Exercise 11.2

Question 1:

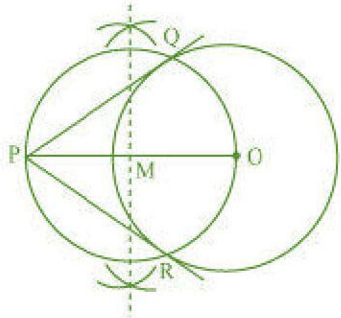
Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. Give the justification of the construction.

Answer 1:

A pair of tangents to the given circle can be constructed as follows.

Step 1:

Taking any point O of the given plane as centre, draw a circle of 6 cm radius. Locate a point P, 10 cm away from O. Join OP.



Step 2

Bisect OP. Let M be the mid-point of PO.

Step 3

Taking M as centre and MO as radius, draw a circle.

Step 4

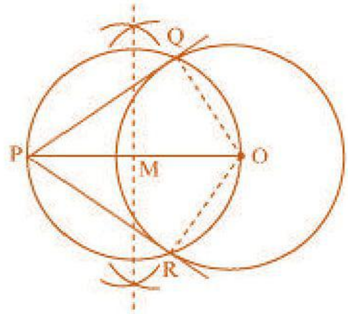
Let this circle intersect the previous circle at point Q and R.

Step 5

Join PQ and PR. PQ and PR are the required tangents.

The lengths of tangents PQ and PR are 8 cm each.

Justification



The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 6 cm). For this, join OQ and OR.

$\angle PQO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$\therefore \angle PQO = 90° \Rightarrow OQ \perp PQ$

Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle

Question 2:

Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation. Give the justification of the construction.

Answer 2:

Tangents on the given circle can be drawn as follows.

Step 1

Draw a circle of 4 cm radius with centre as O on the given plane.

Step 2

Draw a circle of 6 cm radius taking O as its centre. Locate a point P on this circle and join OP.

Step 3

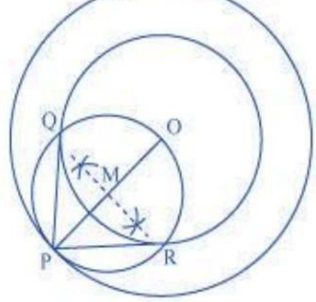
Bisect OP. Let M be the mid-point of PO.

Step 4

Taking M as its centre and MO as its radius, draw a circle. Let it intersect the given circle at the points Q and R.

Step 5

Join PQ and PR. PQ and PR are the required tangents.



It can be observed that PQ and PR are of length 4.47 cm each.

In ΔPQO,

Since PQ is a tangent,

$\angle PQO = 90°$

$PO = 6$ cm

$QO = 4$ cm

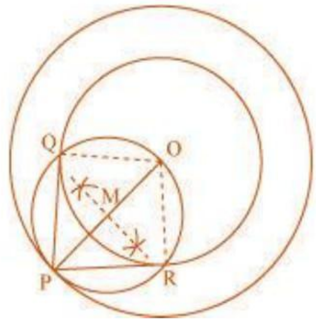
Applying Pythagoras theorem in ΔPQO, we obtain

$PQ^2 + QO^2 = PQ^2 \quad \Rightarrow PQ^2 + (4)^2 = (6)^2 \Rightarrow PQ^2 + 16 = 36$

$PQ^2 = 36 - 16 \quad \Rightarrow PQ^2 = 20 \quad \Rightarrow PQ = 2\sqrt{5}$

$PQ = 4.47$ cm

Justification



The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 4 cm). For this, let us join OQ and OR.

$\angle PQO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

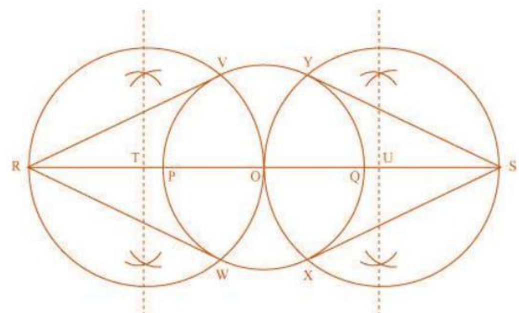
$\therefore \angle PQO = 90° \Rightarrow OQ \perp PQ$

Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle

Question 3:

Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q. Give the justification of the construction.

Answer 3:



The tangent can be constructed on the given circle as follows.

Step 1

Taking any point O on the given plane as centre, draw a circle of 3 cm radius.

