Mathematics

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(Chapter 4) (Quadratic Equations)
(Class 10)
Exercise 4.4

Question 1:

Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:
(i). $2x^2 - 3x + 5 = 0$
(ii). $3x^2 - 4\sqrt{3}x + 4 = 0$
(iii). $2x^2 - 6x + 3 = 0$

Answer 1:

(i) $2x^2 - 3x + 5 = 0$
The given equations is of the form $ax^2 + bx + c = 0$, in which $a = 2, b = -3, c = 5$.
Therefore, $D = b^2 - 4ac = (-3)^2 - 4 \times 2 \times 5 = 9 - 40 = -31 < 0$
Hence, there is no real roots for this quadratic equation.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$
The given equations is of the form $ax^2 + bx + c = 0$ in which $a = 3, b = -4\sqrt{3}, c = 4$ Therefore, $D = b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4 \times 3 \times 4 = 48 - 48 = 0$
So, the roots of quadratic equation are real and equal.
Hence, $x = \frac{4\sqrt{3} \pm \sqrt{0}}{6} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3}$ $\left[\text{As } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$
Hence, the roots of the quadratic equation are $\frac{2\sqrt{3}}{3}$ and $\frac{2\sqrt{3}}{3}$.

(iii) $2x^2 - 6x + 3 = 0$
The given equations is of the form $ax^2 + bx + c = 0$ in which $a = 2, b = -6, c = 3$. Therefore, $D = b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3 = 36 - 24 = 12 > 0$
So, the roots of quadratic equation are real and unequal.
Hence, $x = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$ $\left[\text{As } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$
Either $x = \frac{3+\sqrt{3}}{2}$ or $x = \frac{3-\sqrt{3}}{2}$
Hence, the roots of the quadratic equation are $\frac{3+\sqrt{3}}{2}$ and $\frac{3-\sqrt{3}}{2}$.

Question 2:

Find the values of k for each of the following quadratic equations, so that they have two equal roots.
(i). $2x^2 + kx + 3 = 0$
(ii). $kx(x - 2) + 6 = 0$

Answer 2:

(i) $2x^2 + kx + 3 = 0$
For the quadratic equation $2x^2 + kx + 3 = 0$ we have $a = 2,\ b = k,\ c = 3$.
Therefore,
$b^2 - 4ac = (k)^2 - 4 \times 2 \times 3 = k^2 - 24$
For two equal roots, we have $k^2 - 24 = 0$
$\Rightarrow k^2 = 24$ $\Rightarrow k = \pm\sqrt{24}$ $\Rightarrow k = \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$
On simplification, we get $kx^2 - 2kx + 6 = 0$
For the quadratic equation $kx^2 - 2kx + 6 = 0$ we have $a = k,\ b = -2k,\ c = 6$.
Therefore,

$b^2 - 4ac = (-2k)^2 - 4 \times k \times 6 = 4k^2 - 24k$

For equal roots, we have $4k^2 - 24k = 0$

$\Rightarrow 4k(k-6) = 0$ $\Rightarrow 4k = 0$ or $(k-6) = 0$ $\Rightarrow k = 0$ or $k = 6$

But $k \neq 0$, as it doesn't satisfies the equation $kx(x-2) + 6 = 0$.

Hence, $k = 6$

Question 3:

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m2? If so, find its length and breadth.

Answer 3:

Let, the breadth of mango grove = x m

Therefore, the length = $2x$ m

So, the area = $x \times 2x = 2x^2$

According to question, $2x^2 = 800$

$\Rightarrow x^2 = 400$ $\Rightarrow x = \pm 20$

Since, the breadth of the mango grove can't be negative, so the breadth = 20 m

Hence, the length of the mango grove = $2 \times 20 = 40$ m

Question 4:

Is the following situation possible? If so, determine their present ages.

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Answer 4:

Let, the age of the first friend = x years, so, the age of the other friend = $20 - x$ years

Four years ago: Age of the first friend = $x - 4$ years

Age of the second friend = $20 - x - 4 = 16 - x$ years

According to question, $(x-4)(16-x) = 48$

$\Rightarrow 16x - x^2 - 64 + 4x = 48$ $\Rightarrow x^2 - 20x + 112 = 0$

For the quadratic equation $x^2 - 20x + 112 = 0$, we have $a = 1,\ b = -20,\ c = 112$.

Therefore, $D = b^2 - 4ac = (-20)^2 - 4 \times 1 \times 112 = 400 - 448 = -48 < 0$

So, there is no real roots of this quadratic equation.

Hence, this situation is not possible.

Question 5:

Is it possible to design a rectangular park of perimeter 80 m and area 400 m2? If so, find its length and breadth.

Answer 5:

Let the length of the park = x

Perimeter = 80 m

Therefore, the breadth = $40 - x$ m [As Perimeter = 2(Length + Breadth)]

According to question,

Area = $x(40-x) = 400$

$\Rightarrow 40x - x^2 = 400$ $\Rightarrow x^2 - 40x + 400 = 0$

$\Rightarrow x^2 - 20x - 20x + 400 = 0$

$\Rightarrow x(x-20) - 20(x-20) = 0$ $\Rightarrow (x-20)(x-20) = 0$

$\Rightarrow (x-12)^2 = 0$ or $(x-13) = 0$ $\Rightarrow (x-20) = 0$ $\Rightarrow x = 20$

Hence the length = 20 m and the breadth of the park = $40 - 20 = 20$ m.