Exercise 12.3

Question 1:

Find the area of the shaded region in Figure, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle. [Use $\pi = 22/7$]

Answer 1:

QR is diameter of circle. Therefore, $\angle RPQ = 90°$ [Angle in semicircle is right angle]

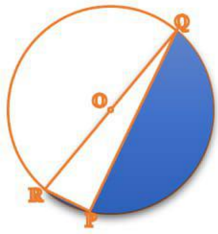
In ΔPQR, by Pythagoras theorem,

$RP^2 + PQ^2 = RQ^2$

$(7)^2 + (24)^2 = RQ^2$

$\Rightarrow RQ^2 = 576 + 49 = 625$

$\Rightarrow RQ = \sqrt{625} = 25$



Therefore, the radius of circle = RQ/2 = 25/2 cm

Area of shaded region = Area of semicircle – Area of ΔPQR

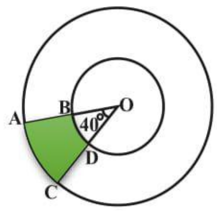
$$= \frac{1}{2} \times \pi r^2 - \frac{1}{2} \times PR \times PQ = \frac{1}{2} \times \pi \left(\frac{25}{2}\right)^2 - \frac{1}{2} \times 7 \times 24 = \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} - 7 \times 12$$

$$= \frac{6875}{28} - 84 = \frac{6875 - 2352}{28} = \frac{4523}{28} \text{ cm}^2$$

Question 2:

Find the area of the shaded region in Figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40°$. [Use $\pi = 22/7$]

Answer 2:



Radius of smaller circle = 7 cm

Radius of larger circle = 14 cm

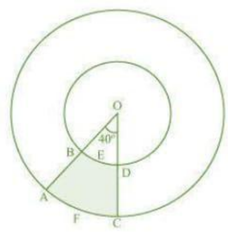
Area of shaded region

= Area of sector OAFCO – Area of sector OBEDO

$$= \frac{40°}{360°} \times \pi(14)^2 - \frac{40°}{360°} \times \pi(7)^2$$

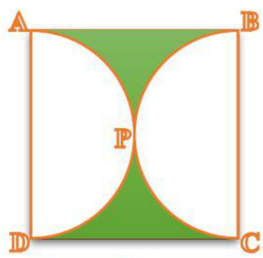
$$= \frac{1}{9} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{9} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{616}{9} - \frac{154}{9} = \frac{462}{9} = \frac{154}{3}\ cm^2$$



Question 3:

Find the area of the shaded region in Figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles. [Use $\pi = 22/7$]

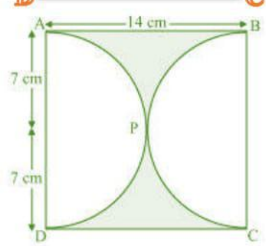


Answer 3:

Side of square is 14 cm, therefore, the radius of semicircle is 7 cm.

Area of semicircle

$$= \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \pi(7)^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2$$



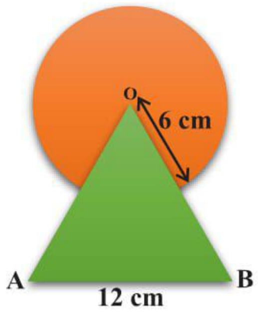
Area of square

$= (\text{side})^2 = (14)^2 = 196 \text{ cm}^2$

Area of shaded region = Area of square – Area of two semicircles

$= 196 - 2 \times 77 = 196 - 154 = 42 \text{ cm}^2$

Question 4:
Find the area of the shaded region in Figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre. [Use π = 22/7]

Answer 4:


We know that each angle of equilateral triangle is 60°.
Area of sector OCDE

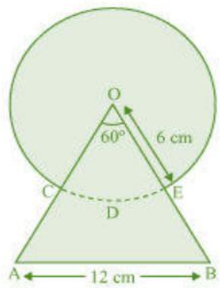
$= \frac{60°}{360°} \times \pi r^2 \quad = \frac{1}{6} \times \pi (6)^2 \quad = \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 \quad = \frac{132}{7} \text{ cm}^2$

Area of equilateral triangle OAB

$= \frac{\sqrt{3}}{4}(12)^2 \quad = 36\sqrt{3} \text{ cm}^2$

Area of circle

$= \pi r^2 \quad = \pi (6)^2 \quad = \frac{22}{7} \times 6 \times 6 \quad = \frac{792}{7} \text{ cm}^2$

