Mathematics

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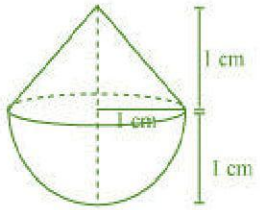
(Chapter – 13) (Surface Areas and Volumes)
(Class 10)

Exercise 13.2

Question 1:

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Answer 1:



Height of conical part (h) = radius of conical part (r) = 1 cm
Radius of conical part (r) = radius of hemispherical part (r) = 1 cm
Volume of Solid = Volume of conical part + Volume of hemispherical part

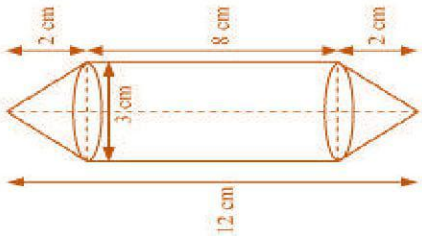
$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi.1^2.1 + \frac{2}{3}\pi.1^3 = \pi\ cm^3$$

Question 2:

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Answer 2:



Height of conical part (h_1) = 2 cm
Radius of conical part (r) = Radius of cylindrical part (r) = 3/2 cm
Height of cylindrical part (h_2) = 12 – 2 × Height of conical part
= 12 – 2 × 2 = 8 cm
The volume of air contained in the model
= Volume of cylindrical part + 2 × Volume of conical part

$$= \pi r^2 h_2 + 2 \times \frac{1}{3}\pi r^2 h_1$$

$$= \pi\left(\frac{3}{2}\right)^2.8 + 2 \times \frac{1}{3}\pi\left(\frac{3}{2}\right)^2.2$$

$$= 18\pi + 3\pi = 21\pi = 21 \times \frac{22}{7} = 66\ cm^3$$

Question 3:

A *gulab jamun,* contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns,* each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Figure).

Answer 3:

Radius of hemispherical part (r) = Radius of cylindrical part (r) = 2.8/2 = 1.4 cm
Height of hemispherical part = Radius of hemispherical part = 1.4 cm
Height of cylindrical part (h_2) = 5 – 2 × height of hemispherical part = 5 – 2 × 1.4 = 2.2 cm
Volume of 1 *Gulab Jamun* = Volume of cylindrical part + 2 × Volume of hemispherical part

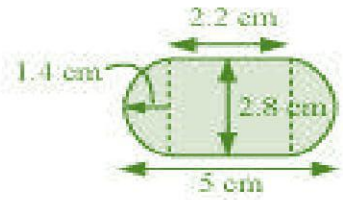
$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi(1.4)^2(2.2) + \frac{4}{3}\pi(1.4)^3$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$$

$= 13.552 + 11.498$

$= 25.05\ cm^3$



Volume of 45 *Gulab Jamun*

$= 45 \times 25.05$

$= 1{,}127.25\ cm^3$

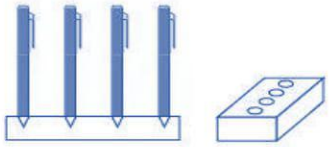
Volume of sugar syrup = 30% of volume of 45 *Gulab Jamun*

$= \frac{30}{100} \times 1127.25$

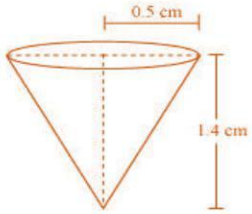
$= 338.17\ cm^3$

Question 4:

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Figure).



Answer 4:



Height of depression (h) = 1.4 cm

Radius of depression (r) = 0.5 cm

Volume of wood in the entire stand = Volume of cuboid − 4 × Volume of depression

$= lbh - 4 \times \frac{1}{3}\pi r^2 h$

$= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 1.4$

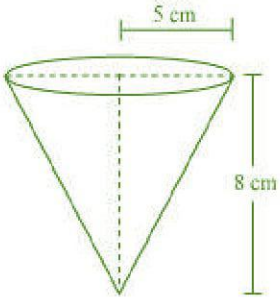
$= 525 - 1.47$

$= 523.53\ cm^3$

Question 5:

A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Answer 5:



Height of vessel (h) = 8 cm

Radius of vessel (r_1) = 5 cm

Radius of lead shots (r_2) = 0.5 cm

Let the number of lead shots dropped = n

Therefore, volume of water flows out = n × Volume of 1 lead shot

$\Rightarrow \frac{1}{4} \times$ Volume of conical vessel $= n \times$ Volume of 1 lead shot

$\Rightarrow \frac{1}{4} \times \frac{1}{3}\pi r_1^2 h = n \times \frac{4}{3}\pi r_2^3$

$\Rightarrow r_1^2 h = n \times 16 r_2^3$

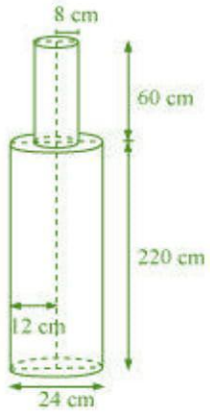
$\Rightarrow 5^2 \times 8 = n \times 16 \times (0.5)^3$

$\Rightarrow n = \frac{25 \times 8}{16 \times (0.5)^3} = 100$

Hence, the number of lead shots dropped in the vessel is 100.

Question 6:

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use π = 3.14)



Answer 6:

Radius of larger cylinder (r_1) = 12 cm
Height of larger cylinder (h_1) = 220 cm
Radius of smaller cylinder (r_2) = 8 cm
Height of smaller cylinder (h_2) = 60 cm
Volume of pole = Volume of larger cylinder + Volume of smaller cylinder

$= \pi r_1^2 h_1 + \pi r_2^2 h_2$

$= \pi(12)^2 \times 220 + \pi(8)^2 \times 60$

$= \pi[144 \times 220 + 64 \times 60] = 3.14 \times 35520 = 111532.8\ cm^3$

Mass of 1 cm^3 of iron = 8 g
Therefore, the mass of 111532.8 cm^3 of iron
= 111532.8 × 8 g = 892262.4g = 892.262 Kg

Question 7:

A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

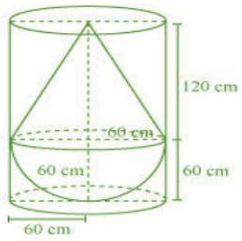
Answer 7:

Radius of hemispherical part = Radius of conical part = Radius of cylindrical part = 60 cm
Height of cylindrical part (h1) = 180 cm, Height of conical part (h2) = 120 cm
Volume of water left in the cylinder
= Volume of cylinder – (Volume of hemisphere + Volume of cone)

$= \pi r^2 h_1 - \left(\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h_2\right) = \pi(60)^2 \times 180 - \left[\frac{2}{3}\pi(60)^3 + \frac{1}{3}\pi(60)^2 \times 120\right]$

$= \pi(60)^2[180 - (40 + 40)] = \frac{22}{7} \times 60 \times 60 \times 100$

$= 1131428.57\ cm^3 = 1.131\ m^3$



Question 8:

A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm3. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Answer 8:

Radius of spherical part (r_1) = 4.25 cm
Height of cylindrical part (h) = 8 cm
Radius of cylindrical part (r_2) = 1 cm
Volume of vessel = Volume of spherical part + Volume of cylindrical part

$= \frac{4}{3}\pi r_1^3 + \pi r_2^2 h = \frac{4}{3}\pi\left(\frac{8.5}{2}\right)^2 + \pi(1)^2(8)$

$= \frac{4}{3} \times 3.14 \times 76.77 + 8 \times 3.14 = 321.392 + 25.12 = 346.51\ cm^3$

Hence, her answer is incorrect.