Mathematics

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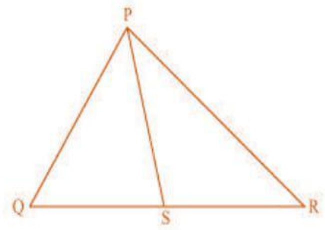
(Chapter – 6) (Triangles)

(Class 10)

Exercise 6.6 (Optional)

Question 1:

In Figure, PS is the bisector of ∠QPR of Δ PQR. Prove that $\frac{QS}{SR} = \frac{PQ}{QR}$.



Answer 1:

A line RT is drawn parallel to SP, which intersects QP produced at T.

Given that, SP bisects angle QPR, therefore

∠QPS = ∠SPR ... (1)

By construction,

∠SPR = ∠PRT (As PS || TR) ... (2)

∠QPS = ∠QTR (As PS || TR) ... (3)

From the above equations, we have

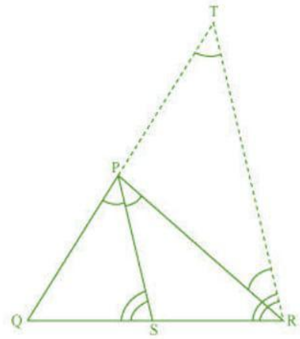
∠PRT = ∠QTR

∴ PT = PR

By construction, PS || TR

In ΔQTR, by Thales theorem

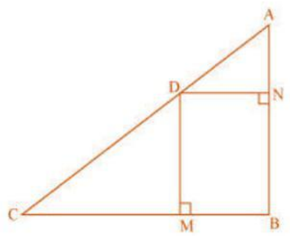
$\frac{QS}{SR} = \frac{QP}{PT} \Rightarrow \frac{QS}{SR} = \frac{PQ}{QR}$ [∵ PT = TR]



Question 2:

In Figure, D is a point on hypotenuse AC of Δ ABC, DM ⊥ BC and DN ⊥ AB. Prove that:

(i) $DM^2 = DN.MC$ **(ii)** $DN^2 = DM.AN$



Answer 2:

(i) Join B and D.

Given that, DN || CB, DM || AB and ∠B = 90°, ∴ DMBN is a rectangle.

∴ DN = MB and DM = NB

Given that, BD ⊥ AC, ∴ ∠CDB = 90°

⇒ ∠2 + ∠3 = 90° ... (1)

In ΔCDM, ∠1 + ∠2 + ∠DMC = 180°

⇒ ∠1 + ∠2 = 90° ... (2)

In ΔDMB, ∠3 + ∠DMB + ∠4 = 180°

⇒ ∠3 + ∠4 = 90° ... (3)

From the equations (1) and (2), we have, ∠1 = ∠3

From the equations (1) and (3), we have, ∠2 = ∠4

In ΔDCM and ΔBDM,

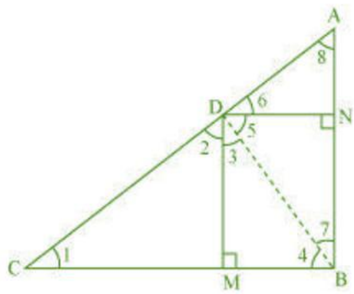
∠1 = ∠3 [Proved above]

∠2 = ∠4 [Proved above]

∴ ΔDCM ~ ΔBDM [AA similarity]

$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC} \Rightarrow \frac{DN}{DM} = \frac{DM}{MC}$ $[\because BM = DN]$

$\Rightarrow DM^2 = DN \times MC$



(ii) In ΔDBN, ∠5 + ∠7 = 90° ... (4)

In ΔDAN, ∠6 + ∠8 = 90° ... (5)

BD ⊥ AC, ∴ ∠ADB = 90°

⇒ ∠5 + ∠6 = 90° ... (6)

From the equations (4) and (6), we have, ∠6 = ∠7

From the equations (5) and (6), we have, ∠8 = ∠5

In ΔDNA and ΔBND,

$\angle 6 = \angle 7$ [Proved above]

$\angle 8 = \angle 5$ [Proved above]

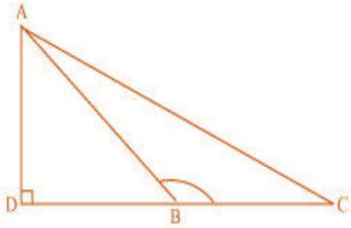
$\therefore$ ΔDNA ~ ΔBND [AA similarity]

$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB} \quad \Rightarrow DN^2 = AN \times NB$

$\Rightarrow DN^2 = AN \times DM$ [$\because NB = DM$]

Question 3:

In Figure, ABC is a triangle in which $\angle ABC > 90°$ and AD ⊥ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.



