Exercise 6.5

Question 1:
Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm
(ii) 3 cm, 8 cm, 6 cm
(iii) 50 cm, 80 cm, 100 cm
(iv) 13 cm, 12 cm, 5 cm

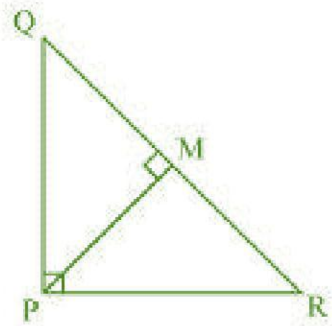
Answer 1:
(i) Sides of triangle: 7 cm, 24 cm and 25 cm.
Squaring these sides, we get 49, 576 and 625.
49 + 576 = 625 $\Rightarrow 7^2 + 24^2 = 25^2$
These sides satisfy the Pythagoras triplet, hence these are sides of right angled triangle.
We know that the hypotenuses is the longest side in right angled triangle.
Hence, its length is 25 cm.

(ii) Sides of triangle: 3 cm, 6 cm and 8 cm.
Squaring these sides, we get 9, 36 and 64.
9 + 36 ≠ 64 $\Rightarrow 3^2 + 6^2 \neq 8^2$
These sides do not satisfy the Pythagoras triplet, hence these are not the sides of right angled triangle.

(iii) Sides of triangle: 50 cm, 80 cm and 100 cm.
Squaring these sides, we get 2500, 6400 and 10000.
2500 + 6400 ≠ 10000 $\Rightarrow 50^2 + 80^2 \neq 100^2$
These sides do not satisfy the Pythagoras triplet, hence these are not the sides of right angled triangle.

(iv) Sides of triangle: 5 cm, 12 cm and 13 cm.
Squaring these sides, we get 25, 144 and 169.
25 + 144 = 169 $\Rightarrow 5^2 + 12^2 = 13^2$
These sides satisfy the Pythagoras triplet, hence these are sides of right angled triangle.
We know that the hypotenuses is the longest side in right angled triangle.
Hence, its length is 13 cm.

Question 2:
PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that PM^2 = QM.MR.

Answer 2:


Let $\angle MPR = x$
In ΔMPR,
$\angle MRP = 180° - 90° - x$
Similarly,
In ΔMPR,
$\angle MPQ = 90° - \angle MPR = 90° - x$
$\angle MQP = 180° - 90° - (90° - x) = x$
In ΔQMP and ΔPMR,
∠MPQ = ∠MRP
∠PMQ = ∠RMP
∠MQP = ∠MPR
⇒ΔQMP ~ ΔPMR [AAA similarity]
We know that the corresponding sides of similar triangles are proportional.
Therefore,
$$\frac{QM}{PM} = \frac{MP}{MR}$$
$\Rightarrow PM^2 = MQ \times MR$

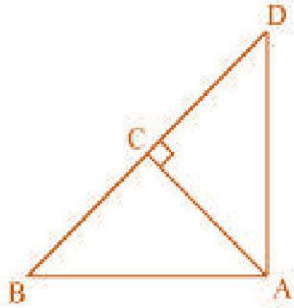
Question 3:

In Figure, ABD is a triangle right angled at A and AC ⊥ BD. Show that

(i) $AB^2 = BC \times BD$

(ii) $AC^2 = BC \times DC$

(iii) $AD^2 = BD \times CD$



Answer 3:

(i) In ΔADB and ΔCAB,

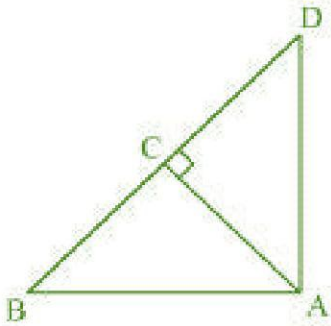
∠DAB = ∠ACB [Each 90°]

∠ABD = ∠CBA [Common]

∴ ΔDCM ~ ΔBDM [AA similarity]

