Exercise 3.6

Question 1:

Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2;\ \frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2;\ \frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}+3y=14;\ \frac{3}{x}-4y=23$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2;\ \frac{6}{x-1}+\frac{3}{y-2}=1$

(v) $\frac{7x-2y}{xy}=5;\ \frac{8x+7y}{xy}=15$

(vi) $6x+3y=6xy;\ 2x+4y=5xy$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4;\ \frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4};\ \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Answer 1:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$ ... (1)

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$ ... (2)

Let, $\frac{1}{x}=a$ and $\frac{1}{y}=b$ ... (3)

From the equation (1), we get

$\frac{a}{2}+\frac{b}{3}=2 \Rightarrow 3a+2b=12$

$\Rightarrow a=\frac{12-2b}{3}$ ... (4)

From the equation (2), we get

$\frac{a}{3}+\frac{b}{2}=\frac{13}{6} \Rightarrow 2a+3b=13$

Putting the value of a from equation (4), we get

$2\left(\frac{12-2b}{3}\right)+3b=13$

$\Rightarrow 24-4b+9b=39$

$\Rightarrow 5b=15 \quad \Rightarrow b=3$

Putting the value of b in equation (4), we get

$a=\frac{12-2(3)}{3}=2$

From the equation (3), we get

$\frac{1}{x}=2$ and $\frac{1}{y}=3$

$\Rightarrow x=\frac{1}{2}$ and $y=\frac{1}{3}$

Hence, $x=\frac{1}{2}$ and $y=\frac{1}{3}$.

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$ ... (1)

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$ ... (2)

Let, $\frac{1}{\sqrt{x}}=a$ and $\frac{1}{\sqrt{y}}=b$ ... (3)

From the equation (1), we get

$2a+3b=2$

$\Rightarrow a=\frac{2-3b}{2}$ ... (4)

From the equation (2), we get

$4a-9b=-1$

Putting the value of a from equation (4), we get

$4\left(\frac{2-3b}{2}\right)-9b=-1$

$\Rightarrow 4-6b-9b=-1 \quad \Rightarrow -15b=-5 \quad \Rightarrow b=\frac{1}{3}$

Putting the value of b in equation (4), we get

$a=\frac{2-3\left(\frac{1}{3}\right)}{2}=\frac{1}{2}$

From the equation (3), we get

$\frac{1}{\sqrt{x}}=\frac{1}{2}$ and $\frac{1}{\sqrt{y}}=\frac{1}{3}$

$\Rightarrow x=4$ and $y=9$

Hence, $x=4$ and $y=9$.

(iii) $\frac{4}{x}+3y=14$ ... (1)

$\frac{3}{x}-4y=23$ ... (2)

Let, $\frac{1}{x}=a$ ... (3)

From the equation (1), we get

$4a+3y=14$

$\Rightarrow a=\frac{14-3y}{4}$ ... (4)

From the equation (2), we get

$3a-4y=23$

Putting the value of a from equation (4), we get

$3\left(\frac{14-3y}{4}\right)-4y=23 \quad \Rightarrow 42-9b-16y=92 \quad \Rightarrow -25y=50 \quad \Rightarrow y=-2$

Putting the value of y in equation (4), we get

$a=\frac{14-3(-2)}{4}=5$

From the equation (3), we get

$\frac{1}{x}=5 \quad \Rightarrow x=\frac{1}{5}$

Hence, $x=\frac{1}{5}$ and $y=-2$.

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$ ... (1)

$\frac{6}{x-1}-\frac{3}{y-2}=1$ ... (2)

Let, $\frac{1}{x-1}=a$ and $\frac{1}{y-2}=b$ ... (3)

From the equation (1), we get

$5a+b=2$

$\Rightarrow b=2-5a$ ... (4)

From the equation (2), we get, $6a-3b=1$

Putting the value of b from equation (4), we get

$6a-3(2-5a)=1$

$\Rightarrow 6a-6+15a=1$

$\Rightarrow 21a=7 \quad \Rightarrow a=\frac{1}{3}$

Putting the value of a in equation (4), we get

$b = 2 - 5\left(\frac{1}{3}\right) = \frac{1}{3}$

From the equation (3), we get

$\frac{1}{x-1} = \frac{1}{3}$ and $\frac{1}{y-2} = \frac{1}{3}$

$\Rightarrow x - 1 = 3$ and $y - 2 = 3$

$\Rightarrow x = 4$ and $y = 5$

Hence, $x = 4$ and $y = 5$.

