Mathematics

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(Chapter 3) (Pair of Linear Equations in two variables)

(Class 10)

Exercise 3.5

Question 1:

Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.

(i) $x - 3y - 3 = 0$; $3x - 9y - 2 = 0$ **(ii)** $2x + y = 5$; $3x + 2y = 8$

(iii) $3x - 5y = 20$; $6x - 10y = 40$ **(iv)** $x - 3y - 7 = 0$; $3x - 3y - 15 = 0$

Answer 1:

(i) $x - 3y - 3 = 0$ ... (1)

$3x - 9y - 2 = 0$ ... (2)

Here, $\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$

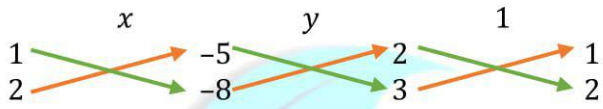
$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, so, pairs of linear equations has no solution.

(ii) $2x + y - 5 = 0$ ... (1)

$3x + 2y - 8 = 0$ ... (2)

Here, $\frac{a_1}{a_2} = \frac{2}{3}$ and $\frac{b_1}{b_2} = \frac{1}{2}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, pairs of linear equations has a unique solution.



By cross-multiplication method,

$$\frac{x}{1 \times (-8) - 2 \times (-5)} = \frac{y}{(-5) \times 3 - (-8) \times 2} = \frac{1}{2 \times 2 - 3 \times 1}$$

$\Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{1}{1} \quad \Rightarrow x = 2 \quad \text{and} \quad y = 1$

Hence, $x = 2$ and $y = 1$.

(iii) $3x - 5y - 20 = 0$ ... (1)

$6x - 10y - 40 = 0$ ... (2)

Here, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$

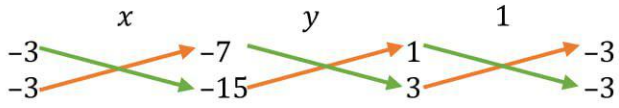
$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, so, the pairs of linear equations has infinite many solutions.

(v) $x - 3y - 7 = 0$ ... (1)

$3x - 3y - 15 = 0$ ... (2)

Here, $\frac{a_1}{a_2} = \frac{1}{3}$ and $\frac{b_1}{b_2} = \frac{-3}{-3} = \frac{1}{1}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, so, the pairs of linear equations has a unique solution.



By cross-multiplication method,

$$\frac{x}{(-3) \times (-15) - (-3) \times (-7)} = \frac{y}{(-7) \times 3 - (-15) \times 1} = \frac{1}{1 \times (-3) - 3 \times (-3)}$$

$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$

$\Rightarrow x = 4 \quad \text{and} \quad y = -1$

Hence, $x = 4$ and $y = -1$.

Question 2:

(i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$$2x + 3y = 7$$
$$(a - b)x + (a + b)y = 3a + b - 2$$

(ii) For which value of k will the following pair of linear equations have no solution?

$$3x + y = 1$$
$$(2k - 1)x + (k - 1)y = 2k + 1$$

Answer 2:

(i) $2x + 3y = 7$ ... (1)

$(a - b)x + (a + b)y = 3a + b - 2$ ... (2)

Here, $\frac{a_1}{a_2} = \frac{2}{a-b}$, $\frac{b_1}{b_2} = \frac{3}{a+b}$ and $\frac{c_1}{c_2} = \frac{7}{3a+b-2}$

Pair of linear equations have an infinite number of solutions, if

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \qquad \Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

$$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} \quad \text{and} \quad \frac{3}{a+b} = \frac{7}{3a+b-2}$$

$$\Rightarrow 2(a + b) = 3(a - b) \quad \text{and} \quad 3(3a + b - 2) = 7(a + b)$$

$$\Rightarrow 2a + 2b = 3a - 3b \quad \text{and} \quad 9a + 3b - 6 = 7a + 7b$$

$$\Rightarrow a = 5b \quad \text{and} \quad 2a = 4b + 6$$

Solving the two equations, we get

$\Rightarrow a = 5$ and $b = 1$

Question 3:

Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$$8x + 5y = 9$$
$$3x + 2y = 4$$

Answer 3:

Substitution method:

