Mathematics

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(Chapter – 6) (Triangles)

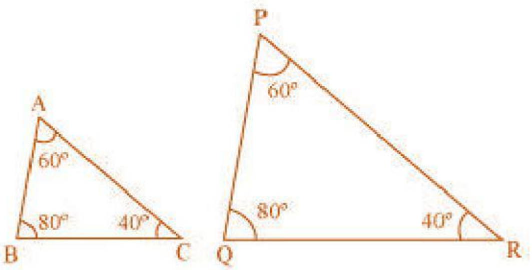
(Class 10)

Exercise 6.3

Question 1:

State which pairs of triangles in Figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

(i)



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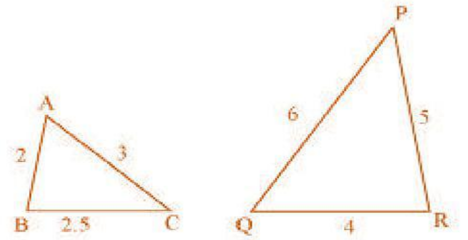

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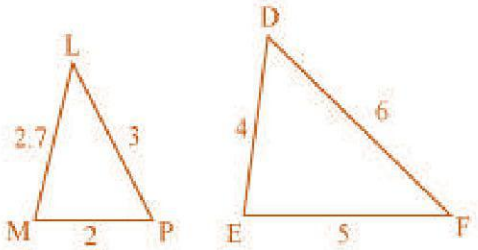

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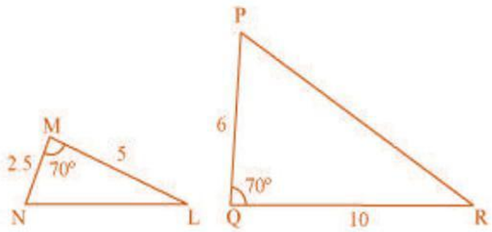

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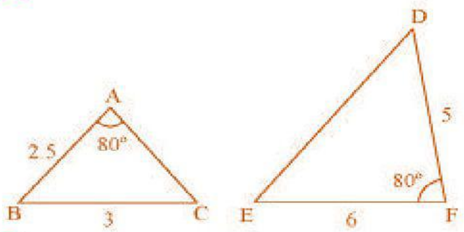

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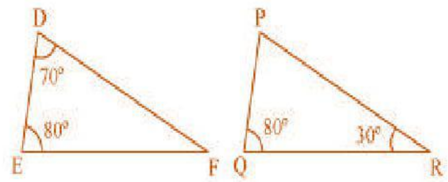


Answer 1:

(i) ∠A = ∠P = 60°, ∠B = ∠Q = 80°, ∠C = ∠R = 40°

Therefore, ΔABC ~ ΔPQR [AAA similarity]

(ii) $\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$,

∴ ΔABC ~ ΔQRP [SSS similarity]

(iii) Triangles are not similar because the corresponding sides are not proportional.

(iv) Triangles are not similar because the corresponding sides are not proportional.

(v) Triangles are not similar because the corresponding sides are not proportional.

(vi) In ΔDEF,

∠D +∠E +∠F = 180°

⇒ 70° + 80° +∠F = 180 ⇒ ∠F = 30°

Similarly, in ΔPQR,

∠P +∠Q +∠R = 180°

⇒ ∠P + 80° +30° = 180° ⇒ ∠P = 70°

In ΔDEF and ΔPQR,

∠D = ∠P [Each 70°]

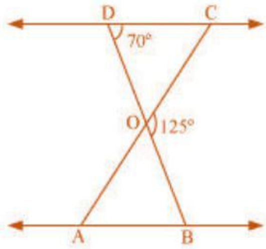
∠E = ∠Q [Each 80°]

∠F = ∠R [Each 30°]

∴ ΔDEF ~ ΔPQR [AAA similarity]

Question 2:

In Figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Answer 2:

