Exercise 1.4

Question 1:

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Answer 1:

(i) $\frac{13}{3125}$

$3125 = 5\times5\times5\times5\times5 = 5^5$

The denominator is of the form 5^m.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$8 = 2\times2\times2 = 2^3$

The denominator is of the form 2^m.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

$455 = 4\times7\times13$

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

$1600 = 2\times2\times2\times2\times2\times2\times5\times = 2^6 5^2$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

$343 = 7\times7\times7 = 7^3$

Since the denominator is not in the form $2^m \times 5^n$, and it has 7 as its factor, the decimal expansion of $\frac{29}{343}$ is non-terminating repeating.

(vi) $\frac{23}{2^3 5^2}$

Denominator $= 2^3 5^2$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{23}{2^3 5^2}$ is terminating.

(vii) $\frac{129}{2^2 5^7 7^5}$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 7 as its factor, the decimal expansion of $\frac{129}{2^2 5^7 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15}$

$\frac{6}{15} = \frac{2\times3}{5\times3} = \frac{2}{5}$

The denominator is of the form 5^n.

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

(ix) $\frac{35}{50}$

$\frac{35}{50} = \frac{5\times7}{2\times5\times5} = \frac{7}{10}$

Denominator $= 10 = 2 \times 5$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{35}{50}$ is terminating.

(x) $\frac{77}{210}$

$\frac{77}{210} = \frac{7\times11}{2\times3\times5\times7} = \frac{11}{30}$

Denominator $= 30 = 2 \times 3 \times 5$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 3 as its factors, the decimal expansion of $\frac{77}{210}$ is non-terminating repeating.

Question 2:

Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer 2:

(i) $\frac{13}{3125}$

$\frac{13}{3125} = \frac{13}{5\times5\times5\times5\times5} = \frac{13}{5^5} \times \frac{2^5}{2^5} = \frac{13\times32}{(5\times2)^5} = \frac{416}{10^5} = 0.00416$

(ii) $\frac{17}{8}$

$\frac{17}{8} = \frac{17}{2\times2\times2} = \frac{17}{2^3} \times \frac{5^3}{5^3} = \frac{17\times125}{(2\times5)^3} = \frac{2125}{10^3} = 2.125$

(iii) $\frac{64}{455}$

Decimal expansion is non-terminating repeating.

(iv) $\frac{15}{1600}$

$\frac{15}{1600} = \frac{3\times5}{2\times2\times2\times2\times2\times2\times5\times5} = \frac{3}{2^6\times5} \times \frac{5^5}{5^5} = \frac{3\times3125}{(2\times5)^6} = \frac{9375}{10^6} = 0.009375$

(v) $\frac{29}{343}$

Decimal expansion is non-terminating repeating.

(vi) $\frac{23}{2^3 5^2}$

$\frac{23}{2^3 5^2} = \frac{23}{2^3\times5^2} \times \frac{5}{5} = \frac{23\times5}{(2\times5)^3} = \frac{115}{10^3} = 0.115$

(vii) $\frac{129}{2^2 5^7 7^5}$

Decimal expansion is non-terminating repeating.

(viii) $\frac{6}{15}$

$\frac{6}{15} = \frac{2\times3}{3\times5} = \frac{2}{5} \times \frac{2}{2} = \frac{2\times2}{2\times5} = \frac{4}{10} = 0.4$

(ix) $\frac{35}{50}$

$\frac{35}{50} = \frac{5\times7}{2\times5\times5} = \frac{7}{2\times5} = \frac{7}{10} = 0.7$

(x) $\frac{77}{210}$

Decimal expansion is non-terminating repeating.

Question 3:

The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factor of q?

(i) 43.123456789 **(ii)** 0.120120012000120000... **(iii)** $43.\overline{123456789}$

Answer 3:

(i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and q is of the form $2^m \times 5^n$

i.e., the prime factors of q will be either 2 or 5 or both.

(ii) 0.120120012000120000 ...

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $43.\overline{123456789}$

Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$ i.e., the prime factors of q will also have a factor other than 2 or 5.