$\Rightarrow \frac{AB}{CB} = \frac{BD}{AB}$

$\Rightarrow AB^2 = CB \times BD$



(ii) Let ∠CAB = x

In ΔCBA,

∠CBA = 180° − 90° − x

⇒∠CBA = 90° − x

Similarly, in ΔCAD,

∠CAD = 90° − ∠CAB

⇒∠CAD = 90° − x

∠CDA = 180° − 90° − (90° − x)

⇒∠CDA = x

In ΔCBA and ΔCAD,

∠CBA = ∠CAD [Proved above]

∠CAB = ∠CDA [Proved above]

∠ACB = ∠DCA [Each 90°]

∴ ΔCBA ~ ΔCAD [AAA similarity]

$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow AC^2 = BC \times DC$

(iii) In ΔDCA and ΔDAB,

∠DCA = ∠DAB [Each 90°]

∠CDA = ∠ADB [Common]

∴ ΔDCA ~ ΔDAB [AA similarity]

$\Rightarrow \frac{DC}{DA} = \frac{DA}{DB}$

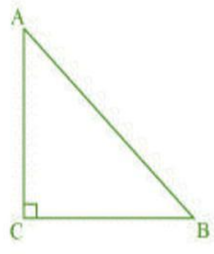
$\Rightarrow AD^2 = BD \times CD$

Question 4:

ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Answer 4:

Given that the triangle ABC is an isosceles triangle such that AC = BC and ∠C = 90°,



In ΔABC, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

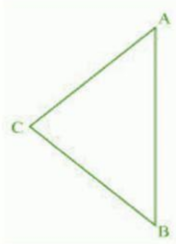
$\Rightarrow AB^2 = AC^2 + AC^2$ [Because AC = BC]

$\Rightarrow AB^2 = 2AC^2$

Question 5:

ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Answer 5:



Given that: $AB^2 = 2AC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$

$\Rightarrow AB^2 = AC^2 + BC^2$ [Because AC = BC]

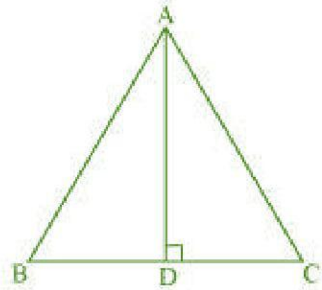
These sides satisfy the Pythagoras theorem.

Hence, the triangle ABC is a right angled triangle.

Question 6:

ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Answer 6:



Let ABC be any equilateral triangle with each sides of length 2a. Perpendicular AD is drawn from A to BC.

We know that the altitude in equilateral triangle, bisects the opposite sides.

Therefore, ∴ BD = DC = a

In ΔADB, by Pythagoras theorem

$AB^2 = AD^2 + BD^2$

$\Rightarrow (2a)^2 = AD^2 + a^2$ [Because AB = 2a]

$\Rightarrow 4a^2 = AD^2 + a^2$

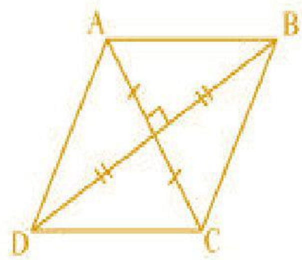
$\Rightarrow AD^2 = 3a^2 \Rightarrow AD = \sqrt{3}a$

Hence, the length of each altitude is $\sqrt{3}a$.