DOB is a straight line. $\therefore \angle DOC + \angle COB = 180°$

$\Rightarrow \angle DOC = 180° - 125° = 55°$

ΔDOC में, $\angle DCO + \angle CDO + \angle DOC = 180°$

$\Rightarrow \angle DCO + 70° + 55° = 180° \Rightarrow \angle DCO = 55°$

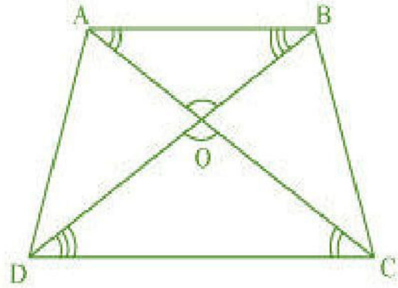
Given that, $\Delta ODC \sim \Delta OBA$.

$\therefore \angle OAB = \angle OCD$ [Corresponding angles of similar triangles]

$\Rightarrow \angle OAB = 55°$

Question 3:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.



Answer 3:

In ΔDOC and ΔBOA,

$\angle DOC = \angle BOA$ [Vertically Opposite Angles]

$\angle CDO = \angle ABO$ [Alternate Angles]

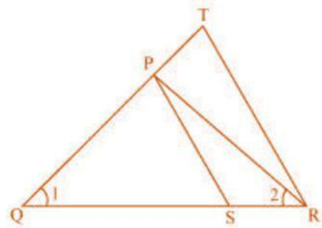
$\angle DCO = \angle BAO$ [Alternate Angles]

$\therefore \Delta DOC \sim \Delta BOA$ [AAA similarity]

$\frac{DO}{BO} = \frac{CO}{AO} \quad \Rightarrow \frac{BO}{DO} = \frac{AO}{CO}$

Question 4:

In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Answer 4:

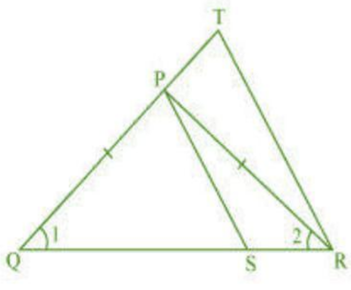
In ΔPQR, $\angle PQR = \angle PRQ$

$\therefore PQ = PR$ ... (1)

Given that,

$\frac{QR}{QS} = \frac{QT}{PR}$

$\Rightarrow \frac{QR}{QS} = \frac{QT}{QP}$ [From equation (1)] ... (2)



In ΔPQS and ΔTQR,

$\frac{QR}{QS} = \frac{QT}{QP}$ [From equation (2)]

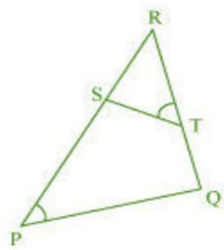
$\angle Q = \angle Q$ [Common]

$\therefore \Delta PQS \sim \Delta TQR$ [SAS similarity]

Question 5:

S and T are points on sides PR and QR of Δ PQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Answer 5:



In ΔRPQ and ΔRST,

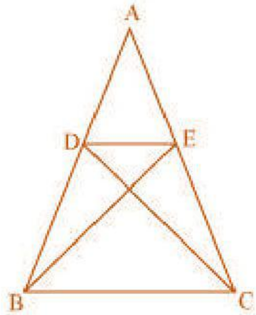
$\angle RTS = \angle QPS$ [Given]

$\angle R = \angle R$ [Common]

$\therefore \Delta RPQ \sim \Delta RTS$ [AA similarity]

Question 6:

In Figure, if Δ ABE ≅ Δ ACD, show that Δ ADE ~ Δ ABC.



Answer 6:

Given that, ΔABE ≅ ΔACD.