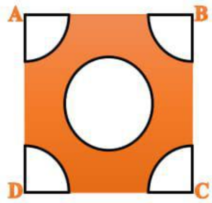


Area of shaded region
= Area of circle + Area of triangle OAB – Area of sector OCDE

$= \left(\frac{792}{7} + 36\sqrt{3} - \frac{132}{7}\right) \text{ cm}^2$

$= \left(36\sqrt{3} + \frac{660}{7}\right) \text{ cm}^2$

Question 5:
From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Figure. Find the area of the remaining portion of the square. [Use π = 22/7]

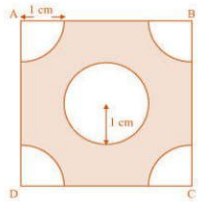
Answer 5:


Radius of each quadrant = 1 cm
Area of each quadrant

$= \frac{90°}{360°} \times \pi r^2 \quad = \frac{1}{4} \times \pi (1)^2 \quad = \frac{1}{4} \times \frac{22}{7} \times 1 \times 1 \quad = \frac{11}{14} \text{ cm}^2$

Area of circle

$= \pi r^2 \quad = \pi (1)^2 \quad = \frac{22}{7} \times 1 \times 1 \quad = \frac{22}{7} \text{ cm}^2$

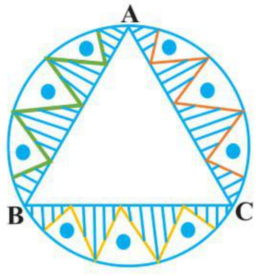


Area of square = $(\text{Side})^2 \quad = (4)^2 \quad = 16 \text{ cm}^2$

Area of shaded region = Area of square – Area of circle – Area of 4 quadrants

$16 - \frac{22}{7} - 4 \times \frac{11}{14} = 16 - \frac{22}{7} - \frac{22}{7} = \frac{112 - 44}{7} = \frac{68}{7} \ cm^2$

Question 6:
In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Figure. Find the area of the design (shaded region). [Use π = 22/7]

Answer 6:


Radius of circle = 32
Centroid O divides median AD into 2:1, therefore AO:OD = 2:1.

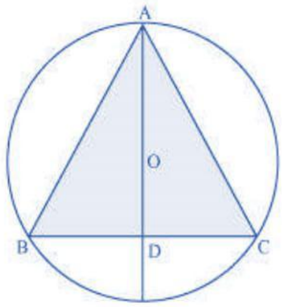
$\Rightarrow \frac{AO}{OD} = \frac{2}{1} \quad \Rightarrow \frac{32}{OD} = \frac{2}{1} \quad \Rightarrow OD = 16$

Therefore, AD = 32 + 16 = 48 cm
In ΔABD,

$AB^2 = AD^2 + BD^2 = (48)^2 + \left(\frac{AB}{2}\right)^2 \Rightarrow \frac{3}{4}AB^2 = (48)^2 \quad \Rightarrow \text{AB} = \frac{48 \times 2}{\sqrt{3}} = \frac{96\sqrt{3}}{3} = 32\sqrt{3}$

Area of equilateral triangle ABC

$= \frac{\sqrt{3}}{4}\left(32\sqrt{3}\right)^2 = 768\sqrt{3}\ \text{cm}^2$



Area of circle

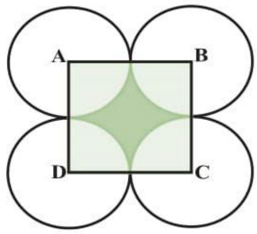
$= \pi r^2 \quad = \pi(32)^2 \quad = \frac{22}{7} \times 32 \times 32 \quad = \frac{22528}{7}\ \text{cm}^2$

Area of design = Area of circle – Area of equilateral triangle ABC

$= \left(\frac{22528}{7} - 768\sqrt{3}\right)\ \text{cm}^2$

Question 7:

In Figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region. [Use π = 22/7]



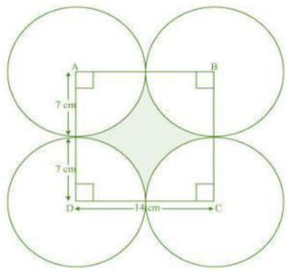
Answer 7:

The circles drawn taking A, B, C and D form quadrants of radius 7 cm in square.

