

# Mathematics

(www.tiwariacademy.in)

(Chapter - 5) (Arithmetic Progressions)

(Class 10)

## Exercise 5.2

### Question 1:

Fill in the blanks in the following table, given that  $a$  is the first term,  $d$  the common difference and  $a_n$  the  $n$ th term of the AP:

	$a$	$d$	$n$	$a_n$
(i)	7	3	8	...
(ii)	-18	...	10	0
(iii)	...	-3	18	-5
(iv)	-18.9	2.5	...	3.6
(v)	3.5	0	105	...

### Answer 1:

(i) Here,  $a = 7$ ,  $d = 3$  and  $n = 8$ , so, putting the values in  $a_n = a + (n - 1)d$ , we get,

$$a_n = 7 + (8 - 1)(3) \Rightarrow a_n = 7 + 21 = 28$$

(ii) Here,  $a = -18$ ,  $n = 10$  and  $a_n = 0$ , so, putting the values in  $a_n = a + (n - 1)d$ , we get,

$$0 = -18 + (10 - 1)d \Rightarrow 18 = 9d \Rightarrow d = 2$$

(iii) Here,  $d = -3$ ,  $n = 18$  and  $a_n = -5$ , so, putting the values in  $a_n = a + (n - 1)d$ , we get,

$$-5 = a + (18 - 1)(-3) \Rightarrow -5 = a - 51 \Rightarrow a = 46$$

(iv) Here,  $a = -18.9$ ,  $d = 2.5$  and  $a_n = 3.6$ , putting the values in  $a_n = a + (n - 1)d$ , we get,

$$3.6 = -18.9 + (n - 1)(2.5) \Rightarrow 3.6 = -18.9 + 2.5n - 2.5 \Rightarrow 2.5n = 3.6 + 21.4 = 25.0 \Rightarrow n = 10$$

(v) Here,  $a = 3.5$ ,  $d = 0$  and  $n = 105$ , putting the values in  $a_n = a + (n - 1)d$ , we get

$$a_n = 3.5 + (105 - 1)(0) \Rightarrow a_n = 3.5 + 0 = 3.5$$

### Question 2:

Choose the correct choice in the following and justify:

(i) 30<sup>th</sup> term of the A.P.: 10, 7, 4, ..., is

(A) 97

(B) 77

(C) -77

(D) -87

(ii) 11<sup>th</sup> term of the A.P.: -3,  $-\frac{1}{2}$ , 2, ..., is

(A) 28

(B) 22

(C) -38

(D)  $-48\frac{1}{2}$

### Answer 2:

(i) Here,  $a = 10$ ,  $d = 7 - 10 = -3$  and  $n = 30$ .

Therefore, putting the values in  $a_n = a + (n - 1)d$ , we get

$$a_{30} = 10 + (30 - 1)(-3) \Rightarrow a_{30} = 10 - 87 = -77$$

Hence, the option (C) is correct.

(ii) Here,  $a = -3$ ,  $d = -\frac{1}{2} - (-3) = \frac{5}{2}$  and  $n = 11$ .

Therefore, putting the values in  $a_n = a + (n - 1)d$ , we get

$$a_{11} = -3 + (11 - 1)\left(\frac{5}{2}\right) \Rightarrow a_{11} = -3 + 25 = 22$$

Hence, the option (B) is correct.

### Question 3:

In the following APs, find the missing terms in the boxes:

(i) 2,   , 26

(ii)   , 13,   , 3

(iii) 5,   ,   ,  $9\frac{1}{2}$

(iv) -4,   ,   ,   ,   , 6

(v)   , 38,   ,   ,   , 22

www.tiwariacademy.in

A Free web support in Education

# Mathematics

(www.tiwariacademy.in)

(Chapter - 5) (Arithmetic Progressions)

(Class 10)

