Mathematics

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(Chapter – 12) (Areas Related to Circles)
(Class 10)

Exercise 12.2

Question 1:

Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°. [Use π = 22/7]

Answer 1:

OACB is a minor sector.

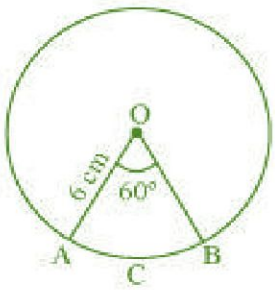
We know that, the area of a sector $= \frac{\theta}{360°} \times \pi r^2$

Therefore, area of sector OACB

$= \frac{60°}{360°} \times \frac{22}{7}(6)^2$

$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6$

$= \frac{132}{7}\ \text{cm}^2$



Question 2:

Find the area of a quadrant of a circle whose circumference is 22 cm. [Use π = 22/7]

Answer 2:

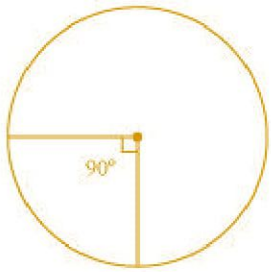
Let the radius of circle = r

Circumference = 22 cm

Therefore, $2\pi r = 22$

$\Rightarrow r = \frac{22}{2\pi} = \frac{11}{\pi}$

Quadrant of circle subtends an angle of 90° at the centre.



Therefore, the area of quadrant

$= \frac{90°}{360°} \times \pi r^2$

$= \frac{1}{4} \times \pi \left(\frac{11}{\pi}\right)^2 = \frac{1}{4} \times \pi \times \frac{11}{\pi} \times \frac{11}{\pi} = \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22} = \frac{77}{8}\ \text{cm}^2$

Question 3:

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes. [Use π = 22/7]

Answer 3:

We know that, the minute hand makes an angle of 360° in one hour.

Hence, angle formed in 5 minutes

$= \frac{360°}{60} \times 5 = 30°$

Therefore, area swept be the minute hand in 5 minutes

= Area of sector of angle 30°

$= \frac{30°}{360°} \times \pi r^2$

$= \frac{1}{12} \times \pi (14)^2$

$= \frac{1}{12} \times \frac{22}{7} \times 14 \times 14$

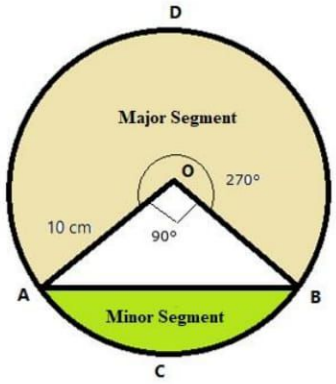
$= \frac{154}{3}\ \text{cm}^2$

Question 4:

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) minor segment **(ii)** major sector. (Use π = 3.14)

Answer 4:



(i) Let the chord AB subtends right angle at the centre O.

Area of segment with an angle 90°

$$= \frac{90°}{360°} \times \pi r^2 = \frac{1}{4} \times \pi(10)^2 = \frac{1}{4} \times 3.14 \times 10 \times 10 = 78.5 \text{ cm}^2$$

Area of triangle OAB

$$= \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$$

Area of minor segment = Area of minor sector – Area of triangle OAB

$$= 78.5 \text{ cm}^2 - 50 \text{ cm}^2 = 28.5 \text{ cm}^2$$

(ii) Area of major segment = Area of cirlce – Area of minor segment

$$= \pi r^2 - \frac{1}{4}\pi r^2 \quad = \left(1 - \frac{1}{4}\right)\pi r^2 \quad = \frac{3}{4}\pi r^2$$

$$= \frac{3}{4} \times \pi(10)^2 \quad = \frac{3}{4} \times 3.14 \times 10 \times 10 \quad = 235.5 \text{ cm}^2$$

Question 5:

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

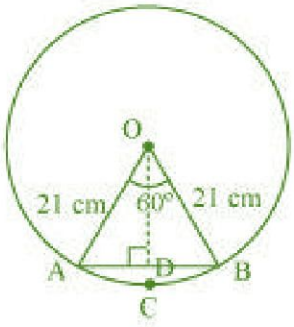
(i) the length of the arc

(ii) area of the sector formed by the arc

(iii) area of the segment formed by the corresponding chord. [Use π = 22/7]

