Exercise 11.1

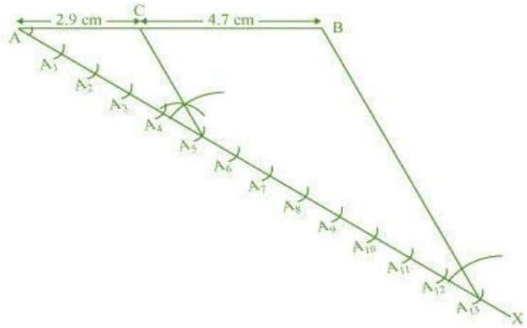
Question 1:

Draw a line segment of length 7.6 cm and divide it in the ratio 5:8. Measure the two parts. Give the justification of the construction.

Answer 1:

A line segment of length 7.6 cm can be divided in the ratio of 5:8 as follows.

Step 1 Draw line segment AB of 7.6 cm and draw a ray AX making an acute angle with line segment AB.



Step 2 Locate 13 (= 5 + 8) points, A_1, A_2, A_3, A_4 …….. A_{13}, on AX such that $AA_1 = A_1A_2 = A_2A_3$ and so on.

Step 3 Join BA_{13}.

Step 4 Through the point A_5, draw a line parallel to BA_{13} (by making an angle equal to $\angle AA_{13}B$) at A_5 intersecting AB at point C.

C is the point dividing line segment AB of 7.6 cm in the required ratio of 5:8.

The lengths of AC and CB can be measured. It comes out to 2.9 cm and 4.7 cm respectively.

Justification

The construction can be justified by proving that $\frac{AC}{CB} = \frac{5}{8}$

By construction, we have $A_5C \parallel A_{13}B$. By applying Basic proportionality theorem for the triangle $AA_{13}B$, we obtain

$\frac{AC}{CB} = \frac{AA_5}{A_5A_{13}}$ … (1)

From the figure, it can be observed that AA_5 and A_5A_{13} contain 5 and 8 equal divisions of line segments respectively.

$\frac{AA_5}{A_5A_{13}} = \frac{5}{8}$ … (2)

On comparing equations (1) and (2), we obtain $\frac{AC}{CB} = \frac{5}{8}$

This justifies the construction.

Question 2:

Construct a triangle of sides 4 cm, 5cm and 6cm and then a triangle similar to it whose sides are 2/3of the corresponding sides of the first triangle.

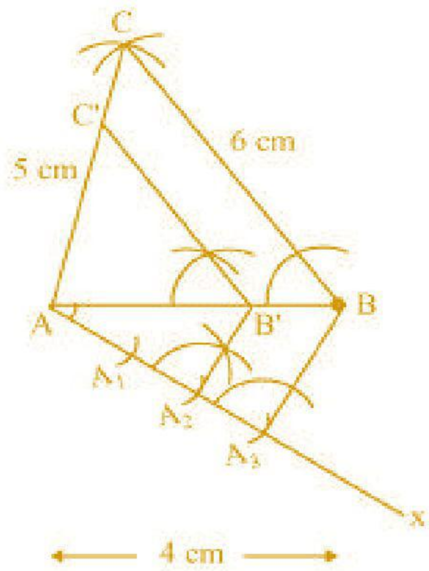
Give the justification of the construction.

Answer 2:

Step 1

Draw a line segment AB = 4 cm. Taking point A as centre, draw an arc of 5 cm radius. Similarly, taking point B as its centre, draw an arc of 6 cm radius. These arcs will intersect each other at point C. Now, AC = 5 cm and BC = 6 cm and ΔABC is the required triangle.

Step 2
Draw a ray AX making an acute angle with line AB on the opposite side of vertex C.
Step 3
Locate 3 points A_1, A_2, A_3 (as 3 is greater between 2 and 3) on line AX such that $AA_1 = A_1A_2 = A_2A_3$.
Step 4
Join BA_3 and draw a line through A_2 parallel to BA_3 to intersect AB at point B'.
Step 5
Draw a line through B' parallel to the line BC to intersect AC at C'.
ΔAB'C' is the required triangle.



