Exercise 5.3

Question 1:

Find the sum of the following APs:

(i) $2, 7, 12, \ldots,$ to 10 terms **(ii)** $-37, -33, -29, \ldots,$ to 12 terms

(iii) $0.6, 1.7, 2.8, \ldots,$ to 100 terms **(iv)** $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots,$ to 11 terms

Answer 1:

(i) A. P.: $2, 7, 12, \ldots$

Here, $a = 2$ and $d = 7 - 2 = 5$.

The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow S_{10} = \frac{10}{2}[2(2) + (10-1)(5)]$ $\Rightarrow S_{10} = 5[4 + 45] = 245$

(ii) A. P.: $-37, -33, -29, \ldots$

Here, $a = -37$ and $d = -33 - (-37) = 4$.

The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow S_{12} = \frac{12}{2}[2(-37) + (12-1)(4)]$ $\Rightarrow S_{12} = 6[-74 + 44] = -180$

(iii) A. P.: $0.6, 1.7, 2.8, \ldots$

Here, $a = 0.6$ and $d = 1.7 - 0.6 = 1.1$.

The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow S_{100} = \frac{100}{2}[2(0.6) + (100-1)(1.1)]$ $\Rightarrow S_{100} = 50[1.2 + 99 \times 1.1] = 50[110.1] = 5505$

(iv) A. P.: $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$

Here, $a = \frac{1}{15}$ and $d = \frac{1}{12} - \left(\frac{1}{15}\right) = \frac{1}{60}$.

The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\left(\frac{1}{60}\right)\right]$ $\Rightarrow S_{11} = \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{9}{30}\right] = \frac{33}{20}$

Question 2:

Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + \cdots + 84$ **(ii)** $34 + 32 + 30 + \cdots + 10$ **(iii)** $-5 + (-8) + (-11) + \cdots + (-230)$

Answer 2:

(i) Here, $a = 7$ and $d = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$.

Let, the nth term of the A.P. is 84.

Therefore, $a_n = 84 \Rightarrow a + (n-1)d = 84$

$\Rightarrow 7 + (n-1)\left(\frac{7}{2}\right) = 84$ $\Rightarrow (n-1)\left(\frac{7}{2}\right) = 77$ $\Rightarrow n - 1 = 22$ $\Rightarrow n = 23$

The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[a + l]$$

$\Rightarrow S_{23} = \frac{23}{2}[7 + 84]$ $\quad\Rightarrow S_{23} = \frac{23}{2}[91] = \frac{2093}{2} = 1046\frac{1}{2}$

(ii) Here, $a = 34$ and $d = 32 - 34 = -2$.
Let, the nth term of the A.P. is 10.
Therefore, $a_n = 10$
$\Rightarrow a + (n-1)d = 10$ $\quad\Rightarrow 34 + (n-1)(-2) = 10$
$\Rightarrow (n-1)(-2) = -24$ $\quad\Rightarrow n - 1 = 12$ $\quad\Rightarrow n = 13$
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[a + l]$$

$\Rightarrow S_{13} = \frac{13}{2}[34 + 10]$ $\quad\Rightarrow S_{13} = \frac{13}{2}[44] = 13 \times 22 = 286$

(iii) Here, $a = -5$ and $d = -8 - (-5) = -3$.
Let, the nth term of the A.P. is -230.
Therefore, $a_n = -230 \Rightarrow a + (n-1)d = -230$
$\Rightarrow -5 + (n-1)(-3) = -230$ $\quad\Rightarrow (n-1)(-3) = -225$ $\quad\Rightarrow n - 1 = 75$ $\quad\Rightarrow n = 76$
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[a + l]$$

$\Rightarrow S_{76} = \frac{76}{2}[-5 - 230]$ $\quad\Rightarrow S_{76} = \frac{76}{2}[-235] = -38 \times 235 = -8930$

Question 3:

In an AP:

(i) given a = 5, d = 3, a_n = 50, find n and S_n.
(ii) given a = 7, a_{13} = 35, find d and S_{13}.
(iii) given a_{12} = 37, d = 3, find a and S_{12}.
(iv) given a_3 = 15, S_{10} = 125, find d and a_{10}.
(v) given d = 5, S_9 = 75, find a and a_9.
(vi) given a = 2, d = 8, S_n = 90, find n and a_n.
(vii) given a = 8, a_n = 62, S_n = 210, find n and d.
(viii) given a_n = 4, d = 2, S_n = –14, find n and a.
(ix) given a = 3, n = 8, S = 192, find d.
(x) given l = 28, S = 144, and there are total 9 terms. Find *a*.