Answer 5:

Radius of circle = 21 cm



(i) The length of arc

$$= \frac{\theta}{360°} \times 2\pi r \quad = \frac{60°}{360°} \times 2\pi r$$

$$= \frac{1}{6} \times 2 \times \pi \times 21 \quad = \frac{1}{6} \times 2 \times \frac{22}{7} \times 21 \quad = 22 \text{ cm}$$

(ii) Area of the sector formed by the arc

$$= \frac{60°}{360°} \times \pi r^2$$

$$= \frac{1}{6} \times \pi(21)^2 \quad = \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 \quad = 231 \text{ cm}^2$$

(iii) Area of the segment formed by the corresponding chord

= Area of sector – Area of ΔOAB

$= 231 \text{ cm}^2 - \frac{\sqrt{3}}{4}(21)^2 \text{ cm}^2$ [As triangle OAB is an equilateral triangle]

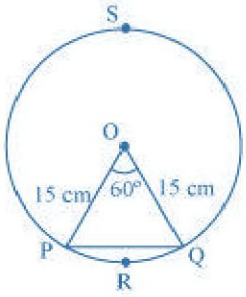
$= 231 \text{ cm}^2 - 441\frac{\sqrt{3}}{4} \text{ cm}^2$

$= \left[231 - \frac{441\sqrt{3}}{4}\right] \text{ cm}^2$

Question 6:

A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and $\sqrt{3} = 1.73$)

Answer 6:



Radius of circle = 15 cm

Area of sector OPRQ

$= \frac{60°}{360°} \times \pi r^2 \quad = \frac{1}{6} \times \pi(15)^2 \quad = \frac{1}{6} \times 3.14 \times 15 \times 15 \quad = 117.75 \text{ cm}^2$

Area of minor segment

= Area of minor sector OPRQ - Area of ΔOPQ

$= 117.75 \text{ cm}^2 - \frac{\sqrt{3}}{4}(15)^2 \text{ cm}^2$ [As triangle OPQ is an equilateral triangle]

$= 117.75 \text{ cm}^2 - 225\frac{\sqrt{3}}{4} \text{ cm}^2$

$= 117.75 \text{ cm}^2 - 56.25 \times 1.73 \text{ cm}^2$

$= 231 \text{ cm}^2 - 97.3125 \text{ cm}^2$

$= 20.4375 \text{ cm}^2$

Area of major segment = Area of circle – Area of minor segment

$= \pi r^2 - 20.4375 \text{ cm}^2$

$= [\pi(15)^2 - 20.4375] \text{ cm}^2$

$= [3.14 \times 15 \times 15 - 20.4375] \text{ cm}^2$

$= [706.5 - 20.4375] \text{ cm}^2$

$= 686.0625 \text{ cm}^2$

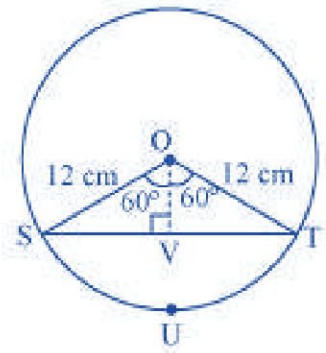
Question 7:

A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and $\sqrt{3} = 1.73$)

Answer 7:

The perpendicular OV drown form O to chord ST bisects ST.

Therefore, SV = VT



In ΔOVS,

$\frac{OV}{OS} = \cos 60° \quad \Rightarrow \frac{OV}{12} = \frac{1}{2} \quad \Rightarrow OV = 6 \text{ cm}$

$\frac{SV}{OS} = \sin 60° \quad \Rightarrow \frac{SV}{12} = \frac{\sqrt{3}}{2} \quad \Rightarrow OV = 6\sqrt{3} \text{ cm}$

$ST = 2 \times SV = 2 \times 6\sqrt{3} = 12\sqrt{3} \text{ cm}$

Area of ΔOST

$= \frac{1}{2} \times ST \times OV \quad = \frac{1}{2} \times 12\sqrt{3} \times 6 \quad = 36\sqrt{3} \quad = 36 \times 1.73 \quad = 62.28 \text{ cm}^2$