Radius of each quadrant = 7 cm

Area of each quadrant

$= \frac{90°}{360°} \times \pi r^2 \quad = \frac{1}{4} \times \pi(7)^2 \quad = \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 \quad = \frac{77}{2}\ \text{cm}^2$



Area of square = $(\text{Side})^2 \quad = (14)^2 \quad = 196\ \text{cm}^2$

Area of shaded region = Area of square – Area of 4 quadrants

$= 196 - 4 \times \frac{77}{2} \ = 196 - 154 \ = 42\ \text{cm}^2$

Question 8:

Figure depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) the distance around the track along its inner edge

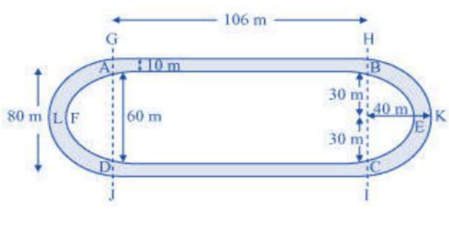
(ii) the area of the track.

[Use π = 22/7]

Answer 8:

(i) The distance around the track along its inner edge

= AB + arc BEC + CD + arc DFA



$= 106 + \frac{1}{2} \times 2\pi r + 106 + + \frac{1}{2} \times 2\pi r$

$= 106 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30 + 106 + + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30$

$= 212 + 2 \times \frac{22}{7} \times 30 = 212 + \frac{1320}{7} = \frac{2804}{7}$

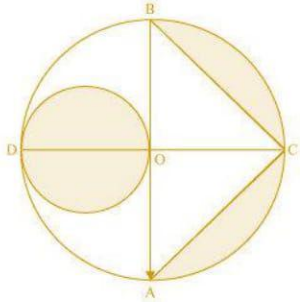
(ii) Area of Track = (Area of GHIJ – Area of ABCD) + (Area of semicircle HKI – Area of semicircle BEC) + (Area of semicirlce GLJ – Area of semicircle AFD)

$= (106 \times 80 - 106 \times 60) + \frac{1}{2} \times \frac{22}{7} \times [(40)^2 - (30)^2] + \frac{1}{2} \times \frac{22}{7} \times [(40)^2 - (30)^2]$

$= 106(80 - 60) + \frac{1}{2} \times \frac{22}{7} \times (700) + \frac{1}{2} \times \frac{22}{7} \times (700) = 2120 + \frac{22}{7} \times (700) = 2120 + +2200 = 4320$

Question 9:

In Figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region. [Use π = 22/7]



Answer 9:

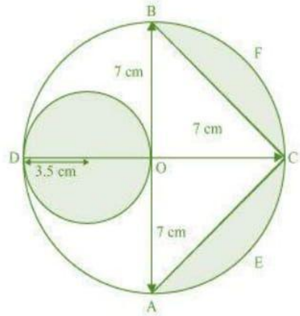
Radius of smaller circle = 7/2 cm

Area of smaller circle = $\pi r^2 = \pi\left(\frac{7}{2}\right)^2 = \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2} = \frac{77}{2}$ cm^2

Radius of larger circle = 7 cm

Area of semicircle AECFB

$= \frac{1}{2}\times\pi r^2 = \pi(7)^2 = \frac{1}{2}\times\frac{22}{7}\times 7\times 7 = 77$ cm^2



Area of triangle ACB

$= \frac{1}{2}\times AB\times OC = \frac{1}{2}\times 14\times 7 = 49$ cm^2

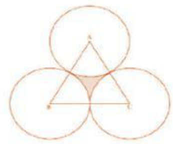
Area of shaded region

= Area of smaller circle + Area of semicircle AECFB – Area of triangle ACB

$= \left(\frac{77}{2}+77-49\right)$ cm^2 = (38.5 + 28) cm^2 = 66.5 cm^2

Question 10:

The area of an equilateral triangle ABC is 17320.5 cm2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Figure). Find the area of the shaded region. [Use π = 3.14 and $\sqrt{3} = 1.732.5$]

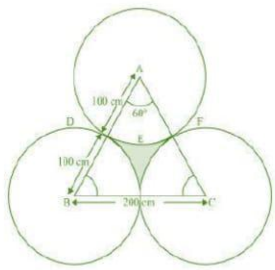


Answer 10:

Let the each side of equilateral triangle = a

Area of equilateral triangle = 17320.5 cm^2

$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 17320.5 \quad \Rightarrow \frac{1.73205}{4}a^2 = 17320.5 \quad \Rightarrow a^2 = 40000 \quad \Rightarrow a = 200$