Answer 3:

(i) Here, $a = 5$, $d = 3$ and $a_n = 50$.
The nth term of the A.P. is 50.
Therefore, $a_n = 50$ $\quad\Rightarrow a + (n-1)d = 50$ $\Rightarrow 5 + (n-1)(3) = 50$
$\Rightarrow (n-1)(3) = 45$ $\quad\Rightarrow n - 1 = 15$ $\quad\Rightarrow n = 16$
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[a + l]$$

$\Rightarrow S_{16} = \frac{16}{2}[5 + 50]$
$\Rightarrow S_{16} = 8[55] = 440$

(ii) Here, $a = 7$ and $a_{13} = 35$.
The 13th term of the A.P. is 35.
Therefore, $a_{13} = 35 \quad \Rightarrow a + (13 - 1)d = 35$
$\Rightarrow 7 + 12d = 35 \qquad \Rightarrow 12d = 28 \qquad \Rightarrow d = \frac{28}{12} = \frac{7}{3}$
The sum of n terms of an AP is given by $\quad S_n = \frac{n}{2}[a + l]$
$\Rightarrow S_{13} = \frac{13}{2}[7 + 35] \quad \Rightarrow S_{13} = \frac{13}{2}[42] = 273$

(iii) Here, $d = 3$ and $a_{12} = 37$.
The 12th term of the A.P. is 37.
Therefore, $a_{12} = 50$
$\Rightarrow a + (12 - 1)d = 37 \qquad \Rightarrow a + 11(3) = 37 \qquad \Rightarrow a = 37 - 33 = 4$
The sum of n terms of an AP is given by $\quad S_n = \frac{n}{2}[a + l]$
$\Rightarrow S_{12} = \frac{12}{2}[4 + 37] \qquad \Rightarrow S_{12} = 6[41] = 246$

(iv) Here, $a_3 = 15$ and $S_{10} = 125$.
$a_3 = 15 \quad \Rightarrow a + (3 - 1)d = 15 \quad \Rightarrow a + 2d = 15$
$\Rightarrow a = 15 - 2d \qquad \ldots (1)$
The sum of n terms of an AP is given by $\quad S_n = \frac{n}{2}[2a + (n - 1)d]$
$\Rightarrow S_{10} = \frac{10}{2}[2a + (10 - 1)d] \qquad \Rightarrow 125 = 5[2a + 9d] \qquad \Rightarrow 2a + 9d = 25$
Putting the value of a from equation (1), we get
$2(15 - 2d) + 9d = 25 \qquad \Rightarrow 30 - 4d + 9d = 25 \qquad \Rightarrow 5d = -5 \qquad \Rightarrow d = -1$
Putting the value of d in equation (1), we get, $a = 15 - 2(-1) = 17$
$a_n = a + (n - 1)d \qquad \Rightarrow a_{10} = 17 + (10 - 1)(-1) = 17 - 9 = 8 \quad \Rightarrow a_{10} = 8$

(v) Here, $d = 5$ and $S_9 = 75$.
The sum of n terms of an AP is given by $\quad S_n = \frac{n}{2}[2a + (n - 1)d]$
$\Rightarrow S_9 = \frac{9}{2}[2a + (9 - 1)(5)] \qquad \Rightarrow 75 = \frac{9}{2}[2a + 40]$
$\Rightarrow 75 = 9a + 180 \qquad \Rightarrow a = -\frac{105}{9} = -\frac{35}{3}$
$a_n = a + (n - 1)d$
$\Rightarrow a_9 = -\frac{35}{3} + (9 - 1)(5) = -\frac{35}{3} + 40 = \frac{85}{3} \quad \Rightarrow a_{10} = \frac{85}{3}$

(vi) Here, $a = 2$, $d = 8$ and $S_n = 90$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$\Rightarrow 90 = \frac{n}{2}[2(2) + (n - 1)(8)] \quad \Rightarrow 90 = \frac{n}{2}[4 + 8n - 8] \qquad \Rightarrow 90 = \frac{n}{2}[8n - 4]$
$\Rightarrow 90 = 4n^2 - 2n \qquad \Rightarrow 2n^2 - n - 45 = 0$
$\Rightarrow 2n^2 - 10n + 9n - 45 = 0 \quad \Rightarrow 2n(n - 5) + 9(n - 5) = 0 \quad \Rightarrow (2n + 9)(n - 5) = 0$
$\Rightarrow n - 5 = 0 \qquad \left[\because\ 2n + 9 \neq 0 \text{ as } n \neq -\frac{9}{2}\right]$
$\Rightarrow n = 5$
$a_n = a + (n - 1)d \quad \Rightarrow a_5 = 2 + (5 - 1)(8) = 2 + 32 = 34 \quad \Rightarrow a_{10} = 34$