Area of sector OSUT

$= \frac{120°}{360°} \times \pi r^2 \quad = \frac{1}{3} \times \pi(12)^2$

$= \frac{1}{3} \times 3.14 \times 12 \times 12 \quad = 150.72 \text{ cm}^2$

Area of minor segment

= Area of sector OSUT – Area of ΔOST

$= (150.72 - 62.28) \text{ cm}^2 = 88.44 \text{ cm}^2$

Question 8:

A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Figure). Find

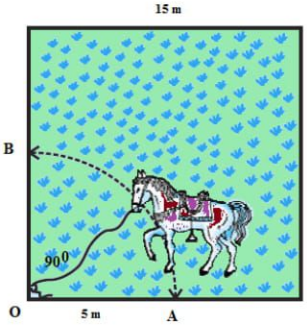
(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

Answer 8:

The shape of grass field, where the horse can graze is a sector with central angle 90°.

(i) The area of the field, where the horse can graze



= Area of sector OABO with radius 5

$$= \frac{90°}{360°} \times \pi r^2 \quad = \frac{1}{4} \times \pi (5)^2$$

$$= \frac{1}{4} \times 3.14 \times 25 \quad = 19.625 \text{ m}^2$$

(ii) If the rope were 10 m long instead of 5 m, the area of field where the horse can graze = Area of sector OABO with radius 10

$$= \frac{90°}{360°} \times \pi r^2 \quad = \frac{1}{4} \times \pi (10)^2$$

$$= \frac{1}{4} \times 3.14 \times 100 \quad = 78.50 \text{ m}^2$$

The increase in grazing area = $(78.50 - 19.625)\ \text{m}^2 = 58.875\ \text{m}^2$

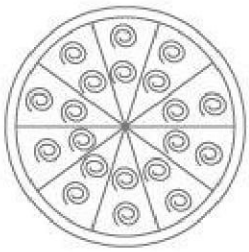
Question 9:

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Figure. Find:

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch. [Use π = 22/7]

Answer 9:



Diameter = 35 mm

Therefore, radius = 35/2 mm

(i) Total length of wire

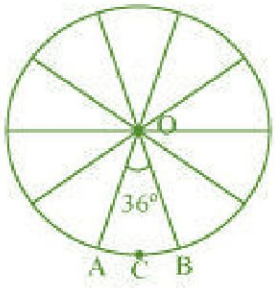
= 5 × Diameter + Circumference = $5 \times 35 + 2\pi r$

$$= 175 + 2 \times \frac{22}{7} \times \frac{35}{2}$$

= 175 + 110 = 285 mm

(ii) There are total 10 sectors.

Therefore, the angle of each sector = $\frac{360°}{10} = 36°$



Area of each sector

$$= \frac{36°}{360°} \times \pi r^2$$

$$= \frac{1}{10} \times \pi \left(\frac{35}{2}\right)^2$$

$$= \frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$$

$$= \frac{385}{4}\ \text{mm}^2$$

Question 10:

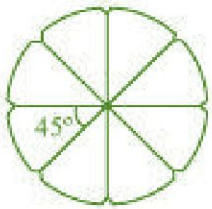
An umbrella has 8 ribs which are equally spaced (see Figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella. [Use π = 22/7]

Answer 10:

Radius = 45 cm

There are total 8 sectors.

Therefore, the angle of each sector = $\frac{360^\circ}{8} = 45^\circ$



the area between the two consecutive ribs = Area of sector

$= \frac{45^\circ}{360^\circ} \times \pi r^2 \quad = \frac{1}{8} \times \pi(45)^2$

$= \frac{1}{8} \times \frac{22}{7} \times 45 \times 45 \quad = \frac{22275}{28}\ \text{cm}^2$

Question 11:

A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades. [Use π = 22/7]

Answer 11:

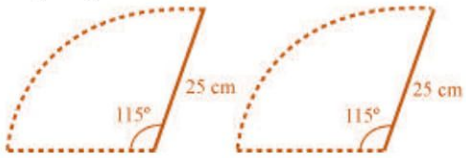
Angle of each wiper = 115°

Radius = 25 cm

Area cleaned at each sweep of the blade = Area of sector formed by wiper

$= \frac{115^\circ}{360^\circ} \times \pi r^2 \quad = \frac{23}{72} \times \pi(25)^2$

$= \frac{23}{72} \times \frac{22}{7} \times 25 \times 25 \quad = \frac{158125}{252}\ \text{cm}^2$



Area cleaned by two wiper's blades = $2 \times \frac{158125}{252} = \frac{158125}{126}\ \text{cm}^2$

Question 12:

To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)

Answer 12:

The light house spreads red light in the form of sector of anlge 80°, whose radius is 16.5 km.

