Exercise 13.1

Question 1:
2 cubes each of volume 64 cm³ are joined end to end. Find the surface area of the resulting cuboid.

Answer 1:
Given that: Volume of cube = 64 cm³
$\Rightarrow$ (Side)³ = 64 cm³ $\Rightarrow$ Side = 4 cm
The sides of cuboids formed by joining the cubes are 4 cm, 4 cm and 8 cm.
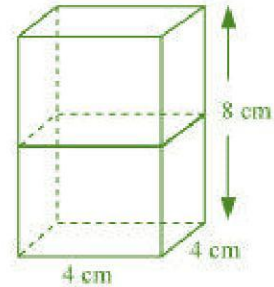Surface area of resulting cuboid
= 2(lb + bh + hl)
= 2(4×4 + 4×8 + 4×8) cm²
= 2(16 + 32 + 32) cm²
= 2(80) cm²
= 160 cm²
Hence, the surface area of the resulting cuboid 160 cm².

Question 2:
A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Answer 2:
Radius of cylinder   = 7 cm
Height of cylinder   = 13 – 7 = 6 cm
Radius of hemi-sphere = 7 cm
Inner surface area of the vessel = CSA of cylinder + CSA of hemisphere
$= 2\pi rh + 2\pi r^2$
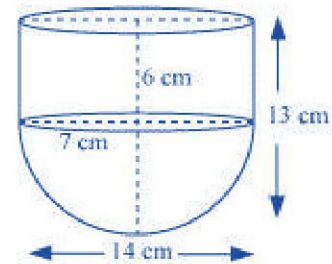$= 2 \times \dfrac{22}{7} \times 7 \times 6 + 2 \times \dfrac{22}{7} \times 7^2$
$= 44(6 + 7)$
$= 44 \times 13$
$= 572$ cm²
Hence, the inner surface area of the vessel is 572 cm².

Question 3:
A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Answer 3:
Radius of cone   = 3.5 cm
Height of cone   = 15.5 – 3.5 = 12 cm
Radius of hemisphere   = 3.5 cm
Slant height of cone (l)
$= \sqrt{r^2 + h^2}$ $= \sqrt{(3.5)^2 + (12)^2}$ $= \sqrt{12.25 + 144}$ $= \sqrt{156.25}$ $= 12.5$
The total surface area of the toy
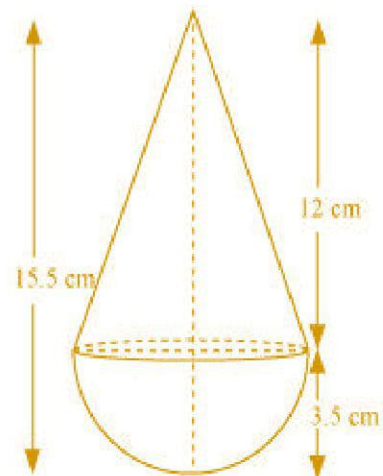= CSA of cone + CSA of hemisphere
$= \pi rl + 2\pi r^2$
$= \dfrac{22}{7} \times 3.5 \times 12.5 + 2 \times \dfrac{22}{7} \times 3.5^2$
$= 137.5 + 77$
$= 214.5$ cm²
Hence, the total surface area of the toy is 572 cm².

Question 4:
A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Answer 4:
The maximum diameter of hemisphere = side of cubical block (a) = 7 cm
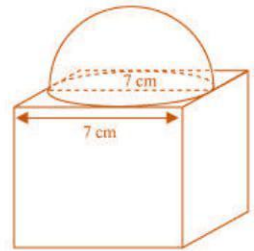Radius of hemisphere = $a/2$ = 3.5 cm
The surface area of the solid
= Surface area of cubical block + CSA of hemisphere – Area of base of hemisphere
$= 6a^2 + 2\pi r^2 - 2\pi r^2$
$= 6a^2 + \pi r^2$
$= 6 \times 7^2 + \frac{22}{7} \times 3.5^2$ $= 294 + 38.5$ $= 332.5$ cm²

Hence, the surface area of the solid is 332.5 cm².

Question 5:
A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Answer 5:
The maximum diameter of hemisphere = side of cubical block = l
Radius of hemisphere (r) = $l/2$
The surface area of the remaining solid
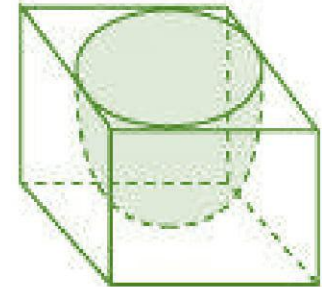= TSA of cubical block + CSA of hemisphere – Area of base of hemisphere
$= 6l^2 + 2\pi r^2 - 2\pi r^2$
$= 6l^2 + \pi r^2$
$= 6l^2 + \pi \left(\frac{l}{2}\right)^2$
$= (6 + \pi/4)l^2$

Hence, the surface area of the remaining solid is $(6 + \pi/4)l^2$.