(vii) Here, $a = 8,\ a_n = 62$ and $S_n = 210$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[a + a_n]$$

$\Rightarrow 210 = \frac{n}{2}[8 + 62]$
$\Rightarrow 210 = 35n$
$\Rightarrow n = \frac{210}{35} = 6$
$a_n = a + (n-1)d$
$\Rightarrow 62 = 8 + (6-1)d$
$\Rightarrow 54 = 5d \quad \Rightarrow d = \frac{54}{5}$

(viii) Here, $a_n = 4, d = 2$ and $S_n = -14$.
$a_n = a + (n-1)d$
$\Rightarrow 4 = a + (n-1)(2)$
$\Rightarrow 4 = a + 2n - 2$
$\Rightarrow a = 6 - 2n$ ... (1)
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow -14 = \frac{n}{2}[2a + (n-1)(2)]$
$\Rightarrow -14 = n[a + n - 1]$
Putting the value of a from equation (1), we get
$-14 = n[6 - 2n + n - 1]$
$\Rightarrow -14 = n[5 - n]$
$\Rightarrow -14 = 5n - n^2$
$\Rightarrow n^2 - 5n - 14 = 0$
$\Rightarrow n^2 - 7n + 2n - 14 = 0$
$\Rightarrow n(n-7) + 2(n-7) = 0$
$\Rightarrow (n-7)(n+2) = 0$
$\Rightarrow n - 7 = 0$ $[\because\ n + 2 \neq 0$ as $n \neq -2\ ]$
$\Rightarrow n = 7$
Putting the value of d in equation (1), we get
$\Rightarrow a = 6 - 2(7) = -8$

(ix) Here, $a = 3,\ n = 8$ and $S = 192$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[a + a_n]$$

$\Rightarrow 192 = \frac{8}{2}[3 + a_n]$
$\Rightarrow 192 = 4[3 + a_n]$
$\Rightarrow 3 + a_n = \frac{192}{4} = 48$
$\Rightarrow a_n = 45$
$a_n = a + (n-1)d \qquad \Rightarrow 45 = 3 + (8-1)d$
$\Rightarrow 42 = 7d \quad \Rightarrow d = 6$

(x) Here, $l = 28$, $S = 144$ and $n = 9$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[a + l]$$

$\Rightarrow 144 = \frac{9}{2}[a + 28]$
$\Rightarrow 144 \times \frac{2}{9} = a + 28$
$\Rightarrow 32 = a + 28$
$\Rightarrow a = 4$

Question 4:

How many terms of the AP: 9, 17, 25 . . . must be taken to give a sum of 636?

Answer 4:

Here, $a = 9$, $d = 17 - 9 = 8$ and $S_n = 636$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$\Rightarrow 636 = \frac{n}{2}[2(9) + (n - 1)(8)]$
$\Rightarrow 636 = n[9 + 4n - 4]$
$\Rightarrow 4n^2 + 5n - 636 = 0$
$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$
$\Rightarrow n(4n + 53) - 12(4n + 53) = 0$
$\Rightarrow (n - 12)(4n + 53) = 0$
$\Rightarrow n - 12 = 0$ $\quad \left[\because\ 4n + 53 \neq 0 \text{ as } n \neq -\frac{53}{4}\right]$
$\Rightarrow n = 12$
Hence, 12 terms of the AP: 9, 17, 25 . . . must be taken to get the sum 636.

Question 5:

The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer 5:

Here, $a = 5$, $a_n = 45$ and $S_n = 400$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[a + a_n]$$

$\Rightarrow 400 = \frac{n}{2}[5 + 45]$
$\Rightarrow 400 = 25n$
$\Rightarrow n = \frac{400}{25} = 16$
$a_n = a + (n - 1)d$
$\Rightarrow 45 = 5 + (16 - 1)d$
$\Rightarrow 40 = 15d$
$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$
Hence, the number of terms are 16 and the common difference is $\frac{8}{3}$.

