Exercise 3.7(Optional)*

Question 1:

The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Answer 1:

Let the Ani's age = x years

Let the Biju's age = y years

According to question,

$x = y + 3$ ... (1) [If Ani is older than Biju]

and $y = x + 3$ ... (2) [If Biju is older than Ani]

Ani's father Dharam is twice as old as Ani, therefore

Age of Dharam = $2x$ years

Biju is twice as old as his sister Cathy, therefore

Age of Cathy = $\frac{y}{2}$ years

According to question,

$2x - \frac{y}{2} = 30$ ... (3)

Putting the value of x from equation (1), we get

$2(y + 3) - \frac{y}{2} = 30 \quad \Rightarrow 4y + 12 - y = 60$

$\Rightarrow 3y = 48 \quad \Rightarrow y = 16$

Putting the value of y in equation (1), we get

$x = 16 + 3 = 19$

Hence, the current age of Ani is 19 years and current age of Biju is 16 years.

In second case, when Biju is older than Ani.

Putting the value of y from equation (2) to equation (3), we get

$2x - \frac{x + 3}{2} = 30$

$\Rightarrow 4x - x - 3 = 60$

$\Rightarrow 3x = 63 \quad \Rightarrow x = 21$

Putting the value of x in equation (2), we get

$y = 21 + 3 = 24$

Hence, the current age of Ani is 21 years and current age of Biju is 24 years.

Question 2:

One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II]

[**Hint**: $x + 100 = 2(y - 100),\ y + 10 = 6(x - 10)$]

Answer 2:

Let the amount with first friend = ₹x

Let the amount with second friend = ₹y

According to first condition,

$x + 100 = 2(y - 100)$

$\Rightarrow x = 2y - 300$ ... (1)

According to second condition, $y + 10 = 6(x - 10)$

$\Rightarrow y - 6x = -70$

Putting the value of x from equation (1), we get

$y - 6(2y - 300) = -70 \quad \Rightarrow y - 12y + 1800 = -70$

$\Rightarrow -11y = -1870 \quad \Rightarrow y = 170$

Putting the value of y in equation (1), we get

$x = 2(170) - 300 = 40$

Hence, one friend have ₹170 and the other have ₹40.

Question 3:

A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Answer 3:

Let the uniform speed of train = x km/h

Let the distance = y km

Therefore, the time taken = $\frac{y}{x}$ hours

According to first condition, if the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time, therefore

$$\frac{y}{x+10} = \frac{y}{x} - 2 \quad \Rightarrow \frac{y}{x} - \frac{y}{x+10} = 2 \quad \Rightarrow y\left(\frac{1}{x} - \frac{1}{x+10}\right) = 2 \quad \Rightarrow y\left(\frac{x+10-x}{x(x+10)}\right) = 2$$

$$\Rightarrow y = \frac{2}{10}(x^2 + 10x) \quad \ldots (1)$$

According to second condition, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Therefore

$$\frac{y}{x-10} = \frac{y}{x} + 3 \quad \Rightarrow \frac{y}{x-10} - \frac{y}{x} = 3 \quad \Rightarrow y\left(\frac{1}{x-10} - \frac{1}{x}\right) = 3 \quad \Rightarrow y\left(\frac{x-x+10}{x(x-10)}\right) = 3$$

$$\Rightarrow y = \frac{3}{10}(x^2 - 10x)$$

Putting the value of y from equation (1), we get

$$\frac{2}{10}(x^2 + 10x) = \frac{3}{10}(x^2 - 10x) \quad \Rightarrow 2x^2 + 20x = 3x^2 - 30x \Rightarrow x^2 = 50x \quad \Rightarrow x = 50$$

Putting the value of x in equation (1), we get

$$y = \frac{2}{10}[(50)^2 + 10(50)] = 600$$

Hence, the distance covered by train is 600 km.

Question 4:

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Answer 4:

Let the number of rows = x

Let the number of students in each row = y

Therefore, the total number of students = xy

According to first condition, if 3 students are extra in a row, there would be 1 row less.

$(x-1)(y+3) = xy \quad \Rightarrow xy + 3x - y - 3 = xy$

$\Rightarrow y = 3x - 3 \quad \ldots (1)$

According to second condition, if 3 students are less in a row, there would be 2 rows more.

