Mathematics

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(Chapter – 10) (Circles)
(Class 10)

Exercise 10.2

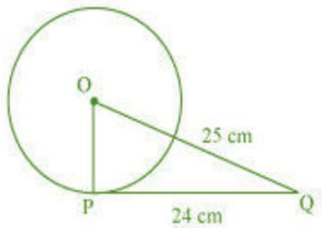
In Q.1 to 3, choose the correct option and give justification.

Question 1:

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm

Answer 1:



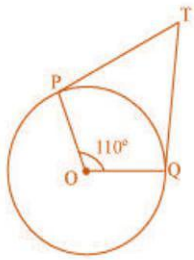
Let O be the centre of the circle.
Given: OQ = 25cm and PQ = 24 cm
We know that the radius is perpendicular to tangent. Therefore, OP ⊥ PQ
In ΔOPQ, By Pythagoras theorem,
$OP^2 + PQ^2 = OQ^2$
$\Rightarrow OP^2 + 24^2 = 25^2$ $\Rightarrow OP^2 = 625 - 576$ $\Rightarrow OP^2 = 49$ $\Rightarrow OP = 7$
Therefore, the radius of circle is 7 cm. Hence, the option (A) is correct.

Question 2:

In Figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to

(A) 60° (B) 70° (C) 80° (D) 90°

Answer 2:



Given: TQ and TP are two tangents of the circle.
We know that the radius is perpendicular to tangent. Therefore, OP ⊥ TP and OQ ⊥ TQ.
⇒ ∠OPT = 90° and ∠OQT = 90°
In quadrilateral POQT, ∠OPT + ∠POQ +∠OQT + ∠PTQ = 360°
⇒ 90° + 110° + 90° +∠PTQ = 360°
⇒ ∠PTQ = 360° - 290° = 70°
Hence, the option (B) is correct.

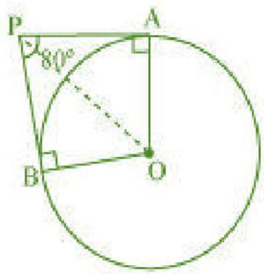
Question 3:

If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to

(A) 50° (B) 60° (C) 70° (D) 80°

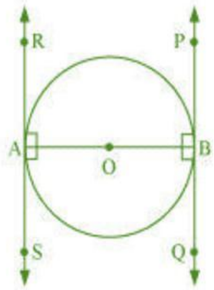
Answer 3:

Given: PA and PB are two tangents of the circle.
We know that the radius is perpendicular to tangent. Therefore, OA ⊥ PA and OB ⊥ PB.
⇒ ∠OBP = 90° and ∠OAP = 90°
In quadrilateral AOBP, ∠OAP + ∠APB +∠PBO + ∠BOA = 360°
⇒ 90° + 80° + 90° +∠BOA = 360°
⇒ ∠BOA = 360° - 260° = 100°



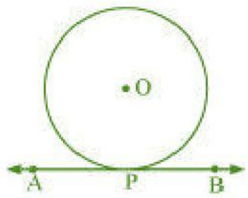
In ΔOPB and ΔOPA,
AP = BP [Tangents drawn from same external point]
OA = OB [Radii]
OP = OP [Common]
Therefore, ΔOPB ≅ ΔOPA [SSS Congruency rule]
Hence, ∠POB = ∠POA
∠POA = ½ ∠AOB = ½ (100°) = 50°. Hence, the option (A) is correct.

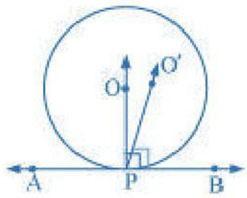
Question 4:
Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer 4:


Let AB is diameter, PQ and RS are tangents drawn at ends of diameter.
We know that the radius is perpendicular to tangent. Therefore, OA ⊥ RS and OB ⊥ PQ.
$\angle OAR = 90°$ and $\angle OAS = 90°$
$\angle OBP = 90°$ and $\angle OBQ = 90°$
From the above, we have
$\angle OAR = \angle OBQ$ [Alternate angles]
$\angle OAS = \angle OBP$ [Alternate angles]
Since, alternate angles are equal. Hence, PQ is parallel to PS.