## Answer 3:

(i) Here,  $a = 2$  and  $a_3 = 26$ . To find:  $a_2$

$$\text{Given that: } a_3 = a + (3 - 1)d = 26 \quad \Rightarrow 2 + 2d = 26 \quad \Rightarrow d = 12$$

$$\text{Therefore, } a_2 = a + (2 - 1)d = 2 + 12 = 14$$

(ii) Here,  $a_2 = 13$  and  $a_4 = 3$ . To find:  $a_1$  and  $a_3$

$$\text{Given that: } a_2 = a + (2 - 1)d = 13 \quad \Rightarrow a + d = 13$$

$$\Rightarrow a = 13 - d \quad \dots (1)$$

$$\text{and } a_4 = 3 \quad \Rightarrow a + 3d = 3$$

$$\text{Putting the value of } a \text{ from equation (1), we get, } 13 - d + 3d = 3 \quad \Rightarrow d = -5$$

$$\text{Putting the value of } d \text{ in equation (1), we get, } a = 13 - (-5) = 18$$

$$\text{Therefore, } a_1 = 18 \text{ and } a_3 = a + (3 - 1)d = 18 + 2(-5) = 8$$

(iii) Here,  $a = 5$  and  $a_4 = 9\frac{1}{2}$ . To find:  $a_2$  and  $a_3$

$$\text{Given that: } a_4 = a + (4 - 1)d = 9\frac{1}{2}$$

$$\Rightarrow 5 + 3d = \frac{19}{2} \quad \Rightarrow 3d = \frac{19}{2} - 5 = \frac{9}{2} \quad \Rightarrow d = \frac{3}{2}$$

$$\text{Therefore, } a_2 = a + d = 5 + \frac{3}{2} = 6\frac{1}{2} \text{ and } a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$$

(iv) Here,  $a = -4$  and  $a_6 = 6$ . To find:  $a_2$ ,  $a_3$ ,  $a_4$  and  $a_5$

$$\text{Given that: } a_6 = a + (6 - 1)d = 6$$

$$\Rightarrow -4 + 5d = 6 \quad \Rightarrow 5d = 10 \quad \Rightarrow d = 2$$

$$\text{Therefore, } a_2 = a + d = -4 + 2 = -2$$

$$a_3 = a + 2d = -4 + 2(2) = 0$$

$$a_4 = a + 3d = -4 + 3(2) = 2$$

$$a_5 = a + 4d = -4 + 4(2) = 4$$

(v) Here,  $a_2 = 38$  and  $a_6 = -22$ . To find:  $a_1$ ,  $a_3$ ,  $a_4$  and  $a_5$

$$\text{Given that: } a_2 = a + (2 - 1)d = 38 \quad \Rightarrow a + d = 38$$

$$\Rightarrow a = 38 - d \quad \dots (1)$$

$$\text{and } a_6 = -22 \quad \Rightarrow a + 5d = -22$$

$$\text{Putting the value of } a \text{ from equation (1), we get, } 38 - d + 5d = -22 \quad \Rightarrow d = -15$$

$$\text{Putting the value of } d \text{ in equation (1), we get, } a = 38 - (-15) = 53$$

$$\text{Therefore, } a_1 = a = 53$$

$$a_3 = a + 2d = 53 + 2(-15) = 23$$

$$a_4 = a + 3d = 53 + 3(-15) = 8$$

$$a_5 = a + 4d = 53 + 4(-15) = -7$$

## Question 4:

Which term of the A.P.: 3, 8, 13, 18, ... is 78?

## Answer 4:

Here,  $a = 3$  and  $d = 8 - 3 = 5$ .

Let,  $n$ th term of the A.P. is 78.

$$\text{Therefore, } a_n = 78$$

$$\Rightarrow a + (n - 1)d = 78 \quad \Rightarrow 3 + (n - 1)(5) = 78$$

$$\Rightarrow (n - 1)(5) = 75$$

$$\Rightarrow n - 1 = 15 \quad \Rightarrow n = 16$$

Hence, 16<sup>th</sup> term of the A.P.: 3, 8, 13, 18, ... is 78.

www.tiwariacademy.in

A Free web support in Education

# Mathematics

(www.tiwariacademy.in)

(Chapter - 5) (Arithmetic Progressions)

(Class 10)

## Question 5:

Find the number of terms in each of the following APs:

(i) 7, 13, 19, ..., 205

(ii)  $18, 15\frac{1}{2}, 13, \dots, -47$

### Answer 5:

(i) Here,  $a = 7$  and  $d = 13 - 7 = 6$ .

Let the total number of terms in the AP is  $n$ .