Question 7:

Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Answer 7:



In ΔAOB, by Pythagoras theorem

$AB^2 = AO^2 + OB^2$ … (i)

In ΔBOC, by Pythagoras theorem

$BC^2 = BO^2 + OC^2$ … (ii)

In ΔCOD, by Pythagoras theorem

$CD^2 = CO^2 + OD^2$ … (iii)

In ΔAOD, by Pythagoras theorem

$AD^2 = AO^2 + OD^2$ … (iv)

Adding the equations (i), (ii), (iii) and (iv), we have

$AB^2 + BC^2 + CD^2 + AD^2 = OA^2 + OB^2 + OB^2 + OC^2 + OC^2 + OD^2 + OD^2 + OA^2$

$= 2[OA^2 + OB^2 + OC^2 + OD^2]$

= 2[2OA2 + 2OB2] [Because OA = OC, OB = OD]

= 4[OA2 + OB2]

$= 4\left[\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right]$ [Because OA = ½ AC, OB = ½ BD]

$= 4\left[\frac{AC^2}{4} + \frac{BD^2}{4}\right]$

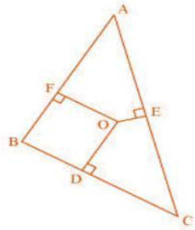
$= AC^2 + BD^2$

Question 8:

In Figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$



Answer 8:

Join OA, OB and OC.

(i) In ΔAOF, by Pythagoras theorem

$OA^2 = OF^2 + AF^2$ … (i)

In ΔBOD, by Pythagoras theorem

$OB^2 = OD^2 + BD^2$ … (ii)

In ΔCOE, by Pythagoras theorem

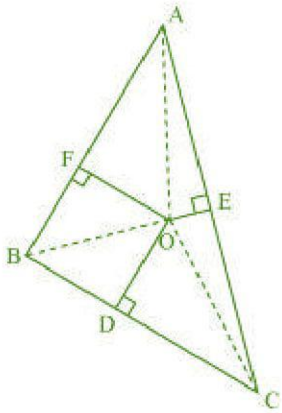
$OC^2 = OE^2 + EC^2$ … (iii)

Adding equations (i), (ii) and (iii), we have

$OA^2 + OB^2 + OC^2$

$= OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$

$\Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$ … (iv)



(ii) From the equation (iv), we have

$AF^2 + BD^2 + CE^2$

$= OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

$= (OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2)$

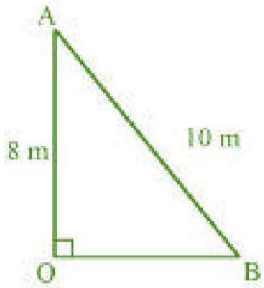
$= AE^2 + CD^2 + BF^2$

Question 9:

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Answer 9:

Let OA is wall and AB is ladder in the figure.



In ΔAOB, by Pythagoras theorem

$AB^2 = OA^2 + OB^2$

$\Rightarrow 10^2 = 8^2 + BO^2$

$\Rightarrow 100 = 64 + BO^2$

$\Rightarrow BO^2 = 36$

$\Rightarrow BO = 6\ m$

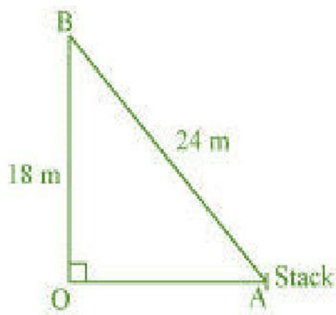
Hence, the distance of the foot of the ladder from the base of the wall is 6 m.

Question 10:

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer 10:

Let OB is vertical pole in the figure.



In ΔAOB, by Pythagoras theorem

$AB^2 = OB^2 + OA^2$

$\Rightarrow 24^2 = 18^2 + OA^2$

$\Rightarrow 576 = 324 + OA^2$

$\Rightarrow OA^2 = 252$

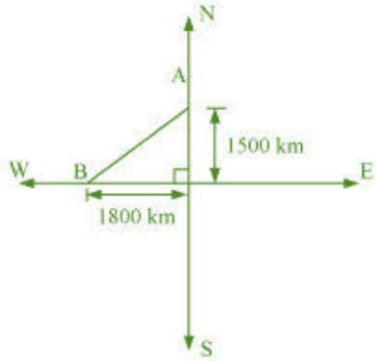
$\Rightarrow OA = 6\sqrt{7}\ m$

Hence, the distance of stake from the base of the pole is $6\sqrt{7}\ m$.