Step 2

Take one of its diameters, PQ, and extend it on both sides. Locate two points on this diameter such that OR = OS = 7 cm

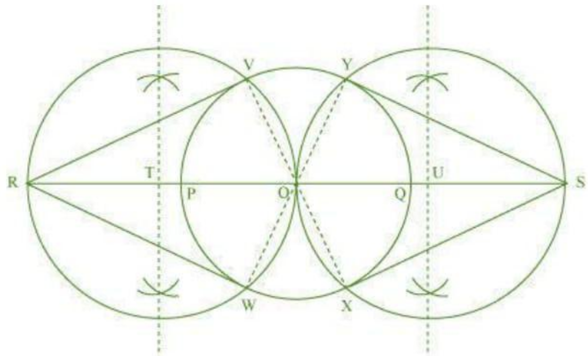
Step 3

Bisect OR and OS. Let T and U be the mid-points of OR and OS respectively.

Step 4

Taking T and U as its centre and with TO and UO as radius, draw two circles. These two circles will intersect the circle at point V, W, X, Y respectively. Join RV, RW, SX, and SY. These are the required tangents.

Justification



The construction can be justified by proving that RV, RW, SY, and SX are the tangents to the circle (whose centre is O and radius is 3 cm). For this, join OV, OW, OX, and OY.

∠RVO is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$\therefore \angle RVO = 90° \Rightarrow OV \perp RV$

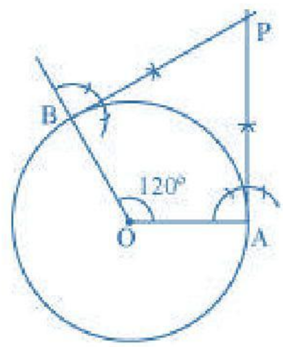
Since OV is the radius of the circle, RV has to be a tangent of the circle. Similarly, OW, OX, and OY are the tangents of the circle

Question 4:

Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°. Give the justification of the construction.

Answer 4:

The tangents can be constructed in the following manner:



Step 1

Draw a circle of radius 5 cm and with centre as O.

Step 2

Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.

Step 3

Draw a radius OB, making an angle of 120° (180° − 60°) with OA.

Step 4

Draw a perpendicular to OB at point B. Let both the perpendiculars intersect at point P. PA and PB are the required tangents at an angle of 60°.

Justification

The construction can be justified by proving that ∠APB = 60°

By our construction

$\angle OAP = 90°$ $\angle OBP = 90°$ and $\angle AOB = 120°$

We know that the sum of all interior angles of a quadrilateral = 360°

$\angle OAP + \angle AOB + \angle OBP + \angle APB = 360° \Rightarrow 90° + 120° + 90° + \angle APB = 360°$

$\angle APB = 60°$

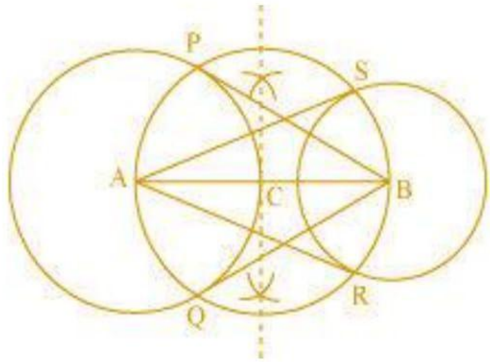
This justifies the construction.

Question 5:

Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle. Give the justification of the construction.

Answer 5:

The tangents can be constructed on the given circles as follows.



Step 1

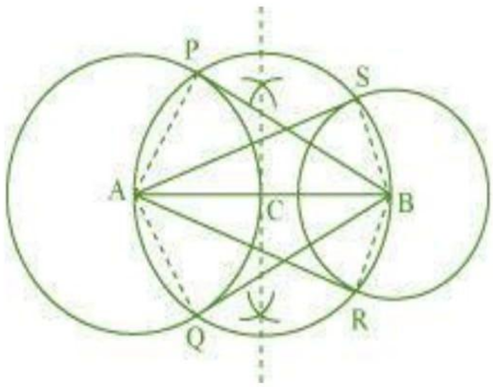
Draw a line segment AB of 8 cm. Taking A and B as centre, draw two circles of 4 cm and 3 cm radius.

Step 2

Bisect the line AB. Let the mid-point of AB be C. Taking C as centre, draw a circle of AC radius which will intersect the circles at points P, Q, R, and S. Join BP, BQ, AS, and AR. These are the required tangents.

Justification

The construction can be justified by proving that AS and AR are the tangents of the circle (whose centre is B and radius is 3 cm) and BP and BQ are the tangents of the circle (whose centre is A and radius is 4 cm). For this, join AP, AQ, BS, and BR.