Justification
The construction can be justified by proving that

$$AB' = \frac{2}{3}AB, B'C' = \frac{2}{3}BC, AC' = \frac{2}{3}AC$$

By construction, we have B'C' || BC
$\therefore \angle AB'C' = \angle ABC$ (Corresponding angles)
In ΔAB'C' and ΔABC,
$\angle AB'C' = \angle ABC$ (Proved above)
$\angle B'AC' = \angle BAC$ (Common)
$\therefore \Delta AB'C' \sim \Delta ABC$ (AA similarity criterion)
$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC}$ ... (1)
In $\Delta AA_2B'$ and ΔAA_3B,
$\angle A_2AB' = \angle A_3AB$ (Common)
$\angle AA_2B' = \angle AA_3B$ (Corresponding angles)
$\therefore \Delta AA_2B' \sim \Delta AA_3B$ (AA similarity criterion)

$$\Rightarrow \frac{AB'}{AB} = \frac{AA_2}{AA_3}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{2}{3} \quad \text{... (2)}$$

From equations (1) and (2), we obtain

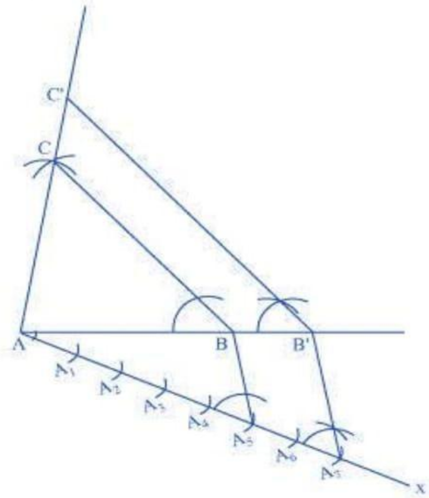
$$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{2}{3}$$

$$\Rightarrow AB' = \frac{2}{3}AB, B'C' = \frac{2}{3}BC, AC' = \frac{2}{3}AC$$

This justifies the construction.

Question 3:
Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are 7/5 of the corresponding sides of the first triangle.
Give the justification of the construction.

Answer 3:



Step 1
Draw a line segment AB of 5 cm. Taking A and B as centre, draw arcs of 6 cm and 5 cm radius respectively. Let these arcs intersect each other at point C. ΔABC is the required triangle having length of sides as 5 cm, 6 cm, and 7 cm respectively.

Step 2
Draw a ray AX making acute angle with line AB on the opposite side of vertex C.

Step 3
Locate 7 points, A_1, A_2, A_3, A_4 A_5, A_6, A_7 (as 7 is greater between 5and 7), on line AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

Step 4
Join BA_5 and draw a line through A_7 parallel to BA_5 to intersect extended line segment AB at point B'.

Step 5
Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. ΔAB'C' is the required triangle.

Justification
The construction can be justified by proving that

$$AB' = \frac{7}{5}AB, B'C' = \frac{7}{5}BC, AC' = \frac{7}{5}AC$$

In ΔABC and ΔAB'C',
∠ABC = ∠AB'C' (Corresponding angles)
∠BAC = ∠B'AC' (Common)
∴ ΔABC ~ ΔAB'C' (AA similarity criterion)

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} \quad \ldots (1)$$

In ΔAA_5B and $\Delta AA_7B'$,
$\angle A_5AB = \angle A_7AB'$ (Common)
$\angle AA_5B = \angle AA_7B'$ (Corresponding angles)
∴ $\Delta AA_5B \sim \Delta AA_7B'$ (AA similarity criterion)

$$\Rightarrow \frac{AB'}{AB} = \frac{AA_5}{AA_7} \Rightarrow \frac{AB'}{AB} = \frac{5}{7} \quad \ldots (2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{7}{5}$$

$\Rightarrow AB' = \frac{7}{5}AB, B'C' = \frac{7}{5}BC, AC' = \frac{7}{5}AC$

This justifies the construction.