$8x + 5y = 9$ ... (1)

$3x + 2y = 4$ ... (2)

From the equation (1), we get

$y = \frac{9-8x}{5}$ ... (3)

Putting the value of y in equation (2), we get

$$3x + 2\left(\frac{9 - 8x}{5}\right) = 4$$

$$\Rightarrow 15x + 18 - 16x = 20$$

$$\Rightarrow -x = 2 \qquad \Rightarrow x = -2$$

Putting the value of x in equation (3), we get

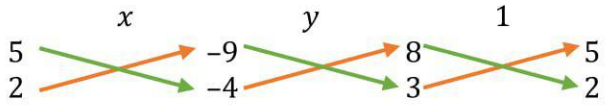
$$y = \frac{9 - 8(-2)}{5} = 5$$

Hence, $x = -2$ and $y = 5$.

Cross-multiplication method:

$8x + 5y - 9 = 0$ ... (1)

$3x + 2y - 4 = 0$ ... (2)



By cross-multiplication method,

$$\frac{x}{5 \times (-4) - 2 \times (-9)} = \frac{y}{(-9) \times 3 - (-4) \times 8} = \frac{1}{8 \times 2 - 3 \times 5}$$

$\Rightarrow \frac{x}{-2} = \frac{y}{5} = \frac{1}{1} \quad \Rightarrow x = -2 \quad \text{and} \quad y = 5$

Hence, $x = -2$ and $y = 5$.

Question 4:

Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Answer 4:

(i) Let the fixed charges = ₹ x

Let the charge for the food per day = ₹ y

If a student A takes food for 20 days she has to pay ₹1000 as hostel charges, therefore

$x + 20y = 1000$

$\Rightarrow x + 20y - 1000 = 0$ ... (1)

If a student B takes food for 26 days she has to pay ₹1180 as hostel charges, therefore

$x + 26y = 1180$

$\Rightarrow x + 26y - 1180 = 0$ ... (2)

By cross-multiplication method,

$$\frac{x}{20 \times (-1180) - 26 \times (-1000)} = \frac{y}{(-1000) \times 1 - (-1180) \times 1} = \frac{1}{1 \times 26 - 1 \times 20}$$

$\Rightarrow \frac{x}{-23600+26000} = \frac{y}{-1000+1180} = \frac{1}{6} \quad \Rightarrow \frac{x}{2400} = \frac{y}{180} = \frac{1}{6}$

$\Rightarrow x = 400 \quad$ and $\quad y = 30$

Hence, the fixed charge is ₹400 and charge for food per day is ₹30.

(ii) Let the numerator = x

Let the denominator = y

Therefore, the fraction = $\frac{x}{y}$

According to first condition,

$\frac{x-1}{y} = \frac{1}{3} \quad \Rightarrow 3x - 3 = y$

$\Rightarrow 3x - y - 3 = 0 \qquad \ldots (1)$

According to second condition,

$\frac{x}{y+8} = \frac{1}{4}$

$\Rightarrow 4x = y + 8$

$\Rightarrow 4x - y - 8 = 0 \qquad \ldots (2)$

$$\begin{array}{ccccccc} x & & y & & 1 \\ -1 & -3 & & 3 & & -1 \\ & \nearrow \quad \searrow & & \nearrow \quad \searrow & & \\ -1 & -8 & & 4 & & -1 \end{array}$$

By cross-multiplication method,

$$\frac{x}{(-1)\times(-8)-(-1)\times(-3)} = \frac{y}{(-3)\times 4-(-8)\times 3} = \frac{1}{3\times(-1)-4\times(-1)}$$

$\Rightarrow \frac{x}{8-3} = \frac{y}{-12+24} = \frac{1}{-3+4}$

$\Rightarrow \frac{x}{5} = \frac{y}{12} = \frac{1}{1}$

$\Rightarrow x = 5 \quad$ and $\quad y = 12$

Hence, the required fraction = $\frac{x}{y} = \frac{5}{12}$.