Answer 3:

In ΔADB, by Pythagoras theorem

$AB^2 = AD^2 + DB^2$ ... (1)

In ΔACD, by Pythagoras theorem

$AC^2 = AD^2 + DC^2$

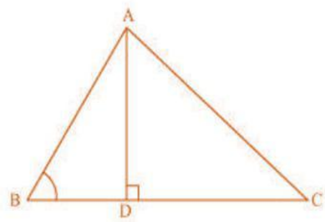
$\Rightarrow AC^2 = AD^2 + (DB + BC)^2$

$\Rightarrow AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

$\Rightarrow AC^2 = AB^2 + BC^2 + 2DB \times BC$ [From the equation (1)]

Question 4:

In Figure, ABC is a triangle in which $\angle ABC < 90°$ and AD ⊥ BC. Prove that
$AC^2 = AB^2 + BC^2 + 2BC.BD$.



Answer 4:

In ΔADB, by Pythagoras theorem

$AD^2 + DB^2 = AB^2$

$\Rightarrow AD2 = AB2 - DB2$ ... (1)

ΔADC में, by Pythagoras theorem, $AD^2 + DC^2 = AC^2$

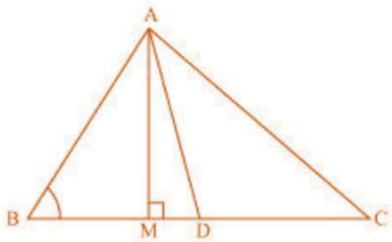
$\Rightarrow AB^2 - BD^2 + DC^2 = AC^2$ [From the equation (1)]

$\Rightarrow AB^2 - BD^2 + (BC - BD)^2 = AC^2$

$\Rightarrow AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD = AB^2 + BC^2 - 2BC \times BD$

Question 5:

In Figure, AD is a median of a triangle ABC and AM ⊥ BC. Prove that:

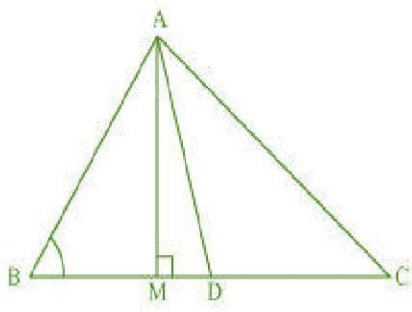


(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$

Answer 5:



(i) In ΔAMD, by Pythagoras theorem

$AM^2 + MD^2 = AD^2$ ... (1)

In ΔAMC, by Pythagoras theorem, $AM^2 + MC^2 = AC^2$

$\Rightarrow AM^2 + (MD + DC)^2 = AC^2$

$\Rightarrow (AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

$\Rightarrow AD^2 + DC^2 + 2MD.DC = AC^2$ [From equation (1)]

$\Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right) = AC^2$ $\left[\because DC = \frac{BC}{2}\right]$

$\Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 + MD.BC = AC^2$

(ii) In ΔABM, by Pythagoras theorem

$AB^2 = AM^2 + MB^2$

$= (AD^2 - DM^2) + MB^2$

$= (AD^2 - DM^2) + (BD - MD)^2$

$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$

$= AD^2 + BD^2 - 2BD \times MD$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right)MD = AC^2 \quad \left[\because BD = \frac{BC}{2}\right]$$

$$\Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 - BC.MD = AC^2$$

(iii) In ΔABM, by Pythagoras theorem, $AM^2 + MB^2 = AB^2$ ... (2)

In ΔAMC, by Pythagoras theorem, $AM^2 + MC^2 = AC^2$ ... (3)

Adding the equations (2) and (3), we have

$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$

$\Rightarrow 2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$

$\Rightarrow 2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$

$\Rightarrow 2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$

$$\Rightarrow 2(AM^2 + MD^2) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