Therefore,  $a_n = 205 \Rightarrow a + (n - 1)d = 205$

$\Rightarrow 7 + (n - 1)(6) = 205 \Rightarrow (n - 1)(6) = 198$

$\Rightarrow n - 1 = 33 \Rightarrow n = 34$

Hence, there are 34 terms in the given AP.

(ii) Here,  $a = 18$  and  $d = 15\frac{1}{2} - 18 = -\frac{5}{2}$ .

Let the total number of terms in the AP is  $n$ .

Therefore,  $a_n = -47$

$\Rightarrow a + (n - 1)d = -47 \Rightarrow 18 + (n - 1)\left(-\frac{5}{2}\right) = -47$

$\Rightarrow (n - 1)\left(-\frac{5}{2}\right) = -65 \Rightarrow n - 1 = 26 \Rightarrow n = 27$

Hence, there are 27 terms in the given AP.

## Question 6:

Check whether -150 is a term of the AP: 11, 8, 5, 2 ...

### Answer 6:

Here,  $a = 11$  and  $d = 8 - 11 = -3$ .

Let, the  $n$ th term of the A.P. is -150.

Therefore,  $a_n = -150 \Rightarrow a + (n - 1)d = -150$

$\Rightarrow 11 + (n - 1)(-3) = -150 \Rightarrow 11 - 3n + 3 = -150$

$\Rightarrow -3n = -164 \Rightarrow n = \frac{164}{3} = 54\frac{2}{3}$

Here,  $n$  is not a natural number, therefore, -150 is not the term of A.P., 11, 8, 5, 2, ...

## Question 7:

Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

### Answer 7:

Here,  $a_{11} = 38$  and  $a_{16} = 73$ .

To find:  $a_{31}$

Given that:  $a_{11} = a + (11 - 1)d = 38$

$\Rightarrow a + 10d = 38$

$\Rightarrow a = 38 - 10d \dots (1)$

and  $a_{16} = 73$

$\Rightarrow a + 15d = 73$

Putting the value of  $a$  from equation (1), we get

$38 - 10d + 15d = 73 \Rightarrow 5d = 35 \Rightarrow d = 7$

Putting the value of  $d$  in equation (1), we get,

$a = 38 - 10(7) = -32$

Therefore,  $a_{31} = a + 30d = -32 + 30(7) = 178$

Hence, the 31<sup>st</sup> term is 178.

www.tiwariacademy.in

A Free web support in Education



# Mathematics

(www.tiwariacademy.in)

(Chapter - 5) (Arithmetic Progressions)

(Class 10)

## Question 8:

An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29<sup>th</sup> term.

### Answer 8:

Here,  $a_3 = 12$  and  $a_{50} = 106$ . To find:  $a_{29}$

Given that:  $a_3 = a + (3 - 1)d = 12$

$$\Rightarrow a + 2d = 12$$

$$\Rightarrow a = 12 - 2d \quad \dots (1)$$

$$\text{and } a_{50} = 106 \Rightarrow a + 49d = 106$$

Putting the value of  $a$  from equation (1), we get

$$12 - 2d + 49d = 106 \Rightarrow 47d = 94 \Rightarrow d = 2$$

Putting the value of  $d$  in equation (1), we get,  $a = 12 - 2(2) = 8$

$$\text{Therefore, } a_{29} = a + 28d = 8 + 28(2) = 64$$

Hence, the 29<sup>th</sup> term of the AP is 64.

## Question 9:

If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?

### Answer 9:

Here,  $a_3 = 4$  and  $a_9 = -8$ . To find:  $n$ , where  $a_n = 0$ .

Given that:  $a_3 = a + (3 - 1)d = 4 \Rightarrow a + 2d = 4$

$$\Rightarrow a = 4 - 2d \quad \dots (1)$$

$$\text{and } a_9 = -8 \Rightarrow a + 8d = -8$$

Putting the value of  $a$  from equation (1), we get

$$4 - 2d + 8d = -8 \Rightarrow 6d = -12 \Rightarrow d = -2$$

Putting the value of  $d$  in equation (1), we get,  $a = 4 - 2(-2) = 8$

Putting the values in  $a_n = 0$ , we get

$$a_n = a + (n - 1)d = 0 \Rightarrow 8 + (n - 1)(-2) = 0 \Rightarrow n - 1 = 4 \Rightarrow n = 5$$

Hence, the 5<sup>th</sup> term of this AP is zero.