Question 6:
The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer 6:
Here, $a = 17$, $a_n = 350$ and $d = 9$.
$a_n = a + (n-1)d$
$\Rightarrow 350 = 17 + (n-1)9$
$\Rightarrow 350 = 8 + 9n$
$\Rightarrow n = \frac{342}{9} = 38$
The sum of n terms of an AP is given by
$$S_n = \frac{n}{2}[a + a_n]$$
$\Rightarrow S_{38} = \frac{38}{2}[17 + 350]$
$\Rightarrow S_{38} = 19 \times 367$
$= 6973$
Hence, there are 38 terms and their sum is 6973.

Question 7:
Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Answer 7:
Here, $d = 7$, $a_n = 149$ and $n = 22$.
$a_n = a + (n-1)d$
$\Rightarrow 149 = a + (22-1)(7)$
$\Rightarrow 149 = a + 147$
$\Rightarrow a = 2$
The sum of n terms of an AP is given by
$$S_n = \frac{n}{2}[a + a_n]$$
$\Rightarrow S_{22} = \frac{22}{2}[2 + 149]$
$\Rightarrow S_{22} = 11 \times 151$
$= 1661$
Hence, the sum of first 22 terms of this AP is 1661.

Question 8:
Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Answer 8:
Here, $a_2 = 14$, $a_3 = 18$ and $n = 51$.
$a_n = a + (n-1)d$
$\Rightarrow a_2 = a + (2-1)d$
$\Rightarrow 14 = a + d$
$\Rightarrow a = 14 - d \qquad \ldots (1)$
and $a_3 = a + (3-1)d$
$\Rightarrow 18 = a + 2d$
Putting the value of a from equation (1), we get
$\Rightarrow 18 = 14 - d + 2d \quad \Rightarrow d = 4$

Putting the value of d in equation (1), we get
$\Rightarrow a = 14 - 4 = 10$
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow S_{51} = \frac{51}{2}[2(10) + (51-1)(4)] \qquad \Rightarrow S_{51} = \frac{51}{2}[220] = 5610$

Question 9:

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Answer 9:

Here, $S_7 = 49$ and $S_{17} = 289$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow S_7 = \frac{7}{2}[2a + (7-1)d]$
$\Rightarrow 49 = \frac{7}{2}[2a + 6d]$
$\Rightarrow 49 = 7(a + 3d)$
$\Rightarrow 7 = a + 3d$
$\Rightarrow a = 7 - 3d \qquad \dots (1)$

and $S_{17} = \frac{17}{2}[2a + (17-1)d]$
$\Rightarrow 289 = \frac{17}{2}[2a + 16d]$
$\Rightarrow 289 = 17(a + 8d)$
$\Rightarrow 17 = a + 8d$

Putting the value of a from equation (1), we have, $17 = 7 - 3d + 8d \Rightarrow 5d = 10 \quad \Rightarrow d = 2$
Putting the value of d in equation (1), we get, $a = 7 - 3 \times 2 = 1$
The sum of n terms of an AP is given by
$S_n = \frac{n}{2}[2a + (n-1)d] \quad = \frac{n}{2}[2(1) + (n-1)(2)] \quad = \frac{n}{2}[2 + 2n - 2] = n^2$

Question 10:

Show that $a_1, a_2, \dots, a_n, \dots$ form an AP where a_n is defined as below:
(i) $a_n = 3 + 4n$ **(ii)** $a_n = 9 - 5n$
Also find the sum of the first 15 terms in each case.

Answer 10:

(i) $a_n = 3 + 4n$
Putting $n = 1$, we get, $a_1 = 3 + 4(1) = 7$
Putting $n = 2$, we get, $a_2 = 3 + 4(2) = 11$
Similarly, $a_3 = 3 + 4(3) = 15$ and $a_4 = 3 + 4(4) = 19$
Difference between the successive terms: $a_2 - a_1 = 11 - 7 = 4$
$a_3 - a_2 = 15 - 11 = 4$
$a_4 - a_3 = 19 - 15 = 4$
The difference between successive terms are same, hence $a_1, a_2, \dots, a_n, \dots$ is an A.P.