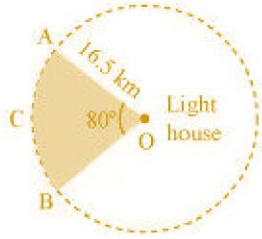
Therefore, the area of sector

$= \frac{80^\circ}{360^\circ} \times \pi r^2$

$= \frac{2}{9} \times \pi(16.5)^2$

$= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5$

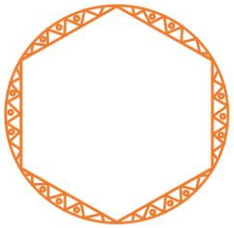
$= 189.97\ \text{km}^2$



Question 13:

A round table cover has six equal designs as shown in Figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs 0.35 per cm².

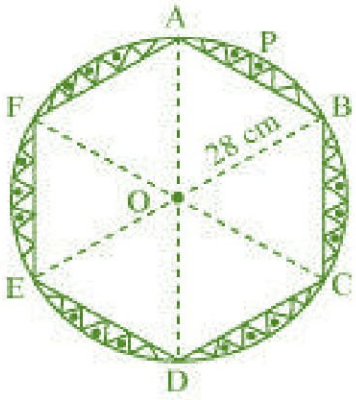
(Use π = 22/7 and $\sqrt{3} = 1.7$)



Answer 13:

6 design of table cover = 6 sectors of circle

Therefore, angle of each sector = $\frac{360°}{6} = 60°$



In ΔOAB,

∠OAB = ∠OBA [Because OA = OB]

∠AOB = 60°

∠OAB + ∠OBA + ∠AOB = 180°

2∠OAB = 180° − 60° = 120°

∠OAB = 60°

Similarly, ΔOAB is an equilateral triangle.

Area of sector OAPB

$= \frac{60°}{360°} \times \pi r^2$

$= \frac{1}{6} \times \pi(28)^2 \quad = \frac{1}{6} \times \frac{22}{7} \times 28 \times 28$

$= \frac{1232}{3}\ \text{cm}^2$

Area of equilateral triangle OAB

$= \frac{\sqrt{3}}{4}(28)^2 \quad = 196\sqrt{3}$

$= 196 \times 1.7 \quad = 333.2\ \text{cm}^2$

Area of segment

= Area of sector OAPB – area of equilateral triangle OAB

$= \left(\frac{1232}{3} - 333.2\right)\ \text{cm}^2$

Area of 6 designs of table cover

$= 6 \times \left(\frac{1232}{3} - 333.2\right)\ \text{cm}^2$

$= (2464 - 1999.2)\ \text{cm}^2 \quad = 464.8\ \text{cm}^2$

Cost of making 1 cm^2 design = ₹ 0.35

Therefore, the cost of making 464.8 cm^2 design = ₹0.35 × 464.8 = ₹ 162.68

Question 14:

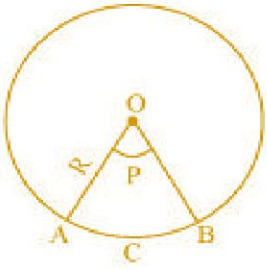
Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\frac{P}{180} \times 2\pi R$ (B) $\frac{P}{180} \times \pi R^2$ (C) $\frac{P}{360} \times 2\pi R$ (D) $\frac{P}{720} \times 2\pi R^2$

Answer 14:

We know that the area of sector with central angle $P°$ is given by



$= \frac{P}{360} \times \pi R^2$

$= \frac{P}{360} \times \pi R^2 \times \frac{2}{2}$

$= \frac{P}{720} \times 2\pi R^2$

Therefore, the option (D) is correct.