Question 11:

An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer 11:



Distance travelled by first aeroplane (due north) in $1\frac{1}{2}$ hours

$= 1000 \times \frac{3}{2} = 1500\ km$

Distance travelled by second aeroplane (due west) in $1\frac{1}{2}$ hours

$= 1200 \times \frac{3}{2} = 1800\ km$

Now, OA and OB are the distance travelled.

Now by Pythagoras theorem, the distance between the two planes

$AB = \sqrt{OA^2 + OB^2}$

$= \sqrt{(1500)^2 + (1800)^2} = \sqrt{2250000 + 3240000}$

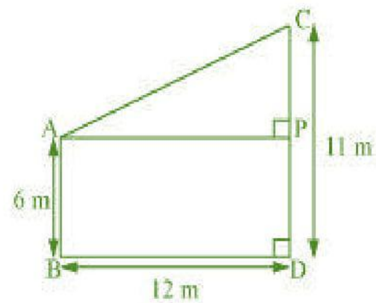
$= \sqrt{5490000} = 300\sqrt{61}\ km$

Hence, $1\frac{1}{2}$ hours, the distance between two planes is $300\sqrt{61}\ km$.

Question 12:

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer 12:



Let AB and CD are the two pole with height 6 m and 11 m respectively.

Therefore, CP = 11 − 6 = 5 m and AP = 12 m

In ΔAPC, by Pythagoras theorem

$AP^2 + PC^2 = AC^2$

$\Rightarrow 12^2 + 5^2 = AC^2$

$\Rightarrow AC^2 = 144 + 25 = 169$

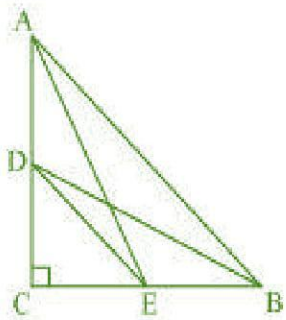
$\Rightarrow AC = 13\ m$

Hence, the distance between the tops of two poles is 13 m.

Question 13:

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Answer 13:



In ΔACE, by Pythagoras theorem

$AC^2 + CE^2 = AE^2$ ... (1)

In ΔBCD, by Pythagoras theorem

$BC^2 + CD^2 = DB^2$ ... (2)

From the equation (1) and (2), we have

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + DB^2$ ... (3)

In ΔCDE, by Pythagoras theorem

$DE^2 = CD^2 + CE^2$ ... (4)

In ΔABC, by Pythagoras theorem

$AB^2 = AC^2 + CB^2$ ... (5)

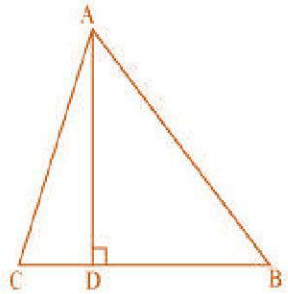
From the equation (3), (4) and (5), we have

$DE^2 + AB^2 = AE^2 + DB^2$

Question 14:

The perpendicular from A on side BC of a Δ ABC intersects BC at D such that DB = 3CD (see Figure). Prove that $2AB^2 = 2AC^2 + BC^2$.

Answer 14:



In ΔACD, by Pythagoras theorem

$AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 - CD^2 = AD^2$ ... (1)

In ΔABD, by Pythagoras theorem

$AB^2 = AD^2 + BD^2$

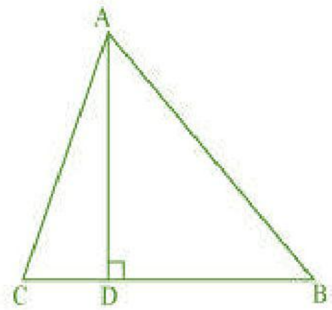
$\Rightarrow AB^2 - BD^2 = AD^2$ ... (2)

From the equation (1) and (2), we have

$AC^2 - CD^2 = AB^2 - BD^2$ ... (3)