$(x + 2)(y - 3) = xy$
$\Rightarrow xy - 3x + 2y - 6 = xy$
$\Rightarrow -3x + 2y = 6$
Putting the value of y from equation (1), we get
$-3x + 2(3x - 3) = 6$
$\Rightarrow -3x + 6x - 6 = 6$
$\Rightarrow 3x = 12 \quad \Rightarrow x = 4$
Putting the value of x in equation (1), we get
$y = 3(4) - 3 = 9$
The total number of students in class $= xy = (4)(9) = 36$
Hence, there are 36 students in the class.

Question 5:

In a ΔABC, $\angle C = 3\ \angle B = 2(\angle A + \angle B)$. Find the three angles.

Answer 5:

Let, $\angle A = x$ and $\angle B = y$
Therefore, $\angle C = 3y$ $[\because \angle C = 3\ \angle B]$
$3\ \angle B = 2(\angle A + \angle B)$
$\Rightarrow 3y = 2(x + y) \quad \Rightarrow y = 2x \quad \ldots (1)$
In ΔABC, $\angle A + \angle B + \angle C = 180°$
$\Rightarrow x + y + 3y = 180°$
$\Rightarrow x + 4y = 180°$
Putting the value of y from equation (1), we get
$x + 4(2x) = 180°$
$\Rightarrow 9x = 180°$
$\Rightarrow x = 20°$
Putting the value of x in equation (1), we get
$y = 2(20°) = 40°$
Hence, $\angle A = 20°$, $\angle B = 40°$ and $\angle C = 120°$.

Question 6:

Draw the graphs of the equations 5x – y = 5 and 3x – y = 3. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y axis.

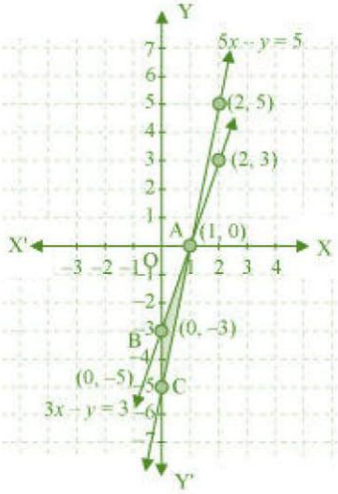
Answer 6:

$5x - y = 5 \quad \ldots (1)$
$3x - y = 3 \quad \ldots (2)$
To get the three solutions for each equations,
From the equation (1), we have
$y = 5x - 5$

x	0	1	2
y	−5	0	5

From the equation (2), we have
$y = 3x - 3$

x	0	1	2
y	−3	0	3



The co-ordinates of the vertices of the triangle ABC formed by these lines and the y axis are $A(1,0), B(0,-3)$ and $C(0,-5)$.

Area of triangle ABC

$= \frac{1}{2} \times BC \times OA$

$= \frac{1}{2} \times 2 \times 1 = 1$

Hence, the area of triangle ABC is 1 square units.

Question 7:

Solve the following pair of linear equations:

(i) $px + qy = p - q\ ;\ qx - py = p + q$

(ii) $ax + by = c\ ;\ bx + ay = 1 + c$

(iii) $\frac{x}{a} - \frac{y}{b} = 0\ ;\ ax + by = a^2 + b^2$

(iv) $(a-b)x + (a+b)y = a^2 - 2ab - b^2;\ (a+b)(x+y) = a^2 + b^2$

(v) $152x - 378y = -74\ ;\ -378x + 152y = -604$

Answer 7:

(i) $px + qy = p - q$ ... (1)

$qx - py = p + q$ ... (2)

Multiplying equation (2) with p and equation (1) with q and subtracting, we get

$$\begin{array}{rrrr} pqx + q^2y = pq - q^2 \\ pqx - p^2y = p^2 + pq \\ - \quad + \quad - \quad - \\ \hline (q^2 + p^2)y = -(p^2 + q^2) \end{array}$$

$$\Rightarrow y = -1$$

Putting the value of y in equation (1), we get

$px + q(-1) = p - q \quad \Rightarrow px = p \quad \Rightarrow x = 1$

Hence, $x = 1$ and $y = -1$.