Area of sector ADEF

$= \frac{\theta}{360°}\times\pi r^2 = \frac{60°}{360°}\times\frac{22}{7}(100)^2 = \frac{1}{6}\times 3.14\times 100\times 100 = \frac{15700}{3}$ cm^2

Area of shaded region = Area of equilateral triangle – Area of three sectors

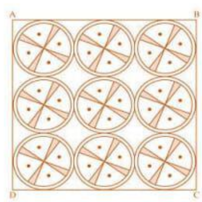
= 17320.5 cm^2 $- 3\times\frac{15700}{3}$ cm^2

= 17320.5 cm^2 $-$ 15700 cm^2

= 1620.5 cm^2

Question 11:

On a square handkerchief, nine circular designs each of radius 7 cm are made (see Figure). Find the area of the remaining portion of the handkerchief. [Use π = 22/7]

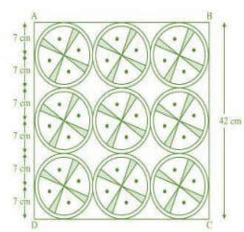


Answer 11:

Radius of circle = 7 cm

Area of one circular design

$= \pi r^2 = \pi(7)^2 = \frac{22}{7}\times 7\times 7 = 154$ cm^2



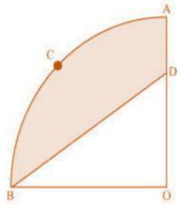
Side of square = 42 cm

Area of squre = (Side)^2 = $(42)^2$ = 1764cm^2

The area of the remaining portion = Area of square – Area of 9 circular designs

= 1764 – 9×154 = 196 – 1386 = 378 cm^2

Question 12:

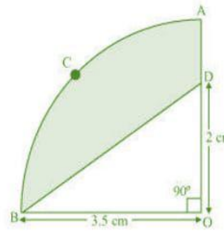


In Figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

(i) quadrant OACB,

(ii) shaded region. [Use π = 22/7]

Answer 12:



(i) Radius of quadrant = 1 cm

Area of quadrant

$= \frac{90°}{360°} \times \pi r^2 \quad = \frac{1}{4} \times \pi (3.5)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 \quad = \frac{77}{8} \text{ cm}^2$

Area of triangle OBD

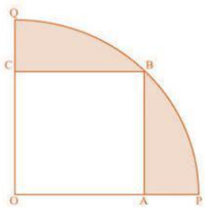
$= \frac{1}{2} \times OB \times OD \quad = \frac{1}{2} \times 3.5 \times 2 \quad = 3.5 \text{ cm}^2$

(ii) Area of shaded region = Area of quadrant – Area of triangle OBD

$= \frac{77}{8} - 3.5 \;= \frac{77}{8} - \frac{7}{2} \;= \frac{77 - 28}{8} \;= \frac{49}{8}\ cm^2$

Question 13:

In Figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14) [Use π = 22/7]



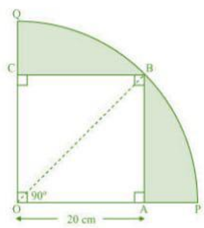
Answer 13:

In ΔOAB,

$OB^2 = OA^2 + AB^2 \;\Rightarrow OB^2 = (20)^2 + (20)^2 \;\Rightarrow OB^2 = 400 + 400$

$\Rightarrow OB^2 = 800 \qquad \Rightarrow OB = \sqrt{800} \qquad \Rightarrow OB = 20\sqrt{2}$

Radius of quadrant = $20\sqrt{2}$ cm



Area of quadrant

$= \frac{90°}{360°} \times \pi r^2 \quad = \frac{1}{4} \times \pi \left(20\sqrt{2}\right)^2$

$= \frac{1}{4} \times 3.14 \times 20\sqrt{2} \times 20\sqrt{2} \quad = 628 \text{ cm}^2$

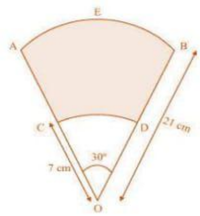
Area of square

$= (\text{Side})^2 \quad = (20)^2 \quad = 400 \text{ cm}^2$

Area of shaded region = Area of quadrant – Area of square

$= 628 - 400 \;= 228 \text{ cm}^2$

Question 14:



AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Figure). If ∠ AOB = 30°, find the area of the shaded region. [Use π = 22/7]