## Question 10:

The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

### Answer 10:

Let the first term =  $a$  and common difference =  $d$

According to question,  $a_{17} = a_{10} + 7$

$$\Rightarrow a + 16d = a + 9d + 7 \Rightarrow 7d = 7 \Rightarrow d = 1$$

Hence, the common difference is 1.

## Question 11:

Which term of the AP: 3, 15, 27, 39 ... will be 132 more than its 54th term?

### Answer 11:

First term = 3 and common difference =  $15 - 3 = 12$

Let the  $n$ th term of AP: 3, 15, 27, 39 ... will be 132 more than its 54th term.

Therefore,  $a_n = a_{54} + 132$

$$\Rightarrow a + (n - 1)d = a + 53d + 132 \Rightarrow (n - 1)(12) = 53 \times 12 + 132 \Rightarrow (n - 1)(12) = 768$$

$$\Rightarrow n - 1 = \frac{768}{12} = 64 \Rightarrow n = 65$$

Hence, 65<sup>th</sup> term of the AP: 3, 15, 27, 39 ... will be 132 more than its 54th term.

www.tiwariacademy.in

A Free web support in Education

# Mathematics

(www.tiwariacademy.in)

(Chapter - 5) (Arithmetic Progressions)

(Class 10)

## Question 12:

Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

### Answer 12:

Let the first term of the first AP =  $A$  and the common difference =  $d$

Let the first term of the second AP =  $a$  and the common difference =  $d$

Difference between their 100th term =  $A_{100} - a_{100} = 100$

$$\Rightarrow (A + 99d) - (a + 99d) = 100$$

$$\Rightarrow A - a = 100$$

Difference between their 1000th term =  $A_{1000} - a_{1000}$

$$= (A + 999d) - (a + 999d)$$

$$= A - a = 100 \quad [\because A - a = 100]$$

Hence, the difference between their 1000th terms is 100.

## Question 13:

How many three-digit numbers are divisible by 7?

### Answer 13:

Three digit numbers divisible by 7: 105, 112, 119, ..., 994

Let the total number of these numbers be  $n$ .

Here,  $a = 105$  and  $d = 112 - 105 = 7$ . To find:  $n$ , where  $a_n = 994$ .

Given that:  $a_n = a + (n - 1)d = 994$

$$\Rightarrow 105 + (n - 1)(7) = 994$$

$$\Rightarrow 7(n - 1) = 889$$

$$\Rightarrow n - 1 = \frac{889}{7} = 127 \quad \Rightarrow n = 128$$

Hence, there are 128 three digits numbers which are divisible by 7.

## Question 14:

How many multiples of 4 lie between 10 and 250?

### Answer 14:

Multiples of 4 lie between 10 and 250: 12, 16, 20, ..., 248

Let the total number of multiples of 4 lie between 10 and 250 be  $n$ .

Here,  $a = 12$  and  $d = 16 - 12 = 4$ . To find:  $n$ , where  $a_n = 248$ .

Given that:  $a_n = a + (n - 1)d = 248$

$$\Rightarrow 12 + (n - 1)(4) = 248 \quad \Rightarrow 4(n - 1) = 236$$

$$\Rightarrow n - 1 = \frac{236}{4} = 59$$

$$\Rightarrow n = 60$$

Hence, the total number of multiples of 4 lie between 10 and 250 is 60.

## Question 15:

For what value of  $n$ , are the  $n$ th terms of two APs: 63, 65, 67 ... and 3, 10, 17 ... equal?

### Answer 15:

First term of first AP =  $A = 63$  and common difference =  $D = 65 - 63 = 2$

Therefore,  $A_n = A + (n - 1)D$

$$\Rightarrow A_n = 63 + (n - 1)2$$

First term of second AP =  $a = 3$  and common difference =  $d = 10 - 3 = 7$

# Mathematics

(www.tiwariacademy.in)

(Chapter – 5) (Arithmetic Progressions)

(Class 10)

Therefore,  $a_n = a + (n - 1)d$

$$\Rightarrow a_n = 3 + (n - 1)7$$

According to question,  $A_n = a_n$

$$\Rightarrow 63 + (n - 1)2 = 3 + (n - 1)7$$

$$\Rightarrow 63 + 2n - 2 = 3 + 7n - 7$$

$$\Rightarrow 65 = 5n$$

$$\Rightarrow n = 13$$

Hence, the 13<sup>th</sup> term of both the APs are equal.

