

Mathematics

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(Chapter – 14) (Statistics)

(Class – 10)

Exercise 14.3

Question 1:

The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 – 85	4
85 – 105	5
105 – 125	13
125 – 145	20
145 – 165	14
165 – 185	8
185 – 205	4

Answer 1:

To find the class marks, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Taking 135 as assumed mean (a), d_i , u_i , $f_i u_i$ are calculated according to step deviation method as follows.

Monthly consumption (in Units)	Number of consumers (f_i)	Class Mark x_i	$d_i = x_i - 135$	$u_i = \frac{d_i}{20}$	$f_i u_i$
65 – 85	4	75	- 60	- 3	-12
85 – 105	5	95	- 40	- 2	- 10
105 – 125	13	115	- 20	- 1	- 13
125 – 145	20	135	0	0	0
145 – 165	14	155	20	1	14
165 – 185	8	175	40	2	16
185 – 205	4	195	60	3	12
Total	68				7

From the table, we obtain

$$\sum f_i = 68, \sum f_i u_i = 7, a = 135 \text{ and } h = 20$$

$$\text{mean}(\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) h = 135 + \left(\frac{7}{68} \right) \times 20 = 135 + 2.058 = 137.058$$

From the table, it can be observed that the maximum class frequency is 20, belonging to class interval 125 – 145.

Modal class = 125 – 145

Lower limit (l) of modal class = 125

Class size (h) = 20

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 13

Frequency (f_2) of class succeeding the modal class = 14

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h = 125 + \left(\frac{20 - 13}{2 \times 20 - 13 - 14} \right) \times 20 = 125 + \frac{7}{13} \times 20 = 125 + 10.76 = 135.76$$

To find the median of the given data, cumulative frequency is calculated as follows.

Monthly consumption (in units)	Number of consumers	Cumulative frequency
65 – 85	4	4
85 – 105	5	4 + 5 = 9
105 – 125	13	9 + 13 = 22
125 – 145	20	22 + 20 = 42
145 – 165	14	42 + 14 = 56
165 – 185	8	56 + 8 = 64
185 – 205	4	64 + 4 = 68

From the table, we obtain $n = 68$

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Cumulative frequency (cf) just greater than $\frac{n}{2}$ (i.e. $\frac{68}{2} = 34$) is 42, belonging to interval 125 - 145.

Therefore, median class = 125 - 145

Lower limit (l) of median class = 125 and class size (h) = 20

Frequency (f) of median class = 20

Cumulative frequency (cf) of class preceding median class = 22

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h = 125 + \left(\frac{34 - 22}{20} \right) \times 20 = 125 + 12 = 137$$

Therefore, median, mode, mean of the given data is 137, 135.76, and 137.05 respectively.

The three measures are approximately the same in this case.

Question 2:

If the median of the distribution is given below is 28.5, find the values of x and y .

Class interval	Frequency
0 - 10	5
10 - 20	x
20 - 30	20
30 - 40	15
40 - 50	y
50 - 60	5
Total	60

Answer 2:

The cumulative frequency for the given data is calculated as follows.

Class interval	Frequency	Cumulative frequency
0 - 10	5	5
10 - 20	x	$5 + x$
20 - 30	20	$25 + x$
30 - 40	15	$40 + x$
40 - 50	y	$40 + x + y$
50 - 60	5	$45 + x + y$
Total (n)	60	

From the table, it can be observed that $n = 60$

$$45 + x + y = 60 \quad \text{or} \quad x + y = 15 \quad \dots\dots\dots (1)$$

Median of the data is given as 28.5 which lies in interval 20 - 30.

Therefore, median class = 20 - 30

Lower limit (l) of median class = 20

Cumulative frequency (cf) of class preceding the median class = $5 + x$

Frequency (f) of median class = 20

Class size (h) = 10

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$\Rightarrow 28.5 = 20 + \left\{ \frac{\frac{60}{2} - (5 + x)}{20} \right\} \times 10$$

$$\Rightarrow 8.5 = \frac{25 - x}{2}$$

$$\Rightarrow 17 = 25 - x$$

$$\Rightarrow x = 8$$

From equation (1),

$$8 + y = 15 \Rightarrow y = 7$$

Hence, the values of x and y are 8 and 7 respectively

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Question 3:

A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Answer 3:

Here, class width is not the same. There is no requirement of adjusting the frequencies according to class intervals. The given frequency table is of less than type represented with upper class limits. The policies were given only to persons with age 18 years onwards but less than 60 years. Therefore, class intervals with their respective cumulative frequency can be defined as below.