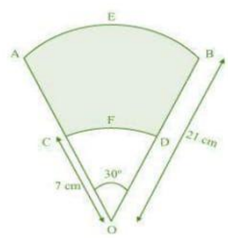
Answer 14:

Area of shaded region = Area of sector OAEB – Area of sector OCFD

$= \frac{30°}{360°} \times \pi \times (21)^2 - \frac{30°}{360°} \times \pi \times 7^2$

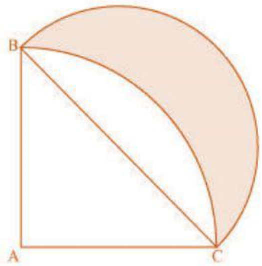
$= \frac{1}{12} \times \pi [441 - 49]$

$= \frac{1}{12} \times \frac{22}{7} \times 392 = \frac{308}{3}\ cm^2$



Question 15:
In Figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region. [Use π = 22/7]

Answer 15:



Radius of sector = 14 cm

Area of sector

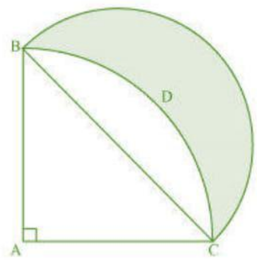
$= \frac{90°}{360°} \times \pi r^2 \quad = \frac{1}{4} \times \pi (14)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 \quad = 154 \text{ cm}^2$

In ΔABC,

$BC^2 = AC^2 + AB^2 \quad \Rightarrow BC^2 = (14)^2 + (14)^2 \quad \Rightarrow BC^2 = 196 + 196$

$\Rightarrow BC^2 = 392 \qquad \Rightarrow BC = \sqrt{392} \qquad \Rightarrow BC = 14\sqrt{2}$



Therefore, the diameter of semicircle = BC = $14\sqrt{2}$

Radius of semicircle = $7\sqrt{2}$ cm

Area of semicircle

$= \frac{1}{2} \times \pi r^2 \quad = \frac{1}{2} \times \pi \left(7\sqrt{2}\right)^2 \quad = \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2} \quad = 154 \text{ cm}^2$

Area of triangle ABC

$= \frac{1}{2} \times AC \times AB \quad = \frac{1}{2} \times 14 \times 14 \quad = 98 \text{ cm}^2$

Area of shaded region

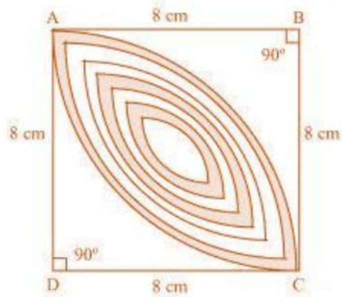
= Aera of triangle ABC + Area of semicircle – Area of quadrant

$= (98 + 154 - 154) \text{ cm}^2$

$= 98 \text{ cm}^2$

Question 16:
Calculate the area of the designed region in Figure common between the two quadrants of circles of radius 8 cm each. [Use π = 22/7]

Answer 16:



Area of sector DAFC

$= \frac{90°}{360°} \times \pi r^2 \quad = \frac{1}{4} \times \pi (8)^2$

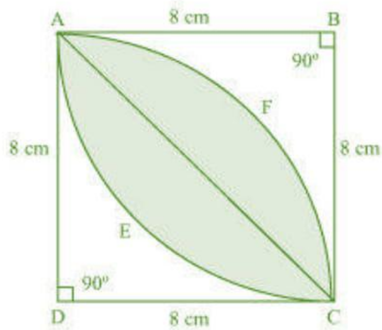
$= \frac{1}{4} \times \frac{22}{7} \times 8 \times 8 \quad = \frac{352}{7} \text{ cm}^2$

Area of triangle ADC

$= \frac{1}{2} \times DC \times AD$

$= \frac{1}{2} \times 8 \times 8$

$= 32 \text{ cm}^2$



Area of segment = Area of sector DAFC – Area of triangle ADC

$= \left(\frac{352}{7} - 32\right) \text{ cm}^2 = \left(\frac{352 - 224}{7}\right) \text{ cm}^2 = \left(\frac{128}{7}\right) \text{ cm}^2$

Area of shaded region = Area of two segments

$= 2 \times \left(\frac{128}{7}\right) \text{ cm}^2 = \frac{256}{7} \text{ cm}^2$