The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow S_{15} = \frac{15}{2}[2(7) + (15-1)(4)]$ $\Rightarrow S_{15} = \frac{15}{2}[70] = 525$

(ii) $a_n = 9 - 5n$
Putting $n = 1$, we get, $a_1 = 9 - 5(1) = 4$
Putting $n = 2$, we get, $a_2 = 9 - 5(2) = -1$
Similarly, $a_3 = 9 - 5(3) = -6$ and $a_4 = 9 - 5(4) = -11$
Difference between the successive terms:
$a_2 - a_1 = -1 - 4 = -5$
$a_3 - a_2 = -6 - (-1) = -5$
$a_4 - a_3 = -11 - (-6) = -5$
The difference between successive terms are same, hence $a_1, a_2, \dots, a_n, \dots$ is an A.P.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow S_{15} = \frac{15}{2}[2(4) + (15-1)(-5)]$ $\Rightarrow S_{15} = \frac{15}{2}[-62] = -465$

Question 11:

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Answer 11:

The sum of n terms of an AP is given by

$$S_n = 4n - n^2$$

Putting $n = 1$, we get
First term = $a_1 = S_1 = 4(1) - (1)^2 = 3$
Putting $n = 2$, we get
Sum of two terms = $a_1 + a_2 = S_2 = 4(2) - (2)^2 = 4$ $\Rightarrow a_1 + a_2 = 4$
$\Rightarrow 3 + a_2 = 4$ [$\because$ the first term $a_1 = 3$]
$\Rightarrow a_2 = 1$
Hence, the second term is 1.
Common difference $d = a_2 - a_1 = 1 - 3 = -2$
Therefore, the tenth term = $a_{10} = a + 9d = 3 + 9(-2) = -16$
Similarly, the nth term = $a_n = a + (n-1)d = 3 + (n-1)(-2) = 5 - 2n$

Question 12:

Find the sum of the first 40 positive integers divisible by 6.

Answer 12:

The first 40 positive integers divisible by 6 are 6, 12, 18, …, 240.
Here, $a = 6, d = 12 - 6 = 6$ and $n = 40$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow S_{40} = \frac{40}{2}[2(6) + (40-1)(6)] = 20[12 + 234] = 20(246) = 4920$
Hence, the sum of the first 40 positive integers divisible by 6 is 4920.

Question 13:
Find the sum of the first 15 multiples of 8.

Answer 13:
The first 15 multiples of 8 are 8, 16, 24, … ,120.
Here, $a = 8, d = 16 - 8 = 8$ and $n = 15$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{15} = \frac{15}{2}[2(8) + (15-1)(8)] = \frac{15}{2}[16 + 112] = \frac{15}{2}(128) = 960$$

Hence, the sum of the first 15 multiples of 8 is 960.

Question 14:
Find the sum of the odd numbers between 0 and 50.

Answer 14:
The odd numbers between 0 and 50: 1, 3, 5, … ,49.
Here, $a = 1, d = 3 - 1 = 2$ and $n = 25$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{25} = \frac{25}{2}[2(1) + (25-1)(2)] = \frac{25}{2}[2 + 48] = \frac{25}{2}(50) = 625$$

Hence, the sum of the odd numbers between 0 and 50 is 625.

Question 15:
A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹250 for the second day, ₹300 for the third day, etc., the penalty for each succeeding day being ₹50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Answer 15:
The amount paid in the form of penalty is in the form of following AP: ₹200, ₹250, ₹300, ₹350, …
Here, $a = 200, d = 250 - 200 = 50$ and $n = 30$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{30} = \frac{30}{2}[2(200) + (30-1)(50)] = 15[400 + 1450] = 15(1850) = 27750$$

Hence, the contractor has to pay ₹27750 as penalty for the delay of 30 days.

Question 16:
A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.

Answer 16:
Let the amount for first prize = ₹ x
Number of prizes = 7, total prize amount = ₹ 700, therefore, the series of 7 prizes are as follows:

$$(x) + (x-20) + (x-40) + (x-60) + (x-80) + (x-100) + (x-120) = 700$$

$$\Rightarrow 7x - 420 = 700 \Rightarrow 7x = 1120 \Rightarrow x = \frac{1120}{7} = 160$$

Hence, the seven prizes are ₹160, ₹140, ₹120, ₹100, ₹80, ₹60 and ₹40.

Question 17:

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Answer 17:

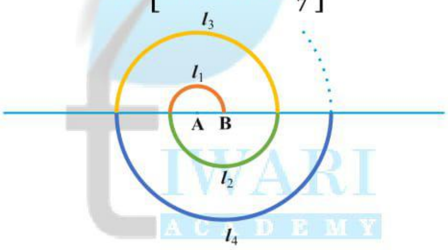
Each section of each class will plant tree = 3 × Class, therefore
Total number of tree planted by class I = 3 × 1 = 3
Total number of tree planted by class II = 3 × 2 = 6
Total number of tree planted by class III = 3 × 3 = 9
Similarly, the series of trees planted by classes are as follows: 3, 6, 9, … , 36
Here, $a = 3, d = 6 - 3 = 3$ and $n = 12$.
The sum of n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\Rightarrow S_{12} = \frac{12}{2}[2(3) + (12-1)(3)] = 6[6 + 33] = 6(39) = 234$$

Hence, the total number of tree planted by the students is 234.