Question 5:
Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Answer 5:


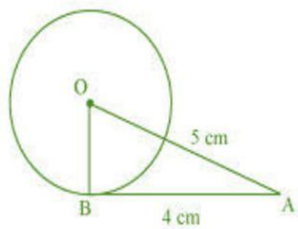


Let, O be the centre of circle and AB is a tangent at P.
We have to prove that the perpendicular at P to AB, passes through O.
Let the perpendicular drawn at P point of AB does not pass through O.
It passes through O'. Join OP and O'P.
Tangent drawn at P passes through O', therefore,
$\angle O'PB = 90°$ ... (1)
We know that the radius is perpendicular to tangent.
Therefore, $\angle OPB = 90°$ ... (2)
Comparing equation (1) and (2), we have
$\angle O'PB = \angle OPB$ ... (3)
From figure, it is clear that,
$\angle O'PB < \angle OPB$ ... (4)

Therefore, $\angle O'PB = \angle OPB$ is not possible. It is possible only when OP and O'P coincident lines.

Therefore, the perpendicular drawn at P passes through the centre O.

Question 6:
The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Answer 6:


Let O be the centre and AB is a tangent at B.
Given: OA = 5cm and AB = 4 cm
We know that the radius is perpendicular to tangent.
Therefore, in ΔABO, OB ⊥ AB.
In ΔABO, by Pythagoras theorem,
$AB^2 + BO^2 = OA^2$
$\Rightarrow 4^2 + BO^2 = 5^2$
$\Rightarrow 16 + BO^2 = 25$
$\Rightarrow BO^2 = 9$
$\Rightarrow BO = 3$
Therefore, the radius of circle is 3 cm.

Question 7:

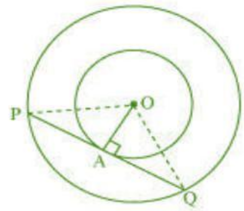
Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer 7:

Let O be the centre of two concentric circles with radius 5 cm (OP) and 3 cm (OA). PQ is chord of larger circle which is a tangent to inner circle.

We know that the radius is perpendicular to tangent. Therefore, in ΔPQO, OA ⊥ PQ.

In ΔAPO, by Pythagoras theorem,



$OA^2 + AP^2 = OP^2$

$\Rightarrow 3^2 + AP^2 = 5^2$

$\Rightarrow 9 + AP^2 = 25 \Rightarrow AP^2 = 16 \Rightarrow AP = 4$

In ΔOPQ, OA ⊥ PQ,

AP = AQ [Perpendicular from the centre bisects the chord]

So, PQ = 2AP = 2 × 4 = 8

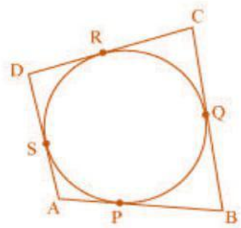
Hence, the length of chord of larger circle is 8 cm.

Question 8:

A quadrilateral ABCD is drawn to circumscribe a circle (see Figure). Prove that: AB + CD = AD + BC

Answer 8:

We know that the tangents drawn from same external point are equal. Therefore,



DR = DS [Tangents drawn from point D] ... (1)

CR = CQ [Tangents drawn from point C] ... (2)

BP = BQ [Tangents drawn from point B] ... (3)

AP = AS [Tangents drawn from point A] ... (4)

Adding (1), (2), (3) and (4), we have

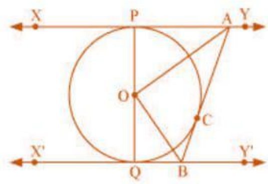
DR + CR + BP + AP = DS + CQ + BQ + AS

⇒ (DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

⇒ CD + AB = AD + BC

Question 9:

In Figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that ∠AOB = 90°.



Answer 9:

Join C from O.

In ΔOPA and ΔOCA,

OP = OC [Radii of same circle]

AP = AC [Tangents drawn from point A]

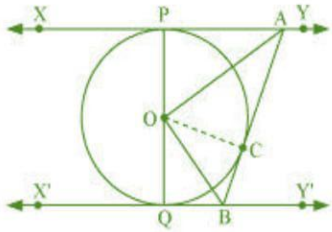
AO = AO [Common]

ΔOPA ≅ ΔOCA [SSS Congruency rule]

Hence, ∠POA = ∠COA ... (i)

Similarly, ΔOQB ≅ ΔOCB

Hence, ∠QOB = ∠COB ... (ii)



As POQ is diameter of circle, therefore it is a straight line.