Given that: 3DC = DB, therefore

$DC = \frac{BC}{4}$ and $BD = \frac{3BC}{4}$ ... (4)



From the equation (3) and (4), we have

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

$$\Rightarrow AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$$\Rightarrow 16AC^2 - BC^2 = 16AB^2 - 9BC^2$$

$$\Rightarrow 16AC^2 = 16AB^2 - 8BC^2$$

$$\Rightarrow 2AC^2 = 2AB^2 - BC^2$$

$$\Rightarrow 2AB^2 = 2AC^2 + BC^2$$

Question 15:

In an equilateral triangle ABC, D is a point on side BC such that BD =1/3 BC. Prove that $9AD^2 = 7AB^2$.

Answer 15:

Triangle ABC is an equilateral triangle with each side a. Draw an altitude AE from A to BC.

We know that the altitude in equilateral triangle, bisects the opposite sides.

Therefore, BE = EC = a/2

In ΔAEB, by Pythagoras theorem

$AB^2 = AE^2 + BE^2$

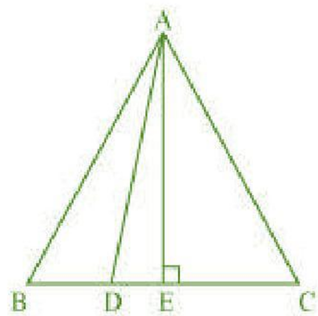
$\Rightarrow (a)^2 = AD^2 + (a/2)^2$ [Because AB = a]

$\Rightarrow a^2 = AD^2 + a^2/4$ $\Rightarrow AD^2 = 3a^2/4$ $\Rightarrow AD = \sqrt{3}a/2$

Given that: BD = 1/3 BC

∴ BD = a/3

DE = BE – BD = a/2 – a/3 = a/6



In ΔADE, by Pythagoras theorem,

$AD^2 = AE^2 + DE^2$

$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$

$$= \frac{3a^2}{4} + \frac{a^2}{36} = \frac{28a^2}{36} = \frac{7}{9}a^2$$

$$\Rightarrow AD^2 = \frac{7}{9}AB^2$$

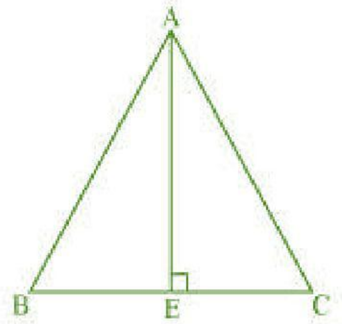
$$\Rightarrow 9AD^2 = 7AB^2$$

Question 16:

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer 16:

Let triangle ABC be an equilateral triangle with side a. Altitude AE is drawn from A to BC.
We know that the altitude in equilateral triangle, bisects the opposite sides.
∴ BE = EC = BC/2 = a/2
In ΔABE, by Pythagoras theorem



$AB^2 = AE^2 + BE^2$

$$\Rightarrow a^2 = AE^2 + \left(\frac{a}{2}\right)^2 = AE^2 + \frac{a^2}{4}$$

$$\Rightarrow AE^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

$4AE^2 = 3a^2$

⇒ 4 × (Altitude) = 3 × (Side)

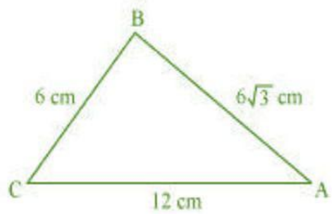
Question 17:

Tick the correct answer and justify: In Δ ABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. The angle B is:

(A) 120° (B) 60°
(C) 90° (D) 45°

Answer 17:

Given that: AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.
Therefore, $AB^2 = 108$, $AC^2 = 144$ and $BC^2 = 36$
Now,



$AB^2 + BC^2$
= 108 + 36
= 144
= AC^2
The sides are satisfying the Pythagoras triplet in ΔABC. Hence, these are the sides of a right angled triangle.
∴ ∠B = 90°
Hence, the option (C) is correct.