∴ AB = AC [CPCT] ... (1)

and, AD = AE [CPCT] ... (2)

In ΔADE and ΔABC,

$\frac{AD}{AB} = \frac{AE}{AC}$ [From the equation (1) and (2)]

∠A = ∠A [Common]

∴ ΔADE ~ ΔABC [SAS similarity]

Question 7:

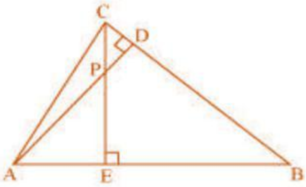
In Figure, altitudes AD and CE of Δ ABC intersect each other at the point P. Show that:

(i) ΔAEP ~ ΔCDP

(ii) ΔABD ~ ΔCBE

(iii) ΔAEP ~ ΔADB

(iv) ΔPDC ~ ΔBEC



Answer 7:

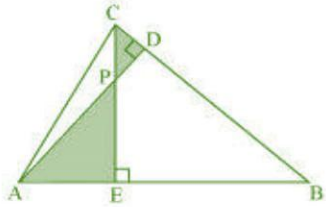
(i)

In ΔAEP and ΔCDP,

∠APE = ∠CPD [Vertically Opposite Angles]

∠AEP = ∠CDP [Each 90°]

∴ ΔAEP ~ ΔCDP [AA similarity]



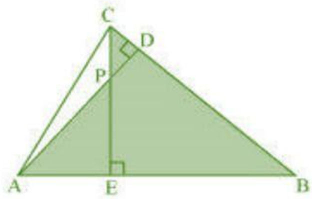
(ii)

In ΔABD and ΔCBE,

∠ADB = ∠CEB [Each 90°]

∠ABD = ∠CBE [Common]

∴ ΔABD ~ ΔCBE [AA similarity]



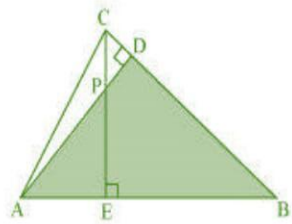
(iii)

In ΔAEP and ΔADB,

∠AEP = ∠ADB [Each 90°]

∠PAE = ∠DAB [Common]

∴ ΔAEP ~ ΔADB [AA similarity]



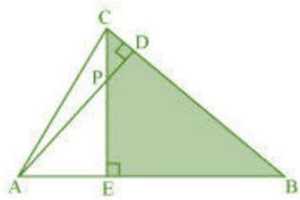
(iv)

In ΔPDC and ΔBEC,

∠PDC = ∠BEC [Each 90°]

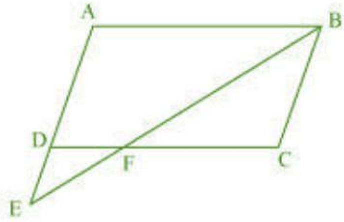
∠PCD = ∠BCE [Common]

∴ ΔPDC ~ ΔBEC [AA similarity]



Question 8:

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.



Answer 8:

In ΔABE and ΔCFB,

∠A = ∠C [Opposite angles of parallelogram]

∠AEB = ∠CBF [Alternate angles as AE || BC]

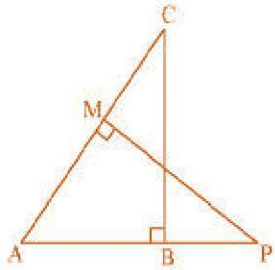
∴ ΔABE ~ ΔCFB [AA similarity]

Question 9:

In Figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

(i) ΔABC ~ ΔAMP

(ii) $\frac{AB}{PQ} = \frac{AD}{PM}$



Answer 9:

(i) In ΔABC and ΔAMP,

∠ABC = ∠AMP [Each 90°]

∠A = ∠A [Common]

∴ ΔABC ~ ΔAMP [AA similarity]

(ii) ΔABC ~ ΔAMP [Proved above]

⇒ $\frac{AB}{PQ} = \frac{AD}{PM}$ [Corresponding parts of similar triangles.]

