}{7}, -\frac{20}{7}\right)$$

Hence, the coordinates of P is $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

Question 9:

Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

Answer 9:

Let the points P, Q and R divides the line segment joining the points $A(-2,2)$ and $B(2,8)$ in to four equal parts.

$A(-2,2)$ 1 P 1 Q 1 R 1 $B(2,8)$

Therefore, $AP: PB = 1:3$

Using section formula

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

The coordinates of P is given by

$$= \left(\frac{1\times 2 + 3\times(-2)}{1+3}, \frac{1\times 8 + 3\times 2}{1+3}\right) = \left(\frac{-4}{4}, \frac{14}{4}\right) = \left(-1, \frac{7}{2}\right)$$

Since, $AQ: QB = 2: 2$, therefor the coordinates of point Q is given by

$$= \left(\frac{2 \times 2 + 2 \times (-2)}{2 + 2}, \frac{2 \times 8 + 2 \times 2}{2 + 2}\right)$$

$$= \left(\frac{0}{4}, \frac{20}{4}\right) = (0, 5)$$

Since, $AR: RB = 3: 1$, therefore the coordinates of Q is given by

$$= \left(\frac{3 \times 2 + 1 \times (-2)}{3 + 1}, \frac{3 \times 8 + 1 \times 2}{3 + 1}\right)$$

$$= \left(\frac{4}{4}, \frac{26}{4}\right) = \left(1, \frac{13}{2}\right)$$

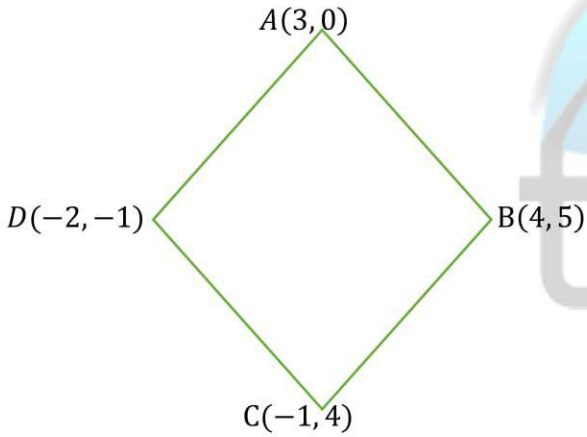
Hence, the points $P\left(-1, \frac{7}{2}\right)$, $Q(0, 5)$ and $R\left(1, \frac{13}{2}\right)$ divides AB in four equal parts.

Question 10:

Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order. [**Hint:** Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)]

Answer 10:

The vertices of rhombus $ABCD$ are $A(3, 0)$, $B(4, 5)$, $C(-1, 4)$ and $D(-2, -1)$.



Diagonal $AC = \sqrt{(-1 - 3)^2 + (4 - 0)^2} = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$

Diagonal $BD = \sqrt{(-2 - 4)^2 + (-1 - 5)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$

Area of rhombus = $\frac{1}{2}$ (Product of two diagonals)

$$= \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$

$= 24$ square units