Question 18:

A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm . . . as shown in Figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? $\left[\text{Take } \pi = \frac{22}{7}\right]$



[**Hint**: Length of successive semicircles is $l_1,\ l_2,\ l_3,\ l_4$, … with centres at A, B, A, B. . . respectively.]

Answer 18:

Circumference of semi-circle $= \frac{1}{2}(2\pi r) = \pi r$
Radii of semi-circles: 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, …
Therefore, the length of first spiral $l_1 = \pi(0.5)\ cm$
Length of second spiral $l_2 = \pi(1.0)\ cm$
Similarly, the lengths of spirals $l_1,\ l_2,\ l_3,\ l_4$, … are as follows:
$\pi(0.5)\ cm, \pi(1.0)\ cm, \pi(1.5)\ cm, \pi(2.0)\ cm, \pi(2.5)\ cm, \ldots$
Here, $a = 0.5\pi, d = 1.0\pi - 0.5\pi = 0.5\pi$ and $n = 13$.
The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{13} = \frac{13}{2}[2(0.5\pi) + (13-1)(0.5\pi)]$$

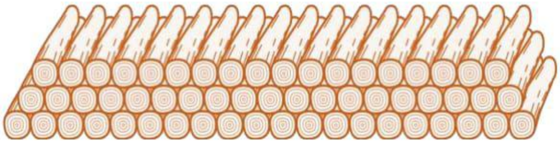
$$= 6.5[\pi + 6\pi] = 6.5\left(7 \times \frac{22}{7}\right)$$

$$= 6.5 \times 22 = 143\ cm$$

Hence, the total length of spirals made up of thirteen semicircles is 143 cm.

Question 19:

200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Figure). In how many rows are the 200 logs placed and how many logs are in the top row?



Answer 19:

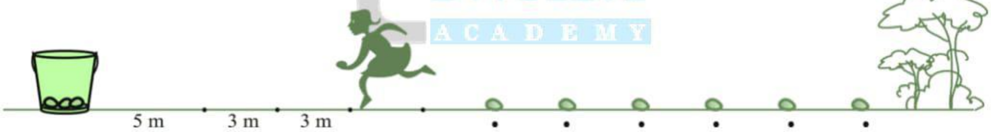
Number of logs in bottom row $= 20$, logs in next row $= 19$, logs in next row $= 18$
Similarly, the series of number of logs is 20, 19, 18, 17, …
Here, $a = 20, d = 19 - 20 = -1$ and $S_n = 200$.
The sum of n terms of an AP is given by $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 200 = \frac{n}{2}[2(20) + (n-1)(-1)]$

$\Rightarrow 400 = n[40 - n + 1] \qquad \Rightarrow 400 = 41n - n^2$
$\Rightarrow n^2 - 41n + 400 = 0 \qquad \Rightarrow n^2 - 16n - 25n + 400 = 0$
$\Rightarrow n(n-16) - 25(n-16) = 0 \qquad \Rightarrow (n-16)(n-25) = 0$
$\Rightarrow n - 16 = 0$ या $n - 25 = 0$
$\Rightarrow n = 16$ या $n = 25$
If, $n = 16, a_{16} = a + 15d = 20 + 15(-1) = 5$
If, $n = 25, a_{25} = a + 24d = 20 + 24(-1) = -4$, which is not possible.
Hence, the 200 logs are placed in 16 rows and 5 logs are in the top row.

Question 20:

In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Figure).



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?
[**Hint:** To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2 × 5 + 2 × (5 + 3)]

Answer 20:

Total distance travel to pick the first potato $= 2 \times 5 = 10$
Total distance travel to pick the second potato $= 2 \times (5 + 3) = 16$
Similarly, the series of distances travelled to pick the potatoes are 10, 16, 22, 28, …
Here, $a = 10,\ d = 16 - 10 = 6$ and $n = 10$.
The sum of n terms of an AP is given by $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2(10) + (10-1)(6)] \qquad = 5[20 + 54] \qquad = 5(74) \quad = 370\ m$

Hence, total distance travelled by the competitor is 370 m.