(v) $\frac{7x-2y}{xy} = 5 \Rightarrow \frac{7x}{xy} - \frac{2y}{xy} = 5 \Rightarrow \frac{7}{y} - \frac{2}{x} = 5$ ... (1)

$\frac{8x+7y}{xy} = 15 \Rightarrow \frac{8x}{xy} + \frac{7y}{xy} = 15 \Rightarrow \frac{8}{y} + \frac{7}{x} = 15$ ... (2)

Let, $\frac{1}{x} = a$ and $\frac{1}{y} = b$ ... (3)

From the equation (1), we get, $7b - 2a = 5$

$\Rightarrow 7b = 5 + 2a$

$\Rightarrow b = \frac{5+2a}{7}$ ... (4)

From the equation (2), we get, $8b + 7a = 15$

Putting the value of b from equation (4), we get

$8\left(\frac{5+2a}{7}\right) + 7a = 15$

$\Rightarrow 40 + 16a + 49a = 105$

$\Rightarrow 65a = 65 \Rightarrow a = 1$

Putting the value of a in equation (4), we get

$b = \frac{5+2(1)}{7} = 1$

From the equation (3), we get

$\frac{1}{x} = 1$ and $\frac{1}{y} = 1 \Rightarrow x = 1$ and $y = 1$

Hence, $x = 1$ and $y = 1$.

(vi) $6x + 3y = 6xy \Rightarrow \frac{6x+3y}{xy} = 6 \Rightarrow \frac{6x}{xy} + \frac{3y}{xy} = 6 \Rightarrow \frac{6}{y} + \frac{3}{x} = 6$ ... (1)

$2x + 4y = 5xy \Rightarrow \frac{2x+4y}{xy} = 5 \Rightarrow \frac{2x}{xy} + \frac{4y}{xy} = 5 \Rightarrow \frac{2}{y} + \frac{4}{x} = 5$ ... (2)

Let, $\frac{1}{x} = a$ and $\frac{1}{y} = b$ ... (3)

From the equation (1), we get, $6b + 3a = 6$

$\Rightarrow 6b = 6 - 3a$

$\Rightarrow b = \frac{6-3a}{6}$ ... (4)

From the equation (2), we get,

$2b + 4a = 5$

Putting the value of b from equation (4), we get

$2\left(\frac{6-3a}{6}\right) + 4a = 5 \Rightarrow 12 - 6a + 24a = 30 \Rightarrow 18a = 18 \Rightarrow a = 1$

Putting the value of a in equation (4), we get

$b = \frac{6-3(1)}{6} = \frac{1}{2}$

From the equation (3), we get
$\frac{1}{x} = 1$ and $\frac{1}{y} = \frac{1}{2}$ $\Rightarrow x = 1$ and $y = 2$
Hence, $x = 1$ and $y = 1$.

(vii) $\frac{10}{x+y} + \frac{2}{x-y} = 4$ ... (1)

$\frac{15}{x+y} - \frac{5}{x-y} = -2$ ... (2)

Let, $\frac{1}{x+y} = a$ and $\frac{1}{x-y} = b$ ... (3)

From the equation (1), we get,
$10a + 2b = 4$
$\Rightarrow 2b = 4 - 10a$
$\Rightarrow b = \frac{4-10a}{2}$ ... (4)

From the equation (2), we get, $15a - 5b = -2$

Putting the value of b from equation (4), we get
$15a - 5\left(\frac{4-10a}{2}\right) = -2 \quad \Rightarrow 30a - 20 + 50a = -4 \quad \Rightarrow 80a = 16 \quad \Rightarrow a = \frac{1}{5}$
Putting the value of a in equation (4), we get

$$b = \frac{4 - 10\left(\frac{1}{5}\right)}{2} = 1$$

From the equation (3), we get
$\frac{1}{x+y} = \frac{1}{5}$ $\Rightarrow x + y = 5$ ... (5)
and $\frac{1}{x-y} = 1$ $\Rightarrow x - y = 1$ ... (6)
Adding equation (5) and (6), we get
$2x = 6 \quad \Rightarrow x = 3$
From the equation (5), we get
$3 + y = 5 \quad \Rightarrow y = 2$
Hence, $x = 3$ and $y = 2$.

(viii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$ ... (1)

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$ ... (2)

Let, $\frac{1}{3x+y} = a$ and $\frac{1}{3x-y} = b$ ... (3)

From the equation (1), we get,
$a + b = \frac{3}{4} \quad \Rightarrow b = \frac{3}{4} - a$
$\Rightarrow b = \frac{3-4a}{4}$ ... (4)
From the equation (2), we get,
$\frac{a}{2} - \frac{b}{2} = -\frac{1}{8}$

Putting the value of b from equation (4), we get
$\frac{a}{2} - \frac{1}{2}\left(\frac{3-4a}{4}\right) = -\frac{1}{8} \quad \Rightarrow 4a - 3 + 4a = -1 \quad \Rightarrow 8a = 2 \quad \Rightarrow a = \frac{1}{4}$
Putting the value of a in equation (4), we get

