Exercise 4.3

Question 1:

Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i). $2x^2 - 7x + 3 = 0$ **(ii).** $2x^2 + x - 4 = 0$

(iii). $4x^2 + 4\sqrt{3}x + 3 = 0$ **(iv).** $2x^2 + x + 4 = 0$

Answer 1:

(i) $2x^2 - 7x + 3 = 0$

Dividing both sides by 2

$x^2 - \frac{7}{2}x + \frac{3}{2} = 0 \qquad \Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2}$

Adding $\left[\frac{1}{2}\left(\frac{7}{2}\right)\right]^2$ on both the sides, we get

$x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 = -\frac{3}{2} + \left(\frac{7}{4}\right)^2 \qquad \left[\text{As } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right]$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = -\frac{3}{2} + \frac{49}{16} \qquad \Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{-24+49}{16}$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16} \qquad \Rightarrow x - \frac{7}{4} = \pm\frac{5}{4}$

Either $x - \frac{7}{4} = \frac{5}{4}$ or $x - \frac{7}{4} = -\frac{5}{4}$

$\Rightarrow x = \frac{5}{4} + \frac{7}{4}$ or $x = -\frac{5}{4} + \frac{7}{4}$

$\Rightarrow x = \frac{5+7}{4} = \frac{12}{4} = 3$ or $x = \frac{-5+7}{4} = \frac{2}{4} = \frac{1}{2}$

Hence, the roots of the quadratic equation are 3 and $\frac{1}{2}$.

(ii) $2x^2 + x - 4 = 0$

Dividing both sides by 2

$x^2 + \frac{1}{2}x - 2 = 0 \qquad \Rightarrow x^2 + \frac{1}{2}x = 2$

Adding $\left[\frac{1}{2}\left(\frac{1}{2}\right)\right]^2$ on both the sides, we get

$x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2 \qquad \left[\text{As } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right]$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = 2 + \frac{1}{16} \qquad \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{32+1}{16}$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16} \qquad \Rightarrow x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

Either $x + \frac{1}{4} = \frac{\sqrt{33}}{4}$ or $x + \frac{1}{4} = -\frac{\sqrt{33}}{4}$

$\Rightarrow x = \frac{\sqrt{33}}{4} - \frac{1}{4}$ or $x = -\frac{\sqrt{33}}{4} - \frac{1}{4}$

$\Rightarrow x = \frac{\sqrt{33}-1}{4}$ or $x = \frac{-\sqrt{33}-1}{4}$

Hence, the roots of the quadratic equation are $\frac{\sqrt{33}-1}{4}$ and $\frac{-\sqrt{33}-1}{4}$.

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Dividing both sides by 4

$x^2 + \sqrt{3}x + \frac{3}{4} = 0 \qquad \Rightarrow x^2 + \sqrt{3}x = -\frac{3}{4}$

Adding $\left[\frac{1}{2}\left(\sqrt{3}\right)\right]^2$ on both the sides, we get

$x^2 + \sqrt{3}x + \left(\frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \left(\frac{\sqrt{3}}{2}\right)^2$ $\left[\text{As } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right]$

$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \frac{3}{4}$ $\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$

$\Rightarrow x + \frac{\sqrt{3}}{2} = 0 \quad \Rightarrow x = -\frac{\sqrt{3}}{2}$

Hence, the roots of the quadratic equation are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

(iv) $2x^2 + x + 4 = 0$

Dividing both sides by 2

$x^2 + \frac{1}{2}x + 2 = 0$ $\Rightarrow x^2 + \frac{1}{2}x = -2$

Adding $\left[\frac{1}{2}\left(\frac{1}{2}\right)\right]^2$ on both the sides, we get

$x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 = -2 + \left(\frac{1}{4}\right)^2$ $\left[\text{As } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right]$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -2 + \frac{1}{16}$ $\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{-32+1}{16}$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -\frac{31}{16} < 0$

We know that the square of any real number can't be negative or $\left(x + \frac{1}{4}\right)^2$ can't be negative. Hence, this quadratic equation has no real roots.

Question 2:

Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Answer 2:

(i) $2x^2 - 7x + 3 = 0$

For the quadratic equation $2x^2 - 7x + 3 = 0$, we have $a = 2$, $b = -7$ and $c = 3$.

Therefore, $b^2 - 4ac = (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25 > 0$

Hence, $x = \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4}$ $\left[\text{As } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right]$

Either $x = \frac{7+5}{4} = \frac{12}{4} = 3$ or $x = \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}$

Hence, the roots of the quadratic equation are 3 and $\frac{1}{2}$.