Question 4:

Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose side are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Give the justification of the construction.

Answer 4:

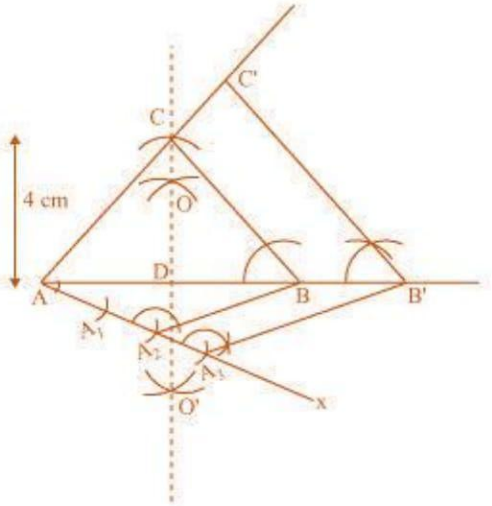
Let us assume that ΔABC is an isosceles triangle having CA and CB of equal lengths, base AB of 8 cm, and AD is the altitude of 4 cm.

A ΔAB'C' whose sides are 3/2 times of ΔABC can be drawn as follows.

Step 1

Draw a line segment AB of 8 cm. Draw arcs of same radius on both sides of the line segment while taking point A and B as its centre.

Let these arcs intersect each other at O and O'. Join OO'. Let OO' intersect AB at D.



Step 2

Taking D as centre, draw an arc of 4 cm radius which cuts the extended line segment OO' at point C. An isosceles ΔABC is formed, having CD (altitude) as 4 cm and AB (base) as 8 cm.

Step 3

Draw a ray AX making an acute angle with line segment AB on the opposite side of vertex C.

Step 4

Locate 3 points (as 3 is greater between 3 and 2) A_1, A_2, and A_3 on AX such that $AA_1 = A_1A_2 = A_2A_3$.

Step 5

Join BA_2 and draw a line through A_3 parallel to BA_2 to intersect extended line segment AB at point B'.

Step 6

Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. ΔAB'C' is the required triangle.

Justification

The construction can be justified by proving that

$$AB' = \frac{3}{2}AB, B'C' = \frac{3}{2}BC, AC' = \frac{3}{2}AC$$

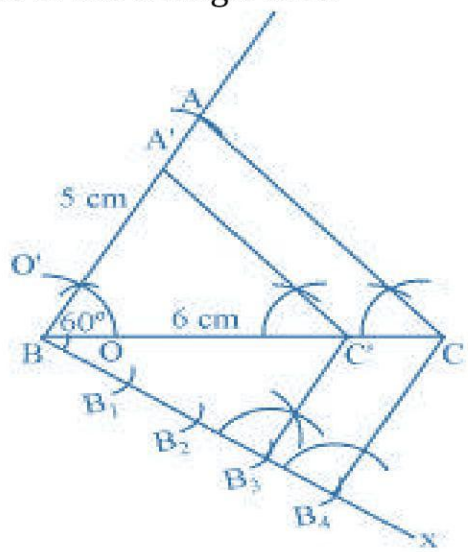
In ΔABC and ΔAB'C',
∠ABC = ∠AB'C' (Corresponding angles)
∠BAC = ∠B'AC' (Common)
∴ ΔAB'C' ~ ΔABC (AA similarity criterion)
$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC}$ ... (1)
In ΔAA_2B and $\Delta AA_3B'$,
$\angle A_2AB = \angle A_3AB'$ (Common)
$\angle AA_2B = \angle AA_3B'$ (Corresponding angles)
∴ $\Delta AA_2B \sim \Delta AA_3B'$ (AA similarity criterion)
$\Rightarrow \frac{AB}{AB'} = \frac{AA_2}{AA_3} \Rightarrow \frac{AB}{AB'} = \frac{2}{3}$ ... (2)
On comparing equations (1) and (2), we obtain

$$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{3}{2}$$
$$\Rightarrow AB' = \frac{3}{2}AB, B'C' = \frac{3}{2}BC, AC' = \frac{3}{2}AC$$

This justifies the construction.