(ii) $ax + by = c$ ... (1)

$bx + ay = 1 + c$ ... (2)

Multiplying equation (2) with a and equation (1) with b and subtracting, we get

$$\begin{array}{rrrr} abx + b^2y = bc \\ abx + a^2y = a + ac \\ - \quad - \quad - \quad - \\ \hline (b^2 - a^2)y = bc - a - ac \end{array}$$

$$\Rightarrow y = \frac{bc - a - ac}{b^2 - a^2}$$

Putting the value of y in equation (1), we get

$$ax + b\left(\frac{bc - a - ac}{b^2 - a^2}\right) = c \quad \Rightarrow ax = c - \frac{b^2c - ab - abc}{b^2 - a^2}$$

$$\Rightarrow ax = \frac{b^2c - a^2c - b^2c + ab + abc}{b^2 - a^2}$$

$$\Rightarrow x = \frac{-ac + b + bc}{b^2 - a^2}$$

Hence, $x = \frac{-ac+b+bc}{b^2-a^2}$ and $y = \frac{bc-a-ac}{b^2-a^2}$.

(iii) $\frac{x}{a} - \frac{y}{b} = 0$ ... (1)

$ax + by = a^2 + b^2$ ... (2)

From the equation (1), we have

$x = \frac{ay}{b}$ ... (3)

Putting the value of x in equation (2), we get

$a\left(\frac{ay}{b}\right) + by = a^2 + b^2$

$\Rightarrow a^2y + b^2y = a^2b + b^3$

$\Rightarrow y(a^2 + b^2) = b(a^2 + b^2)$

$\Rightarrow y = b$

Putting the value of y in equation (3), we get

$x = \frac{a(b)}{b} = a$

Hence, $x = a$ and $y = b$.

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$ ... (1)

$(a + b)(x + y) = a^2 + b^2 \quad \Rightarrow (a + b)x + (a + b)y = a^2 + b^2$ ... (2)

Subtracting equation (2) form equation (1), we get

$$\begin{array}{r} (a-b)x + (a+b)y = a^2 - 2ab - b^2 \\ (a+b)x + (a+b)y = a^2 + b^2 \\ - \quad - \quad - \quad - \\ \hline (a - b - a - b)x = -2ab - 2b^2 \end{array}$$

$$\Rightarrow -2bx = -2b(a + b)$$

$$\Rightarrow x = a + b$$

Putting the value of x in equation (1), we get

$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$

$\Rightarrow a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$

$\Rightarrow (a + b)y = -2ab$

$\Rightarrow y = \frac{-2ab}{a + b}$

Hence, $x = a + b$ and $y = -\frac{2ab}{a + b}$.

(v) $152x - 378y = -74$ ... (1)

$-378x + 152y = -604$ ... (2)

Subtracting equation (2) from equation (1), we get

$$\begin{array}{r} 152x - 378y = -74 \\ -378x + 152y = -604 \\ + \quad - \quad + \\ \hline 530x - 530y = 530 \end{array}$$

$\Rightarrow x - y = 1$

$\Rightarrow x = 1 + y$ ... (3)

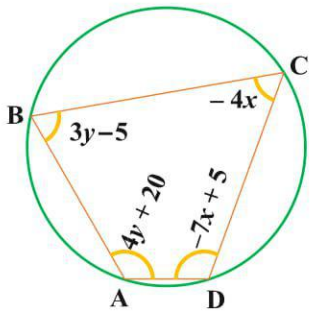
Putting the value of x in equation (1), we get

$152(1 + y) - 378y = -74$

$\Rightarrow 152 + 152y - 378y = -74$
$\Rightarrow -226y = -226$
$\Rightarrow y = 1$
Putting the value of y in equation (3), we get
$x = 1 + 1 = 2$
Hence, $x = 2$ and $y = 1$.

Question 8:

ABCD is a cyclic quadrilateral (see Figure). Find the angles of the cyclic quadrilateral.



Answer 8:

The opposite angles of cyclic quadrilateral are supplementary. Therefore
$\angle A + \angle C = 180°$ ... (1)
$\angle B + \angle D = 180°$ ... (2)
From the equation (1), we have
$4y + 20 - 4x = 180$
$\Rightarrow y - x = 40$
$\Rightarrow y = 40 + x$ ... (3)
From the equation (2), we have
$3y - 5 - 7x + 5 = 180$
$\Rightarrow 3y - 7x = 180$
Putting the value of y in equation (3), we get
$3(40 + x) - 7x = 180$
$\Rightarrow 120 + 3x - 7x = 180$
$\Rightarrow -4x = 60$
$\Rightarrow x = -15$
Putting the value of x in equation (3), we get
$y = 40 + (-15) = 25$
Hence,
$\angle A = 4y + 20 = 4(25) + 20 = 120°$
$\angle B = 3y - 5 = 3(25) - 5 = 70°$
$\angle C = -4x = -4(-15) = 60°$
$\angle D = -7x + 5 = -7(-15) + 5 = 110°$