(ii) $2x^2 + x - 4 = 0$

For the quadratic equation $2x^2 + x - 4 = 0$, we have $a = 2$, $b = 1$ and $c = -4$.

Therefore, $b^2 - 4ac = (1)^2 - 4 \times 2 \times (-4) = 1 + 32 = 33 > 0$

Hence, $x = \frac{-1 \pm \sqrt{33}}{4}$ $\left[\text{As } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right]$

Either $x = \frac{\sqrt{33}-1}{4}$ or $x = \frac{-\sqrt{33}-1}{4}$

Hence, the roots of the quadratic equation are $\frac{\sqrt{33}-1}{4}$ and $\frac{-\sqrt{33}-1}{4}$.

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

For the quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$, we have $a = 4$, $b = 4\sqrt{3}$ and $c = 3$.

Therefore, $b^2 - 4ac = \left(4\sqrt{3}\right)^2 - 4 \times 4 \times 3 = 48 - 48 = 0$

Hence, $x = \frac{-4\sqrt{3}\pm\sqrt{0}}{8} = -\frac{4\sqrt{3}}{8} = -\frac{\sqrt{3}}{2}$ $\left[\text{As } x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$

or $x = -\frac{\sqrt{3}}{2}$

Hence, the roots of the quadratic equation are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

(iv) $2x^2 + x + 4 = 0$

For the quadratic equation $2x^2 + x + 4 = 0$, we have $a = 2,\ b = 1$ and $c = 4$.

Therefore, $b^2 - 4ac = (1)^2 - 4 \times 2 \times 4 = 1 - 32 = -31 < 0$

We know that the square of any real number can't be negative, so the value of $\sqrt{b^2 - 4ac}$ cannot be real.

Hence, the roots of the quadratic equation is not real.

Question 3:

Find the roots of the following equations:

(i). $x - \frac{1}{x} = 3,\ x \neq 0$

(ii). $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30},\ x \neq -4, 7$

Answer 3:

(i). $x - \frac{1}{x} = 3,\ x \neq 0$

$\Rightarrow x^2 - 1 = 3x \quad \Rightarrow x^2 - 3x - 1 = 0$

For the quadratic equation $x^2 - 3x - 1 = 0$, we have $a = 1,\ b = -3$ and $c = -1$.

Therefore, $b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-1) = 9 + 4 = 13 > 0$

Hence, $x = \frac{3\pm\sqrt{13}}{2}$ $\left[\text{As } x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$

Either $x = \frac{3+\sqrt{13}}{2}$ or $x = \frac{3-\sqrt{13}}{2}$

Hence, the roots of the quadratic equation are $\frac{3+\sqrt{13}}{2}$ and $\frac{3-\sqrt{13}}{2}$.

(ii). $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30},\ x \neq -4, 7$

$\Rightarrow \frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30} \quad \Rightarrow \frac{-11}{x^2-3x-28} = \frac{11}{30} \quad \Rightarrow x^2 - 3x - 28 = -30$

$\Rightarrow x^2 - 3x + 2 = 0$

For the quadratic equation $x^2 - 3x + 2 = 0$, we have $a = 1,\ b = -3$ and $c = 1$.

Therefore, $b^2 - 4ac = (-3)^2 - 4 \times 1 \times 2 = 9 - 8 = 1 > 0$

Hence, $x = \frac{3\pm\sqrt{1}}{2} = \frac{3\pm1}{2}$ $\left[\text{As } x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$

Either $x = \frac{3+1}{2} = \frac{4}{2} = 2$ or $x = \frac{3-1}{2} = \frac{2}{2} = 1$

Hence, the roots of the quadratic equation are 2 and 1.

Question 4:

The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Answer 4:

Let the Rehman's current age = x years

Therefore, 3 years ago, age = $x - 3$ years and, after 5 years, age = $x + 5$ years

According to questions,

$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3} \Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)}=\frac{1}{3} \quad \Rightarrow \frac{2x+2}{x^2+2x-15}=\frac{1}{3}$

$\Rightarrow x^2+2x-15=6x+6$

$\Rightarrow x^2-4x-21=0$

For the quadratic equation $x^2-4x-21=0$, we have $a=1$, $b=-4$ and $c=-21$.