Question 10:

CD and GH are respectively the bisectors of ∠ ACB and ∠ EGF such that D and H lie on sides AB and FE of Δ ABC and Δ EFG respectively. If Δ ABC ~ Δ FEG, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) ΔDCB ~ ΔHGE

(iii) ΔDCA ~ ΔHGF

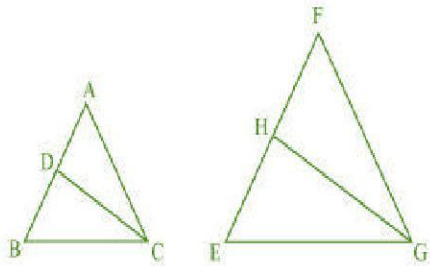
Answer 10:

(i) Given that, ΔABC ~ ΔFEG

∴ ∠A = ∠F, ∠B = ∠E and ∠ACB = ∠FGE, ∠ACB = ∠FGE

∴ ∠ACD = ∠FGH [CD and GH are the bisectors of equal angles]

and, ∠DCB = ∠HGE [CD and GH are the bisectors of equal angles]



In ΔACD and ΔFGH,

∠A = ∠F, and ∠ACD = ∠FGH [Proved above]

∴ ΔACD ~ ΔFGH [AA similarity]

⇒ $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) In ΔDCB and ΔHGE,

∠DCB = ∠HGE [Proved above]

∠B = ∠E [Proved above]

∴ ΔDCB ~ ΔHGE [AA similarity]

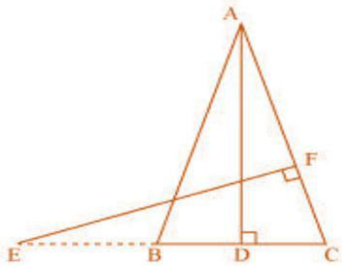
(iii) In ΔDCA and ΔHGF,

∠ACD = ∠FGH [Proved above]

∠A = ∠F [Proved above]

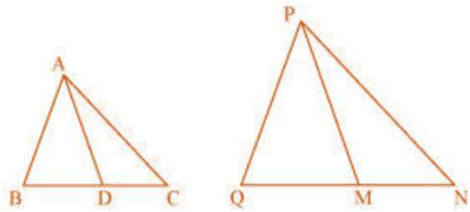
∴ ΔDCA ~ ΔHGF [AA similarity]

Question 11:
In Figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that Δ ABD ~ Δ ECF.

Answer 11:


Given that, ABC is an isosceles triangle.
∴ AB = AC ⇒ ∠ABD = ∠ECF
In ΔABD and ΔECF,
∠ADB = ∠EFC [Each 90°]
∠ABD = ∠ECF [Proved above]
∴ ΔABD ~ ΔECF [AA similarity]

Question 12:
Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of Δ PQR (see Figure). Show that Δ ABC ~ Δ PQR.

Answer 12:


AD and PM are the median of triangle. Therefore,
BD = BC/2 and QM = QR/2
Given that,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In ΔABD and ΔPQM,

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$ [Proved above]

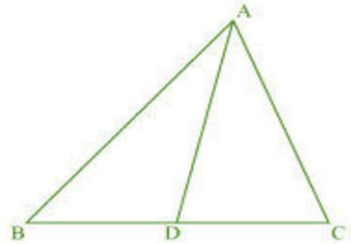
∴ ΔABD ~ ΔPQM [SSS similarity]
⇒ ∠ABD = ∠PQM [Corresponding angles of similar triangles]

In ΔABC and ΔPQR,
∠ABD = ∠PQM [Proved above]

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

∴ ΔABC ~ ΔPQR [SAS similarity]

Question 13:
D is a point on the side BC of a triangle ABC such that ∠ ADC = ∠ BAC. Show that $CA^2 = CB.CD$.

