Question 1:

Find the roots of the following quadratic equations by factorisation:

(i). $x^2 - 3x - 10 = 0$ **(ii).** $2x^2 + x - 6 = 0$ **(iii).** $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv). $2x^2 - x + \frac{1}{8} = 0$ **(v).** $100x^2 - 20x + 1 = 0$

Answer 1:

(i) $x^2 - 3x - 10 = 0$

Solving the quadratic equation, we get

$x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x + 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

$\Rightarrow (x - 5) = 0$ or $(x + 2) = 0$

Either $x = 5$ or $x = -2$

Hence, the roots of the given quadratic equation are 5 and – 2.

(ii) $2x^2 + x - 6 = 0$

Solving the quadratic equation, we get

$2x^2 + x - 6 = 0 \Rightarrow 2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow 2x(x - 2) + 3(x - 2) = 0$

$\Rightarrow (x - 2)(2x + 3) = 0$

$\Rightarrow (x - 2) = 0$ or $(2x + 3) = 0$

Either $x = 2$ or $x = -\frac{3}{2}$

Hence, the roots of the given quadratic equation are 2 and $-\frac{3}{2}$.

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Solving the quadratic equation, we get

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$\Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$\Rightarrow (\sqrt{2}x + 5) = 0$ or $(x + \sqrt{2}) = 0$

Either $x = -\frac{5}{\sqrt{2}}$ or $x = -\sqrt{2}$

Hence, the roots of the given quadratic equation are $-\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$.

(iv) $2x^2 - x + \frac{1}{8} = 0$

Solving the quadratic equation, we get

$16x^2 - 8x + 1 = 0$

$\Rightarrow 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow 4x(4x - 1) - 1(4x - 1) = 0$

$\Rightarrow (4x - 1)(4x - 1) = 0$

$\Rightarrow (4x - 1) = 0$ or $(4x - 1) = 0$

Either $x = \frac{1}{4}$ or $x = \frac{1}{4}$

Hence, the roots of the given quadratic equation are $\frac{1}{4}$ and $\frac{1}{4}$.

(v) $100x^2 - 20x + 1 = 0$

Solving the quadratic equation, we get,

$100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow 10x(10x - 1) - 10(10x - 1) = 0$

$\Rightarrow (10x - 1)(10x - 1) = 0$

$\Rightarrow (10x - 1) = 0 \quad \text{or} \quad (10x - 1) = 0$

Either $x = \frac{1}{10}$ or $x = \frac{1}{10}$

Hence, the roots of the given quadratic equation are $\frac{1}{10}$ and $\frac{1}{10}$.

Question 2:

Solve the problems given in Example 1. [The problems given in the example 1 are $x^2 - 45x + 324 = 0$ and $x^2 - 55x + 750 = 0$.]

Answer 2:

$x^2 - 45x + 324 = 0$

Solving the quadratic equation, we get, $x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0$

$\Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 36)(x - 9) = 0$

$\Rightarrow (x - 36) = 0 \quad \text{or} \quad (x - 9) = 0$

Either $x = 36$ or $x = 9$

Hence, John and Jivanti have 36 and 9 marbles respectively in the beginning.

$x^2 - 55x + 750 = 0$

Solving the quadratic equation, we get

$x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 30x - 25x + 750 = 0$

$\Rightarrow x(x - 30) - 25(x - 30) = 0$

$\Rightarrow (x - 30)(x - 25) = 0$

$\Rightarrow (x - 30) = 0 \quad \text{or} \quad (x - 25) = 0$

Either $x = 30$ or $x = 25$

Hence, the number of toys on that day was 30 or 25.

Question 3:

Find two numbers whose sum is 27 and product is 182.

Answer 3:

Let the first number = x

Therefore, the second number = 27 – x

According to question,

Product = $x(27 - x) = 182$

$\Rightarrow 27x - x^2 = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x - 13) - 14(x - 13) = 0$

$\Rightarrow (x - 13)(x - 14) = 0$

$\Rightarrow (x - 13) = 0 \quad \text{or} \quad (x - 14) = 0$

Either $x = 13$ or $x = 14$

Hence, the two required numbers are 13 and 14.

Question 4:

Find two consecutive positive integers, sum of whose squares is 365.

Answer 4:

Let the first number $= x$, Therefore, the second number $= x + 1$

According to questions, $x^2 + (x+1)^2 = 365$

$\Rightarrow x^2 + x^2 + 2x + 1 = 365$ $\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$ $\Rightarrow x^2 - 13x + 14x + 182 = 0$

$\Rightarrow x(x-13) + 14(x-13) = 0$ $\Rightarrow (x-13)(x+14) = 0$

$\Rightarrow (x-13) = 0$ or $(x+14) = 0$ $\Rightarrow$ Either $x = 13$ or $x = -14$

Hence, the two consecutive positive integers are 13 and 14.

Question 5:

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Answer 5:

Let the base = x cm Therefore, the height = x – 7 cm

Given that: Hypotenuse = 13 cm

Using Pythagoras theorem, $x^2 + (x-7)^2 = 13^2$

$\Rightarrow x^2 + x^2 - 14x + 49 = 169$ $\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow x^2 - 7x - 60 = 0$ $\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x-12) + 5(x-12) = 0$ $\Rightarrow (x-12)(x+5) = 0$

$\Rightarrow (x-12) = 0$ or $(x+5) = 0$

Either $x = 12$ or $x = -5$

But $x \neq -5$, as x is side of triangle.

Therefore, $x = 12$ and the second side $= x - 7 = 12 - 7 = 5$

Hence, the other two sides are 5 cm and 12 cm.

Question 6:

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹90, find the number of articles produced and the cost of each article.

Answer 6:

Let, the number of article $= x$

Therefore, the cost of one article $= 2x + 3$

According to question, the total cost $= x(2x+3) = 90$

$\Rightarrow 2x^2 + 3x = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x(2x+15) - 6(2x+15) = 0$

$\Rightarrow (2x+15)(x-6) = 0$

$\Rightarrow (2x+15) = 0$ or $(x-6) = 0$

Either $x = -\frac{15}{2}$ or $x = 6$

But, $x \neq -\frac{15}{2}$, as x is number of articles.

Therefore, $x = 6$ and the cost of each article $= 2x + 3 = 2 \times 6 + 3 = 15$

Hence, the number of articles = 6 and the cost of each article is ₹15.