Age (in years)	Number of policy holders (f_i)	Cumulative frequency (cf)
18 – 20	2	2
20 – 25	$6 - 2 = 4$	6
25 – 30	$24 - 6 = 18$	24
30 – 35	$45 - 24 = 21$	45
35 – 40	$78 - 45 = 33$	78
40 – 45	$89 - 78 = 11$	89
45 – 50	$92 - 89 = 3$	92
50 – 55	$98 - 92 = 6$	98
55 – 60	$100 - 98 = 2$	100
Total (n)		

From the table, it can be observed that $n = 100$.

Cumulative frequency (cf) just greater than $\frac{n}{2}$ (i.e. $\frac{100}{2} = 50$) is 78, belonging to interval 35 – 40.

Therefore, median class = 35 – 40

Lower limit (l) of median class = 35 and class size (h) = 5

Frequency (f) of median class = 33

Cumulative frequency (cf) of class preceding median class = 45

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h = 35 + \left(\frac{50 - 45}{33} \right) \times 5 = 35 + \frac{25}{33} = 35 + 0.76 = 35.76$$

Therefore, median age is 35.76 years.

Question 4:

The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table:

Length (in mm)	Number of leaves f_i
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Find the median length of the leaves.

(Hint: The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 – 126.5, 126.5 – 135.5... 171.5 – 180.5)

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Answer 4:

The given data does not have continuous class intervals. It can be observed that the difference between two class intervals is 1. Therefore, $1/2 = 0.5$ has to be added and subtracted to upper class limits and lower class limits respectively. Continuous class intervals with respective cumulative frequencies can be represented as follows.

Length (in mm)	Number or leaves f_i	Cumulative frequency
117.5 – 126.5	3	3
126.5 – 135.5	5	$3 + 5 = 8$
135.5 – 144.5	9	$8 + 9 = 17$
144.5 – 153.5	12	$17 + 12 = 29$
153.5 – 162.5	5	$29 + 5 = 34$
162.5 – 171.5	4	$34 + 4 = 38$
171.5 – 180.5	2	$38 + 2 = 40$

From the table, it can be observed that the cumulative frequency just greater than $\frac{n}{2}$ (i.e. $\frac{40}{2} = 20$) is 29, belonging to class interval 144.5 – 153.5.

Median class = 144.5 – 153.5

Lower limit (l) of median class = 144.5 and class size (h) = 9

Frequency (f) of median class = 12

Cumulative frequency (cf) of class preceding median class = 17

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h = 144.5 + \left(\frac{20 - 17}{12} \right) \times 9 = 144.5 + \frac{9}{4} = 144.5 + 2.25 = 146.75$$

Therefore, median length of leaves is 146.75 mm.

Question 5:

Find the following table gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	Number of lamps
1500 – 2000	14
2000 – 2500	56
2500 – 3000	60
3000 – 3500	86
3500 – 4000	74
4000 – 4500	62
4500 – 5000	48

Find the median life time of a lamp.

Answer 5:

The cumulative frequencies with their respective class intervals are as follows.

Life time	Number of lamps (f_i)	Cumulative frequency
1500 – 2000	14	14
2000 – 2500	56	$14 + 56 = 70$
2500 – 3000	60	$70 + 60 = 130$
3000 – 3500	86	$130 + 86 = 216$
3500 – 4000	74	$216 + 74 = 290$
4000 – 4500	62	$290 + 62 = 352$
4500 – 5000	48	$352 + 48 = 400$
Total (n)	400	

It can be observed that the cumulative frequency just greater than $\frac{n}{2}$ (i.e. $\frac{400}{2} = 200$) is 216, belonging to class interval 3000 – 3500.

Median class = 3000 – 3500

Lower limit (l) of median class = 3000

Frequency (f) of median class = 86

Cumulative frequency (cf) of class preceding median class = 130

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Class size (h) = 500

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h = 3000 + \left(\frac{200 - 130}{86} \right) \times 500 = 3000 + \frac{35000}{86} = 3000 + 406.978 = 3406.978$$

Therefore, median life time of lamps is 3406.98 hours.