## Question 16:

Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

### Answer 16:

Let the first term of the AP =  $a$  and common difference =  $d$

Third term = 16

$$\Rightarrow a_3 = 16$$

$$\Rightarrow a + 2d = 16 \quad \dots (1)$$

7th term exceeds the 5th term by 12, therefore  $a_7 = a_5 + 12$

$$\Rightarrow a + 6d = a + 4d + 12$$

$$\Rightarrow 2d = 12$$

$$\Rightarrow d = 6$$

Putting the value of  $d$  in equation (1), we get  $a + 2(6) = 16$

$$\Rightarrow a = 4$$

Hence, the A.P. =  $a, a + d, a + 2d, \dots = 4, 10, 16, \dots$

## Question 17:

Find the 20th term from the last term of the AP: 3, 8, 13... 253.

### Answer 17:

The 20th term from the last term of the AP: 3, 8, 13... 253 = the 20th term from the beginning of the AP: 253, ..., 13, 8, 3.

In the A.P.: 253, ..., 13, 8, 3, first term = 253 and common difference =  $3 - 8 = -5$

Therefore,  $a_{20} = a + 19d$

$$\Rightarrow a_{20} = 253 + 19(-5) = 253 - 95 = 158$$

## Question 18:

The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

### Answer 18:

Let the first term of the AP =  $a$  and common difference =  $d$

According to first condition,

$$a_4 + a_8 = 24$$

$$\Rightarrow a + 3d + a + 7d = 24$$

$$\Rightarrow 2a + 10d = 24$$

$$\Rightarrow a + 5d = 12$$

$$\Rightarrow a = 12 - 5d \quad \dots (1)$$

According to second condition,

$$a_6 + a_{10} = 44$$

$$\Rightarrow a + 5d + a + 9d = 44$$

www.tiwariacademy.in

A Free web support in Education



# Mathematics

(www.tiwariacademy.in)

(Chapter - 5) (Arithmetic Progressions)

(Class 10)

$$\Rightarrow 2a + 14d = 44$$

$$\Rightarrow a + 7d = 22$$

Putting the value of  $a$  from equation (1), we get

$$(12 - 5d) + 7d = 22$$

$$\Rightarrow 2d = 10$$

$$\Rightarrow d = 5$$

Putting the value of  $d$  in equation (1), we get  $a = 12 - 5(5) = -13$

The first three terms of this AP:  $a, a + d, a + 2d = -13, -8, -3$ .

## Question 19:

Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 each year. In which year did his income reach ₹7000?

### Answer 19:

Starting salary =  $a = ₹5000$  annual increment (common difference) =  $d = ₹200$

Let, after  $n$  years, his salary become ₹7000.

Therefore,  $a_n = 7000$

$$\Rightarrow a + (n - 1)d = 7000$$

$$\Rightarrow 5000 + (n - 1)(200) = 7000$$

$$\Rightarrow (n - 1)(200) = 2000$$

$$\Rightarrow n - 1 = 10$$

$$\Rightarrow n = 11$$

Hence, in 11<sup>th</sup> year his salary become ₹7000.

## Question 20:

Ramkali saved ₹5 in the first week of a year and then increased her weekly savings by ₹1.75. If in the  $n$ th week, her weekly savings become ₹20.75, find  $n$ .

### Answer 20:

Savings for the first week =  $a = ₹5$  increment in savings =  $d = ₹1.75$

Let, her saving become ₹20.75 after  $n$  weeks.

Therefore,  $a_n = 20.75$

$$\Rightarrow a + (n - 1)d = 20.75$$

$$\Rightarrow 5 + (n - 1)(1.75) = 20.75$$

$$\Rightarrow (n - 1)(1.75) = 15.75$$

$$\Rightarrow n - 1 = \frac{15.75}{1.75} = 9$$

$$\Rightarrow n = 10$$

Hence, her saving become ₹20.75, after 10 weeks.