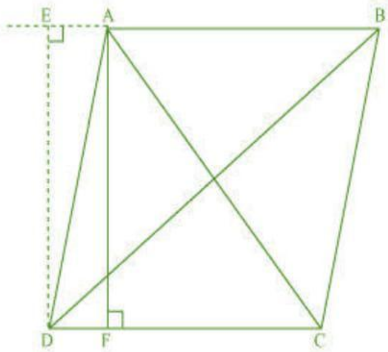
$$\Rightarrow 2AD^2 + \frac{1}{2}BC^2 = AB^2 + AC^2$$

Question 6:

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Answer 6:

In parallelogram ABCD, altitudes AF and DE is drawn on DC and produced BA.



In ΔDEA, by Pythagoras theorem, $DE^2 + EA^2 = DA^2$ ... (i)

In ΔDEB, by Pythagoras theorem, $DE^2 + EB^2 = DB^2$

$\Rightarrow DE^2 + (EA + AB)^2 = DB^2$

$\Rightarrow (DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$

$\Rightarrow DA^2 + AB^2 + 2EA \times AB = DB^2$ ... (ii)

In ΔADF, by Pythagoras theorem, $AD^2 = AF^2 + FD^2$

In ΔAFC, by Pythagoras theorem

$AC^2 = AF^2 + FC^2 = AF^2 + (DC - FD)^2 = AF^2 + DC^2 + FD^2 - 2DC \times FD$

$= (AF^2 + FD^2) + DC^2 - 2DC \times FD$

$\Rightarrow AC^2 = AD^2 + DC^2 - 2DC \times FD$ ... (iii)

ABCD is a parallelogram.

Therefore

AB = CD ... (iv)

and, BC = AD ... (v)

In ΔDEA and ΔADF,

∠DEA = ∠AFD [Each 90°]

∠EAD = ∠ADF [EA || DF]

AD = AD [Common]

∴ ΔEAD ≅ ΔFDA [AAS congruency rule]

⇒ EA = DF ... (vi)

Adding equations (ii) and (iii), we have

$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$

$\Rightarrow DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$

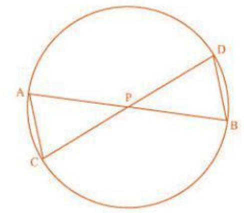
$\Rightarrow BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$ [From the equation (iv) and (vi)]

$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Question 7:

In Figure, two chords AB and CD intersect each other at the point P. Prove that:

(i) $\Delta APC \sim \Delta DPB$ **(ii)** $AP.BP = CP.DP$



Answer 7:

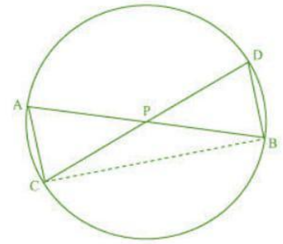
Join CB.

(i) In ΔAPC and ΔDPB,

$\angle APC = \angle DPB$ [Vertically Opposite Angles]

$\angle CAP = \angle BDP$ [Angles in the same segment]

$\Delta APC \sim \Delta DPB$ [AA similarity]



(ii) We have already proved that $\Delta APC \sim \Delta DPB$.

We know that the corresponding sides of similar triangles are proportional. So,

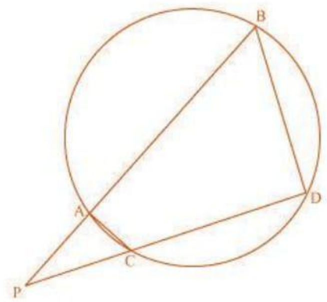
$$\frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD} \quad \Rightarrow \frac{AP}{DP} = \frac{PC}{PB} \quad \Rightarrow AP.PB = PC.DP$$

Question 8:

In Figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) $\Delta PAC \sim \Delta PDB$

(ii) $PA.PB = PC.PD$



Answer 8:

(i) In ΔPAC and ΔPDB,

$\angle P = \angle P$ [Common]

$\angle PAC = \angle PDB$

[The exterior angle of cyclic quadrilateral is equal to opposite interior angle]

$\therefore \Delta PAC \sim \Delta PDB$ [AA similarity]

(ii) We know that the corresponding sides of similar triangles are proportional. Therefore,

$$\frac{PA}{PD} = \frac{AC}{BD} = \frac{PC}{PB} \quad \Rightarrow \frac{PA}{PD} = \frac{PC}{PB} \quad \Rightarrow PA.PB = PC.DP$$

Question 9:

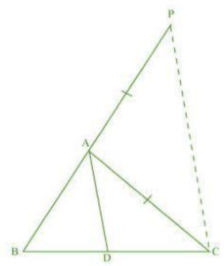
In Figure, D is a point on side BC of Δ ABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of $\angle$ BAC.