Question 5:
Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and ∠ABC = 60°. Then construct a triangle whose sides are ¾ of the corresponding sides of the triangle ABC. Give the justification of the construction.

Answer 5:
A ΔA'BC' whose sides are ¾ of the corresponding sides of ΔABC can be drawn as follows.



Step 1
Draw a ΔABC with side BC = 6 cm, AB = 5 cm and ∠ABC = 60°.
Step 2
Draw a ray BX making an acute angle with BC on the opposite side of vertex A.
Step 3
Locate 4 points (as 4 is greater in 3 and 4), B_1, B_2, B_3, B_4, on line segment BX.
Step 4
Join B_4C and draw a line through B_3, parallel to B_4C intersecting BC at C'.

Step 5

Draw a line through C' parallel to AC intersecting AB at A'. ΔA'BC' is the required triangle.

Justification

The construction can be justified by proving $A'B = \frac{3}{4}AB, BC' = \frac{3}{4}BC, A'C' = \frac{3}{4}AC$

In ΔA'BC' and ΔABC,

∠A'C'B = ∠ACB (Corresponding angles)

∠A'BC' = ∠ABC (Common)

∴ ΔA'BC' ~ ΔABC (AA similarity criterion)

$\Rightarrow \frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC}$ ... (1)

In $\Delta BB_3C'$ and ΔBB_4C,

$\angle B_3BC' = \angle B_4BC$ (Common)

$\angle BB_3C' = \angle BB_4C$ (Corresponding angles)

∴ $\Delta BB_3C' \sim \Delta BB_4C$ (AA similarity criterion)

$\Rightarrow \frac{BC'}{BC} = \frac{BB_3}{BB_4} \Rightarrow \frac{BC'}{BC} = \frac{3}{4}$ ... (2)

From equations (1) and (2), we obtain

$$\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{3}{4}$$

$$\Rightarrow A'B = \frac{3}{4}AB, BC' = \frac{3}{4}BC, A'C' = \frac{3}{4}AC$$

This justifies the construction.

Question 6:

Draw a triangle ABC with side BC = 7 cm, ∠B = 45°, ∠A = 105°. Then, construct a triangle whose sides are 4/3 times the corresponding side of ΔABC. Give the justification of the construction.

Answer 6:

∠B = 45°, ∠A = 105°

Sum of all interior angles in a triangle is 180°.

∠A + ∠B + ∠C = 180° ⇒ 105° + 45° + ∠C = 180° ⇒ ∠C = 180° – 150° ⇒ ∠C = 30°

The required triangle can be drawn as follows.

Step 1

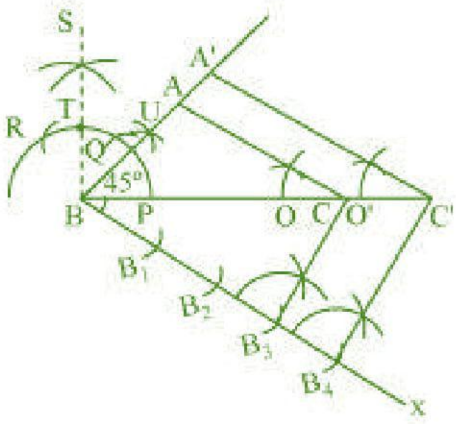
Draw a ΔABC with side BC = 7 cm, ∠B = 45°, ∠C = 30°.

Step 2

Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

Step 3

Locate 4 points (as 4 is greater in 4 and 3), B_1, B_2, B_3, B_4, on BX.



Step 4

Join B_3C. Draw a line through B_4 parallel to B_3C intersecting extended BC at C'.

Step 5

Through C', draw a line parallel to AC intersecting extended line segment at C'.
ΔA'BC' is the required triangle.