Question 6:

100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1 – 4	4 – 7	7 – 10	10 – 13	13 – 16	16 – 19
Number of surnames	6	30	40	6	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

Answer 6:

The cumulative frequencies with their respective class intervals are as follows.

Number of letters	Frequency (f_i)	Cumulative frequency
1 – 4	6	6
4 – 7	30	30 + 6 = 36
7 – 10	40	36 + 40 = 76
10 – 13	16	76 + 16 = 92
13 – 16	4	92 + 4 = 96
16 – 19	4	96 + 4 = 100
Total (n)	100	

It can be observed that the cumulative frequency just greater than $\frac{n}{2}$ (i.e. $\frac{100}{2} = 50$) is 76, belonging to class interval 7 – 10.

Median class = 7 – 10

Lower limit (l) of median class = 7

Cumulative frequency (cf) of class preceding median class = 36

Frequency (f) of median class = 40

Class size (h) = 3

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h = 7 + \left(\frac{50 - 36}{40} \right) \times 3 = 7 + \frac{42}{40} = 7 + 1.05 = 8.05$$

To find the class marks of the given class intervals, the following relation is used.

$$x_i = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Taking 11.5 as assumed mean (a), d_i , u_i , and $f_i u_i$ are calculated according to step deviation method as follows.

Number of letters	Number of surnames f_i	x_i	$d_i = x_i - 11.5$	$u_i = \frac{d_i}{3}$	$f_i u_i$
1 – 4	6	2.5	-9	-3	-18
4 – 7	30	5.5	-6	-2	-60
7 – 10	40	8.5	-3	-1	-40
10 – 13	16	11.5	0	0	0
13 – 16	4	14.5	3	1	4
16 – 19	4	17.5	6	2	8
Total	100				-106

From the table, we obtain

$$\sum f_i = 100, \sum f_i u_i = -106, a = 11.5 \text{ and } h = 3$$

$$\text{mean } (\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) h = 11.5 + \left(\frac{-106}{100} \right) \times 3 = 11.5 - \frac{318}{100} = 11.5 - 3.18 = 8.32$$

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The data in the given table can be written as

Number of letters	Frequency (f_i)
1 – 4	6
4 – 7	30
7 – 10	40
10 – 13	16
13 – 16	4
16 – 19	4
Total (n)	100

From the table, it can be observed that the maximum class frequency is 40 belonging to class interval 7 – 10.

Modal class = 7 – 10

Lower limit (l) of modal class = 7

Class size (h) = 3

Frequency (f_1) of modal class = 40

Frequency (f_0) of class preceding the modal class = 30

Frequency (f_2) of class succeeding the modal class = 16

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h = 7 + \left(\frac{40 - 30}{2 \times 40 - 30 - 16} \right) \times 3 = 7 + \frac{10}{34} \times 3 = 7 + 0.88 = 7.88$$

Therefore, median number and mean number of letters in surnames is 8.05 and 8.32 respectively while modal size of surnames is 7.88.

Question 7:

The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75
Number of students	2	3	8	6	6	3	2

Answer 7:

The cumulative frequencies with their respective class intervals are as follows.

Weight (in kg)	Frequency (f_i)	Cumulative frequency
40 – 45	2	2
45 – 50	3	2 + 3 = 5
50 – 55	8	5 + 8 = 13
55 – 60	6	13 + 6 = 19
60 – 65	6	19 + 6 = 25
65 – 70	3	25 + 3 = 28
70 – 75	2	28 + 2 = 30
Total (n)	30	

Cumulative frequency just greater than $\frac{n}{2}$ (i.e. $\frac{30}{2} = 15$) is 19, belonging to class interval 55 – 60.

Median class = 55 – 60

Lower limit (l) of median class = 55

Frequency (f) of median class = 6

Cumulative frequency (cf) of median class = 13

Class size (h) = 5

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h = 55 + \left(\frac{15 - 13}{6} \right) \times 5 = 55 + \frac{10}{6} = 55 + 1.67 = 56.67$$

Therefore, median weight is 56.67 kg.