Answer 9:

Produce BA to P, such that AP = AC and join P to C.

Given that:

$$\frac{BD}{CD} = \frac{AB}{AC} \quad \Rightarrow \frac{BD}{CD} = \frac{AP}{AC}$$

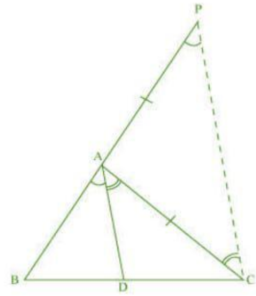


By the converse of Thales theorem, we have

AD || PC $\Rightarrow \angle BAD = \angle APC$ [Corresponding angle] ... (1)

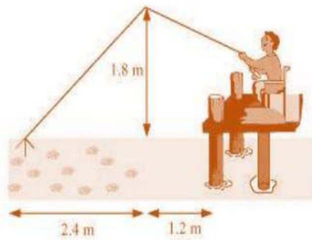
and, $\angle DAC = \angle ACP$ [Alternate angle] ... (2)

By construction,
AP = AC
$\Rightarrow \angle APC = \angle ACP$ ... (3)
From the equations (1), (2) and (3), we have
$\angle BAD = \angle APC$
$\Rightarrow$ AD, bisects angle BAC.



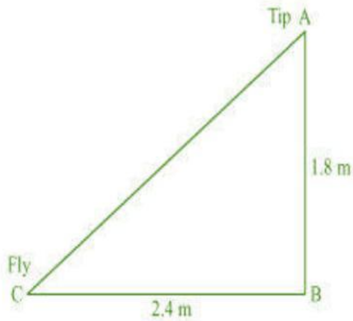
Question 10:

Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?

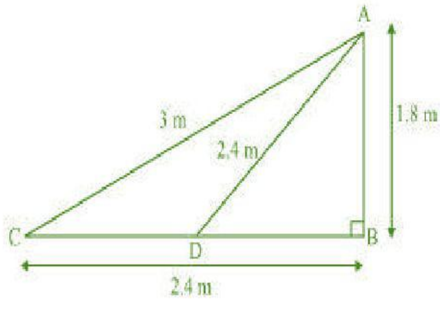


Answer 10:

Let AB be the height of rod tip from the surface of water and BC is the horizontal distance between fly to tip of the rod.
Then, the length of the string is AC.
In ΔABC, by Pythagoras theorem
$AC^2 = AB^2 + BC^2$
$\Rightarrow AB^2 = (1.8 \text{ m})^2 + (2.4 \text{ m})^2$
$\Rightarrow AB^2 = (3.24 + 5.76) \text{ m}^2$
$\Rightarrow AB^2 = 9.00 \text{ m}^2$
$\Rightarrow AB = \sqrt{9} = 3 \text{ m}$
Hence, the length of string, which is out, is 3 m.



If she pulls in the string at the rate of 5 cm/s, then the distance travelled by fly in 12 seconds
= 12 × 5 = 60 cm = 0.6 m
Let, D be the position of fly after 12 seconds.
Hence, AD is the length of string that is out after 12 seconds.
The length of the string pulls in by Nazima = AD = AC − 12
= (3.00 − 0.6) m
= 2.4 m



In ΔADB,
$AB^2 + BD^2 = AD^2$
$\Rightarrow (1.8 \text{ m})^2 + BD^2 = (2.4 \text{ m})^2$
$\Rightarrow BD^2 = (5.76 - 3.24) \text{ m}^2 = 2.52 \text{ m}^2$
$\Rightarrow BD = 1.587 \text{ m}$

Horizontal distance travelled by Fly
= BD + 1.2 m
= (1.587 + 1.2) m
= 2.787 m
= 2.79 m