∠ASB is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

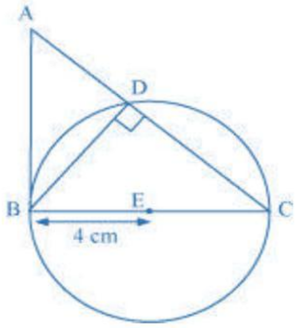
$\therefore \angle ASB = 90° \Rightarrow BS \perp AS$

Since BS is the radius of the circle, AS has to be a tangent of the circle. Similarly, AR, BP, and BQ are the tangents.

Question 6:

Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and ∠B = 90°. BD is the perpendicular from B on AC. The circle through B, C, and D is drawn. Construct the tangents from A to this circle. Give the justification of the construction.

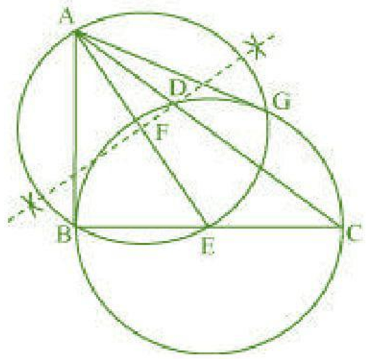
Answer 6:



Consider the following situation. If a circle is drawn through B, D, and C, BC will be its diameter as ∠BDC is of measure 90°. The centre E of this circle will be the mid-point of BC.

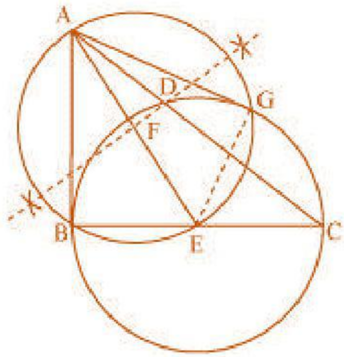
The required tangents can be constructed on the given circle as follows.

Step 1
Join AE and bisect it. Let F be the mid-point of AE.



Step 2
Taking F as centre and FE as its radius, draw a circle which will intersect the circle at point B and G. Join AG.
AB and AG are the required tangents.

Justification
The construction can be justified by proving that AG and AB are the tangents to the circle. For this, join EG.
∠AGE is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

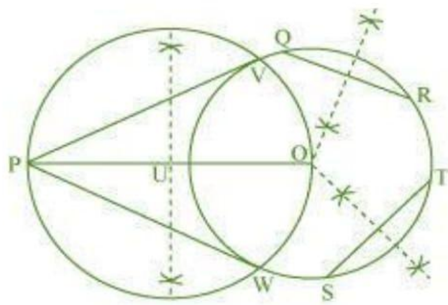


$\therefore \angle AGE = 90°$
$\Rightarrow EG \perp AG$
Since EG is the radius of the circle, AG has to be a tangent of the circle.
Already, $\angle B = 90°$
$\Rightarrow AB \perp BE$
Since BE is the radius of the circle, AB has to be a tangent of the circle.

Question 7:
Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circles. Give the justification of the construction.

Answer 7:
The required tangents can be constructed on the given circle as follows.

Step 1
Draw a circle with the help of a bangle.



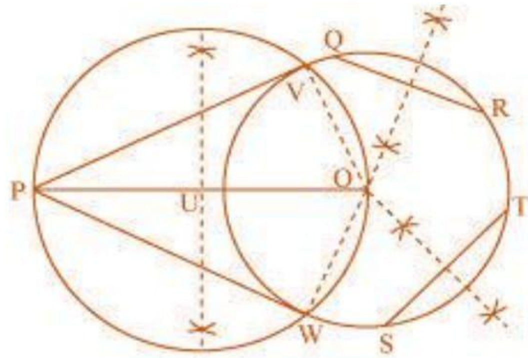
Step 2
Take a point P outside this circle and take two chords QR and ST.

Step 3
Draw perpendicular bisectors of these chords. Let them intersect each other at point O.

Step 4
Join PO and bisect it. Let U be the mid-point of PO. Taking U as centre, draw a circle of radius OU, which will intersect the circle at V and W. Join PV and PW.
PV and PW are the required tangents.

Justification

The construction can be justified by proving that PV and PW are the tangents to the circle. For this, first of all, it has to be proved that O is the centre of the circle. Let us join OV and OW.



We know that perpendicular bisector of a chord passes through the centre. Therefore, the perpendicular bisector of chords QR and ST pass through the centre. It is clear that the intersection point of these perpendicular bisectors is the centre of the circle. ∠PVO is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

$\therefore \angle PVO = 90^\circ$

$\Rightarrow OV \perp PV$

Since OV is the radius of the circle, PV has to be a tangent of the circle. Similarly, PW is a tangent of the circle.