$b = \dfrac{3 - 4\left(\frac{1}{4}\right)}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

From the equation (3), we get

$\frac{1}{3x+y} = \frac{1}{4} \quad \Rightarrow 3x + y = 4 \quad \ldots (5)$

and $\frac{1}{3x-y} = \frac{1}{2} \quad \Rightarrow 3x - y = 2 \quad \ldots (6)$

Adding equation (5) and (6), we get

$6x = 6 \quad \Rightarrow x = 1$

From the equation (5), we get

$3 + y = 4 \quad \Rightarrow y = 1$

Hence, $x = 1$ and $y = 1$.

Question 2:

Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Answer 2:

(i) Let the speed of rowing in still water = x km/h

Let the speed of water = y km/h

Towards downstream, speed = $x + y$ km/h, distance = 20 km, time = 2 hours

According to question, speed = distance/time

$x + y = \frac{20}{2} \quad \Rightarrow y = 10 - x \quad \ldots (1)$

Towards upstream, speed = $x - y$ km/h, distance = 4 km, time = 2 hours

$x - y = \frac{4}{2} \quad \Rightarrow x - y = 2$

Putting the value of y form equation (1), we get

$x - (10 - x) = 2 \quad \Rightarrow 2x = 12 \quad \Rightarrow x = 6$

Putting the value of x in equation (1), we get

$y = 10 - 6 = 4$

Hence, speed of rowing in still water is 6 km/h and speed of water 4 km/h.

(ii) Let the number of days taken by 1 woman to complete the work = x

Let the number of days taken by 1 man to complete the work = y

So, the work done by 1 woman in 1 day = $\frac{1}{x}$

And the work done by 1 man in 1 day = $\frac{1}{y}$

According to first condition,

$4\left(\frac{2}{x} + \frac{5}{y}\right) = 1 \quad \Rightarrow \frac{8}{x} + \frac{20}{y} = 1 \quad \ldots (1)$

According to second condition,

$3\left(\frac{3}{x} + \frac{6}{y}\right) = 1 \quad \Rightarrow \frac{9}{x} + \frac{18}{y} = 1 \quad \ldots (2)$

Let, $\frac{1}{x} = a$ and $\frac{1}{y} = b$ ... (3)

From equation (1), we get

$8a + 20b = 1 \quad \Rightarrow a = \frac{1-20b}{8}$ ... (4)

From equation (2), we get

$9a + 18b = 1$

Putting the value of a form equation (4), we get

$9\left(\frac{1-20b}{8}\right) + 18b = 1$

$\Rightarrow 9 - 180b + 144b = 8 \quad \Rightarrow -36b = -1 \quad \Rightarrow b = \frac{1}{36}$

Putting the value of b in equation (4), we get

$a = \frac{1-20\left(\frac{1}{36}\right)}{8} = \frac{4}{72} = \frac{1}{18}$

From the equation (3), we get

$\frac{1}{x} = \frac{1}{18}$ and $\frac{1}{y} = \frac{1}{36} \quad \Rightarrow x = 18$ and $y = 36$

Hence, 1 woman takes 18 days and 1 man takes 36 days to complete the work.

(iii) Let the speed of train = x km/h

Let the speed of bus = y km/h

If she travels 60 km by train and the remaining by bus, she take 4 hours, so, according to question,

$\frac{60}{x} + \frac{240}{y} = 4$ ... (1)

If she travels 100 km by train and the remaining by bus, she take 10 minutes more, so, according to question,

$\frac{100}{x} + \frac{200}{y} = 4 + \frac{10}{60}$

$\Rightarrow \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$ ... (2)

Let, $\frac{1}{x} = a$ and $\frac{1}{y} = b$ ... (3)

From the equation (1), we get

$60a + 240b = 4 \quad \Rightarrow a = \frac{4-240b}{60}$ ... (4)

From the equation (2), we get

$100a + 200b = \frac{25}{6}$

Putting the value of a from equation (4), we get

$100\left(\frac{4-240b}{60}\right) + 200b = \frac{25}{6} \quad \Rightarrow 40 - 2400b + 1200b = 25$

$\Rightarrow -1200b = -15$

$\Rightarrow b = \frac{1}{80}$

Putting the value of b in equation (4), we get

$a = \frac{4-240\left(\frac{1}{80}\right)}{60} = \frac{1}{60}$

From the equation (3), we get

$\frac{1}{x} = \frac{1}{60}$ and $\frac{1}{y} = \frac{1}{80}$

$\Rightarrow x = 60$ and $y = 80$

Hence, the speed of train is 60 km/h and the speed of bus is 80 km/h.