Answer 13:


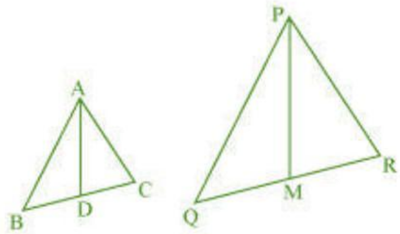
In ΔADC and ΔBAC,
∠ADC = ∠BAC [Given]
∠ACD = ∠BCA [Common]
∴ ΔADC ~ ΔBAC [AA similarity]
We know that the corresponding sides of similar triangles are proportional. Therefore

$$\frac{CA}{CB} = \frac{CD}{CA}$$

$$\Rightarrow CA^2 = CB \times CD$$

Question 14:

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that Δ ABC ~ Δ PQR.



Answer 14:

Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

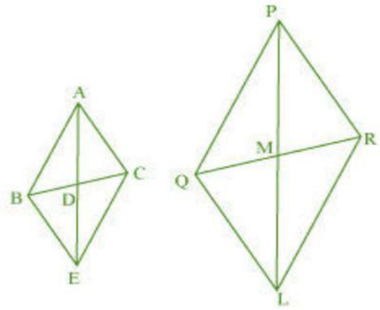
Produce AD and PM to E and L such that AD = DE and PM = DE. Now join B to E, C to E, Q to L and R to L.

AD and PM are medians of triangle, therefore

BD = DC and QM = MR

and, AD = DE [By construction]

and, PM = ML [By construction]



So, the diagonals of ABEC bisecting each other at D, therefore ABEC is a parallelogram.

Similarly, PQLR is also a parallelogram.

∴ AC = BE, AB = EC and PR = QL, PQ = LR

Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM} \quad \Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

∴ ΔABE ~ ΔPQL [SSS similarity]

We know that the corresponding angles of similar triangles are equal. Therefore

∴ ∠BAE = ∠QPL ... (1)

Similarly, ΔAEC ~ ΔPLR and

∠CAE = ∠RPL ... (2)

Adding the equations (1) and (2), we get

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒ ∠CAB = ∠RPQ ... (3)

In ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{AC}{PR}$$ [Given]

∠CAB = ∠RPQ [From the equation (3)]

∴ ΔABC ~ ΔPQR [SAS similarity]

Question 15:

A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer 15:

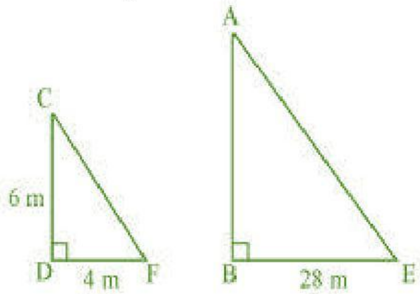
Let CD be the pole and AB is tower. Therefore, DF and BE are the shadows of pole and tower.

In ΔABE and ΔCDF,

∠DCF = ∠BAE [Angle of sun at same place]

∠CDF = ∠ABE [Each 90°]

∴ ΔABE ~ ΔCDF [AA similarity]



$$\frac{AB}{CD} = \frac{BE}{QL}$$

$$\Rightarrow \frac{AB}{6} = \frac{28}{4}$$

$$\Rightarrow AB = 42\ m$$

Hence, the height of the tower is 42 m.

Question 16:

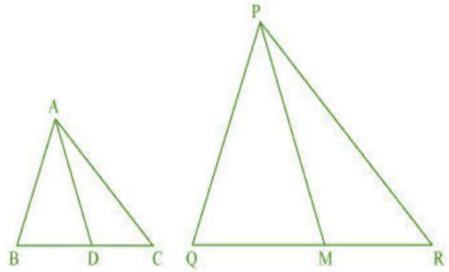
If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Answer 16:

Given that: ΔABC ~ ΔPQR

We know that the corresponding sides of similar triangles are proportional. Therefore

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \quad \ldots (1)$$



and,

∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ... (2)

AD and PM are medians of triangle. Therefore

BD = BC/2 and QM = QR/2 ... (3)

From the equation (1) and (3), we have

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad \ldots (4)$$

In ΔABD and ΔPQM,

∠B = ∠Q [From the equation (2)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [From the equation (4)]

∴ ΔABD ~ ΔPQM [SAS similarity]

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$