(iii) Let the number of right answers = x

Let the number of wrong answers = y

According to first condition,

$3x - y = 40$

$\Rightarrow 3x - y - 40 = 0 \qquad \ldots (1)$

According to second condition,

$4x - 2y = 50$

$\Rightarrow 4x - 2y - 50 = 0 \qquad \ldots (2)$

$$\begin{array}{ccccccc} x & & y & & 1 \\ -1 & -40 & & 3 & & -1 \\ & \nearrow \quad \searrow & & \nearrow \quad \searrow & & \\ -2 & -50 & & 4 & & -2 \end{array}$$

By cross-multiplication method,

$$\frac{x}{(-1)\times(-50)-(-2)\times(-40)} = \frac{y}{(-40)\times 4-(-50)\times 3} = \frac{1}{3\times(-2)-4\times(-1)}$$

$\Rightarrow \frac{x}{50-80} = \frac{y}{-160+150} = \frac{1}{-6+4} \quad \Rightarrow \frac{x}{-30} = \frac{y}{-10} = \frac{1}{-2}$

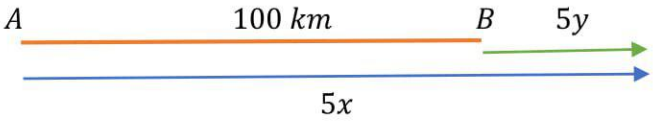
$\Rightarrow x = 15 \quad \text{and} \quad y = 5$

Hence, the total number of questions $= x + y = 15 + 5 = 20$.

(iv) Let the speed of car from station A = $x\ km/h$

Let the speed of car from station B = $y\ km/h$

After 5 hours,



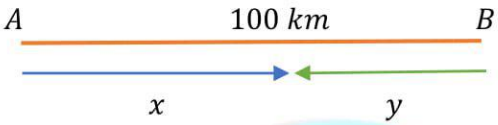
Distance travelled by car A = $5x \ \ km$

Distance travelled by car B = $5y \ \ km$

So, $5x - 5y = 100$

$\Rightarrow x - y - 20 = 0 \qquad \ldots (1)$

After 1 hour,



Distance travelled by car A = $x \ \ km$

Distance travelled by car B = $y \ \ km$

So, $x + y = 100$

$\Rightarrow x + y - 100 = 0 \qquad \ldots (2)$

By cross-multiplication method,

$$\frac{x}{(-1)\times(-100) - 1\times(-20)} = \frac{y}{(-20)\times 1 - (-100)\times 1} = \frac{1}{1\times 1 - 1\times(-1)}$$

$$\Rightarrow \frac{x}{100+20} = \frac{y}{-20+100} = \frac{1}{1+1}$$

$$\Rightarrow \frac{x}{120} = \frac{y}{80} = \frac{1}{2}$$

$\Rightarrow x = 60 \quad \text{and} \quad y = 40$

Hence, the speed of car from A is 60 km/h and the speed of car form B is 40 km/h.

(v) Let the length = x units

Let the breadth = y units

Therefore, the area = xy square units

If its length is reduced by 5 units and breadth is increased by 3 units, then

Area = $(x-5)(y+3)$ Square units

According to question,

$(x-5)(y+3) = xy - 9$

$\Rightarrow xy + 3x - 5y - 15 = xy - 9$

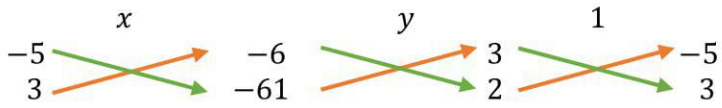
$\Rightarrow 3x - 5y - 6 = 0 \qquad \ldots (1)$

If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units.
Therefore,

$(x + 3)(y + 2) = xy + 67$

$\Rightarrow xy + 2x + 3y + 6 = xy + 67$

$\Rightarrow 2x + 3y - 61 = 0$ ... (2)



By cross-multiplication method,

$$\frac{x}{(-5)\times(-61) - 3\times(-6)} = \frac{y}{(-6)\times 2 - (-61)\times 3} = \frac{1}{3\times 3 - 2\times(-5)}$$

$$\Rightarrow \frac{x}{305 + 18} = \frac{y}{-12 + 183} = \frac{1}{9 + 10}$$

$$\Rightarrow \frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$\Rightarrow x = 17$ and $y = 9$

Hence, the length of rectangle is 17 units and the breadth is 9 units.