Therefore, $b^2-4ac=(-4)^2-4\times 1\times(-21)=16+84=100>0$

Hence, $x=\frac{4\pm\sqrt{100}}{2}=\frac{4\pm 10}{2}$ $\quad \left[\text{As } x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$

Either $x=\frac{4+10}{2}=\frac{14}{2}=7$ or $x=\frac{4-10}{2}=\frac{-6}{2}=-3$

Age of person can't be negative, so Rehman's age = 7 years.

Question 5:

In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Answer 5:

Let, Shefali's marks in Mathematics = x

Therefore, Shefali's marks in English = $30-x$

If she got 2 marks more in Mathematics and 3 marks less in English,

Marks in Mathematics = $x+2$

Marks in English = $30-x-3$

According to questions, Product = $(x+2)(27-x)=210$

$\Rightarrow 27x-x^2+54-2x=210 \quad \Rightarrow -x^2+25x-156=0 \quad \Rightarrow x^2-25x+156=0$

$\Rightarrow x^2-12x-13x+156=0 \quad \Rightarrow x(x-12)-13(x-12)=0 \Rightarrow (x-12)(x-13)=0$

$\Rightarrow (x-12)=0$ or $(x-13)=0$

Either $x=12$ or $x=13$

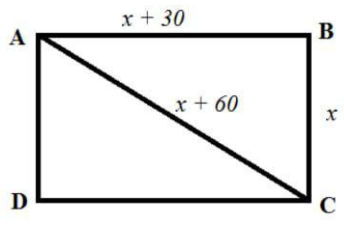
If $x=12$ then, marks in Maths = 12 and marks in English = $30-12=18$

If $x=13$ then, marks in Maths = 13 and marks in English = $30-13=17$

Question 6:

The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Answer 6:



Let the smaller side = x m

Therefore, hypotenuse = $x+60$ m

So, the longer side = $x+30$ m

According to question, $(x+60)^2=x^2+(x+30)^2$

$\Rightarrow x^2+120x+3600=x^2+x^2+60x+900$

$\Rightarrow -x^2+60x+2700=0$

$\Rightarrow x^2-60x-2700=0$

$\Rightarrow x^2-90x+30x-2700=0$

$\Rightarrow x(x-90)+30(x-90)=0$

$\Rightarrow (x-90)(x+30)=0$

$\Rightarrow (x-90)=0$ or $(x+30)=0$

Either $x=90$ or $x=-30$

But $x\neq -30$, as x is side of field which can't be negative.

Therefore, $x=90$ and hence the smaller side = 90 m

So, the longer side = $90+30=120$ m

Question 7:
The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Answer 7:
Let the larger number = x and let the smaller number = y

Therefore, $y^2 = 8x$

According to question, $x^2 - y^2 = 180$

$\Rightarrow x^2 - 8x = 180$ $\quad$ [As $y^2 = 8x$]

$\Rightarrow x^2 - 8x - 180 = 0$ $\quad \Rightarrow x^2 - 18x + 10x - 180 = 0$ $\quad \Rightarrow x(x-18) + 10(x-18) = 0$

$\Rightarrow (x-18)(x+10) = 0$ $\quad \Rightarrow (x-18) = 0$ or $(x+10) = 0$

Either $x = 18$ or $x = -10$

But $x \neq -10$, as x is the larger of two numbers. So, $x = 18$. Therefore, the larger number = 18

Hence, the smaller number = $y = \sqrt{144} = 12$

Question 8:
A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Answer 8:
Let, the speed of train = x km/h and distance = 360 km

Therefore, the time taken $t_1 = \frac{360}{x}$ hours $\quad \left[\text{Because time} = \frac{\text{Distance}}{\text{Speed}}\right]$

If the speed had been 5 km/h more, then the time $t_2 = \frac{360}{x+5}$ hours

According to question,

$$\frac{360}{x} = \frac{360}{x+5} + 1 \quad \Rightarrow \frac{360}{x} - \frac{360}{x+5} = 1 \quad \Rightarrow \frac{360(x+5) - 360x}{x(x+5)} = 1$$

$\Rightarrow 360x + 1800 - 360x = x(x+5)$ $\quad \Rightarrow 1800 = x^2 + 5x$ $\quad \Rightarrow x^2 + 5x - 1800 = 0$

$\Rightarrow x^2 + 45x - 40x - 1800 = 0$ $\quad \Rightarrow x(x+45) - 40(x+45) = 0$

$\Rightarrow (x+45)(x-40) = 0$ $\quad \Rightarrow (x+45) = 0$ or $(x-40) = 0$

Either $x = -45$ or $x = 40$

But, $x \neq -45$, as x is the speed of train which can't be negative. So, $x = 40$

Hence, the speed of train is 40 km/h.