Justification

The construction can be justified by proving that
$A'B = \frac{4}{3}AB, BC' = \frac{4}{3}BC, AC' = \frac{4}{3}AC$
In ΔABC and ΔA'BC',
∠ABC = ∠A'BC' (Common)
∠ACB = ∠A'C'B (Corresponding angles)
∴ ΔABC ~ ΔA'BC' (AA similarity criterion)
$\Rightarrow \frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'}$ … (1)
In ΔBB_3C and $\Delta BB_4C'$,
$\angle B_3BC = \angle B_4BC'$ (Common)
$\angle BB_3C = \angle BB_4C'$ (Corresponding angles)
∴ $\Delta BB_3C \sim \Delta BB_4C'$ (AA similarity criterion)
$\Rightarrow \frac{BC}{BC'} = \frac{BB_3}{BB_4} \Rightarrow \frac{BC}{BC'} = \frac{3}{4}$ … (2)
On comparing equations (1) and (2), we obtain

$$\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{AC'}{AC} = \frac{4}{3}$$

$$\Rightarrow A'B = \frac{4}{3}AB, BC' = \frac{4}{3}BC, AC' = \frac{4}{3}AC$$

This justifies the construction.

Question 7:

Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. the construct another triangle whose sides are 5/3 times the corresponding sides of the given triangle. Give the justification of the construction.

Answer 7:

It is given that sides other than hypotenuse are of lengths 4 cm and 3 cm. Clearly, these will be perpendicular to each other.
The required triangle can be drawn as follows.

Step 1

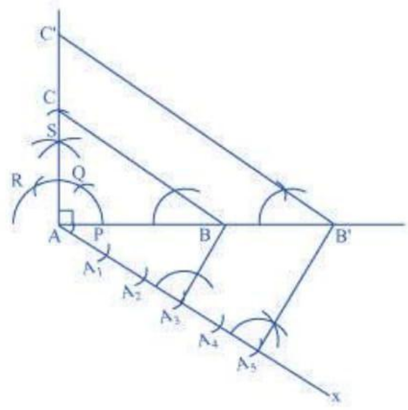
Draw a line segment AB = 4 cm. Draw a ray SA making 90° with it.

Step 2

Draw an arc of 3 cm radius while taking A as its centre to intersect SA at C. Join BC. ΔABC is the required triangle.

Step 3

Draw a ray AX making an acute angle with AB, opposite to vertex C.



Step 4

Locate 5 points (as 5 is greater in 5 and 3), A_1, A_2, A_3, A_4, A_5, on line segment AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

Step 5

Join A_3B. Draw a line through A_5 parallel to A_3B intersecting extended line segment AB at B'.

Step 6

Through B', draw a line parallel to BC intersecting extended line segment AC at C'. ΔAB'C' is the required triangle.

Justification

The construction can be justified by proving that

$$AB' = \frac{5}{3}AB, B'C' = \frac{5}{3}BC, AC' = \frac{5}{3}AC$$

In ΔABC and ΔAB'C',

∠ABC = ∠AB'C' (Corresponding angles)

∠BAC = ∠B'AC' (Common)

∴ ΔABC ~ ΔAB'C' (AA similarity criterion)

$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$ ... (1)

In ΔAA_3B and $\Delta AA_5B'$,

$\angle A_3AB = \angle A_5AB'$ (Common)

$\angle AA_3B = \angle AA_5B'$ (Corresponding angles)

∴ $\Delta AA_3B \sim \Delta AA_5B'$ (AA similarity criterion)

$\Rightarrow \frac{AB}{AB'} = \frac{AA_3}{AA_5} \Rightarrow \frac{AB}{AB'} = \frac{3}{5}$ ... (2)

On comparing equations (1) and (2), we obtain

$$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{5}{3}$$

$$\Rightarrow AB' = \frac{5}{3}AB, B'C' = \frac{5}{3}BC, AC' = \frac{5}{3}AC$$

This justifies the construction.