Question 6:
A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

Answer 6:
Radius of cylindrical part   = 5/2 = 2.5 mm
Length of cylindrical part   = 14 – (2.5 + 2.5) = 9 mm
Radius of hemisphere   = 5/2 = 2.5 mm

Surface area of capsule
= CSA of cylindrical part + 2 × CSA of hemispherical part
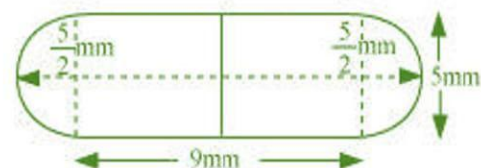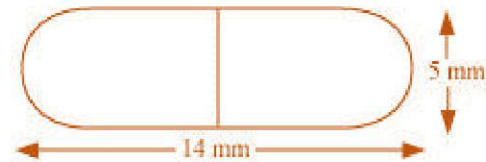$= 2\pi rh + 2(2\pi r^2)$
$= 2\pi rh + 4\pi r^2$
$= 2\pi(2.5)(9) + 4\pi(2.5)^2$
$= 45\pi + 25\pi$ $= 70\pi$
$= 70 \times \frac{22}{7}$
$= 220$ mm²
Hence, the surface area of capsule is 220 mm².

Question 7:

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m². (Note that the base of the tent will not be covered with canvas.)

Answer 7:

Height of cylindrical part (h) = 2.1 m
Diameter of cylindrical part = 4 m
Radius of cylindrical part (r) = 2 m
Height of conical part (l) = 2.8 m

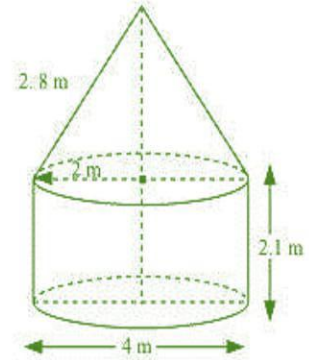Area of canvas used = CSA conical part + CSA of cylindrical part
$= \pi r l + 2\pi r h$
$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$
$= 2\pi(2.8 + 4.2) = 2 \times \frac{22}{7} \times 7 = 44\ m^2$

The cost of 1 m² canvas = ₹ 500
Therefore, the cost of 44 m² canvas = 44 × ₹ 500 = ₹ 22000

Question 8:

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².

Answer 8:

Height of conical part = Height of cylindrical part (h) = 2.4 cm
Diameter of cylindrical part = 1.4 m, so, the radius of cylindrical part (r) = 0.7 m
Slant height of cylindrical part (l) $= \sqrt{r^2 + h^2}$
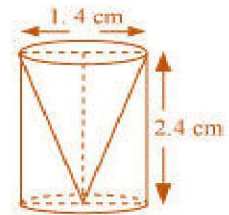$= \sqrt{(0.7)^2 + (2.4)^2}$ $= \sqrt{0.49 + 5.76}$ $= \sqrt{6.25} = 2.5$

The total surface area of the remaining solid
= CSA of cylindrical + CSA of conical part + Area of base of cylinder
$= 2\pi r h + \pi r l + \pi r^2$
$= 2 \times \frac{22}{7} \times 0.7 \times 2.4 + \frac{22}{7} \times 0.7 \times 2.5 + \frac{22}{7} \times 0.7 \times 0.7$
$= 4.4 \times 2.4 + 2.2 \times 2.5 + 2.2 \times 0.7 = 10.56 + 5.50 + 1.56 = 17.60\ cm^2$

Question 9:

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

Answer 9:

Radius of hemispherical part = radius of cylindrical part (r) = 3.5 cm

Height of cylindrical part (h) = 10 m

The total surface area of article
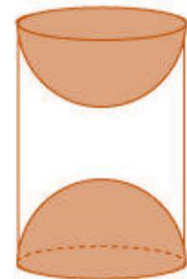
= CSA of cylindrical part + CSA of two hemispherical part
$= 2\pi r h + 2 \times 2\pi r^2$
$= 2\pi \times 3.5 \times 10 + 2 \times 2\pi \times 3.5 \times 3.5$
$= 70\pi + 49\pi = 119\pi$
$= 119 \times \frac{22}{7} = 17 \times 22 = 374\ cm^2$

Hence, the total surface area of the article is $374\ cm^2$.