Question 9:
Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Answer 9:
Let, the time taken by larger tap to fill the tank = x hours

So, the time taken by smaller tap to fill the tank = $x + 10$ hours

Therefore, in 1 hour,

Tank filled by larger tap = $\frac{1}{x}$

And tank filled by smaller tap = $\frac{1}{x+10}$

According to question,

$$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{9\frac{3}{8}} \quad \Rightarrow \frac{x+10+x}{x(x+10)} = \frac{8}{75} \quad \Rightarrow 75(2x+10) = 8x(x+10)$$

$\Rightarrow 150x + 750 = 8x^2 + 80x$ $\quad \Rightarrow 8x^2 - 70x - 750 = 0$ $\quad \Rightarrow 4x^2 - 35x - 375 = 0$
$\Rightarrow 4x^2 - 60x + 25x - 375 = 0$ $\quad \Rightarrow 4x(x - 15) + 25(x - 15) = 0$
$\Rightarrow (x - 15)(x + 25) = 0$ $\quad \Rightarrow (x - 15) = 0$ or $(x + 25) = 0$
Either $x = 15$ or $x = -25$
But, $x \neq -25$, as x is the time taken to fill the tank which can't be negative. So, $x = 15$
Hence, the time taken by larger tap to fill the tank = 15 hours
And the time taken by smaller tap to fill the tank = $15 + 10 = 25$ hours

Question 10:

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.

Answer 10:

Let, the speed of passenger train = x km/h, therefore, the speed of express train = $x + 11$ km/h
Distance travelled = 132 km
Time taken by passenger train $t_1 = \frac{132}{x}$ hours $\quad \left[\text{As, time} = \frac{\text{Distance}}{\text{Speed}}\right]$
Time taken by express train $t_2 = \frac{132}{x+11}$ hours
According to question,

$$\frac{132}{x} = \frac{132}{x+11} + 1 \quad \Rightarrow \frac{132}{x} - \frac{132}{x+11} = 1 \quad \Rightarrow \frac{132(x+11) - 132x}{x(x+11)} = 1$$

$\Rightarrow 132x + 1452 - 132x = x(x + 11)$

$\Rightarrow 1452 = x^2 + 11x$ $\quad \Rightarrow x^2 + 11x - 1452 = 0$ $\quad \Rightarrow x^2 + 44x - 33x - 1452 = 0$
$\Rightarrow x(x + 44) - 33(x + 44) = 0$ $\quad \Rightarrow (x + 44)(x - 33) = 0$ $\quad \Rightarrow (x + 44) = 0$ or $(x - 33) = 0$
Either $x = -44$ or $x = 33$
But $x \neq -44$, as x is the speed of train, which can't be negative. So, $x = 33$
Hence, the speed of passenger train = 33 km/h and the speed of express train = $33 + 11 = 44$ km/h

Question 11:

Sum of the areas of two squares is 468 m². If the difference of their perimeters is 24 m, find the sides of the two squares.

Answer 11:

Let the side of larger square = x m
Let the side of smaller square = y m
According to question, $x^2 + y^2 = 468$ ... (i)
Difference between perimeters, $4x - 4y = 24$ $\quad \Rightarrow x - y = 6$
$\Rightarrow x = 6 + y$ ... (ii)
Putting the value of x in equation (i), we get
$(y + 6)^2 + y^2 = 468$ $\quad \Rightarrow y^2 + 12y + 36 + y^2 = 468$ $\quad \Rightarrow 2y^2 + 12y - 432 = 0$
$\Rightarrow y^2 + 6y - 216 = 0$ $\quad \Rightarrow y^2 + 18y - 12y - 216 = 0$ $\quad \Rightarrow y(y + 18) - 12(y + 18) = 0$
$\Rightarrow (y + 18)(y - 12) = 0$ $\quad \Rightarrow (y + 18) = 0$ or $(y - 12) = 0$
Either $y = -18$ or $y = 12$
But, $y \neq -18$, as x is the side of square, which can't be negative. So, $y = 12$
Hence, the side of smaller square = 12 m
Putting the value of y in equation (ii), we get
Side of larger square = $x = y + 6 = 12 + 6 = 18$ m