Hence, ∠POA + ∠COA + ∠COB + ∠QOB = 180°

From (i) and (ii), we have,

2∠COA + 2 ∠COB = 180°

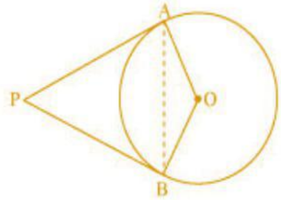
⇒ ∠COA + ∠COB = 90°

⇒ ∠AOB = 90°

Question 10:

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Answer 10:



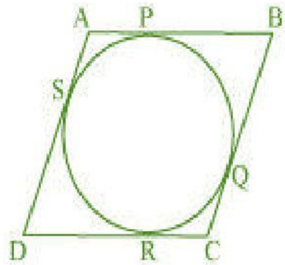
Let PA and PB are two tangents of circle with centre O. Join OA and OB.
We know that the radius is perpendicular to tangent. Therefore
∠OAP = 90° and ∠OBP = 90°.
In quadrilateral OAPB, ∠OAP +∠APB+∠PBO +∠BOA = 360°
⇒ 90° + ∠APB + 90° + ∠BOA = 360° ⇒ ∠APB + ∠BOA = 180°
Hence, it is proved that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Question 11:

Prove that the parallelogram circumscribing a circle is a rhombus.

Answer 11:

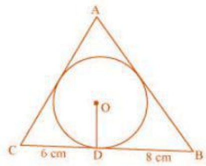
ABCD is a parallelogram, therefore
AB = CD ... (1)
BC = AD ... (2)
We know that the tangent drawn from same external point to circle are equal. So,
DR = DS [Tangents drawn from point D] ... (3)
CR = CQ [Tangents drawn from point C] ... (4)
BP = BQ [Tangents drawn from point B] ... (5)
AP = AS [Tangents drawn from point A] ... (6)



Adding (3), (4), (5) and (6), we have
DR + CR + BP + AP = DS + CQ + BQ + AS
⇒ (DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)
⇒ CD + AB = AD + BC
From (1) and (2), we have
2AB = 2BC ⇒ AB = BC ... (7)
From (1), (2) and (7), we have
AB = BC = CD = DA
Hence, ABCD is a rhombus.

Question 12:

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Figure). Find the sides AB and AC.



Answer 12:

Let the circle touches AB and AC at E and F respectively. Let AF = x cm
We know that the tangents drawn from same external point to the circle are equal. Therefore,
CF = CD = 6cm [Tangents drawn from point C]
BE = BD = 8cm [Tangents drawn from point B]
AE = AF = x [Tangents drawn from point A]
AB = AE + EB = x + 8
BC = BD + DC = 8 + 6 = 14
CA = CF + FA = 6 + x
In ΔABC,
2s = AB + BC + CA = x + 8 + 14 + 6 + x = 28 + 2 x
∴ s = 14 + x

Area of ΔABC

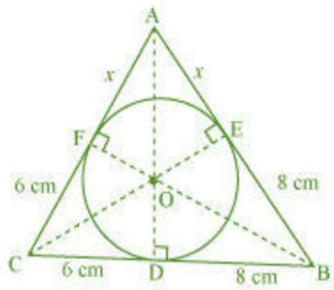
$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\{14+x\}\{(14+x)-14\}\{(14+x)-(8+x)\}\{(14+x)-(6+x)\}}$

$= \sqrt{(14+x)(x)(6)(8)}$

$= \sqrt{48x(14+x)}$ cm²

$= 4\sqrt{3(14x+x^2)}$ cm²



Area of ΔOBC $= \frac{1}{2} \times OD \times BC = \frac{1}{2} \times 4 \times 14 = 28$ cm²

Area of ΔOCA $= \frac{1}{2} \times OF \times AC = \frac{1}{2} \times 4 \times (6+x) = 12 + 2x$ cm²

Area of ΔOAB $= \frac{1}{2} \times OE \times AB = \frac{1}{2} \times 4 \times (8+x) = 16 + 2x$ cm²

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$\Rightarrow 4\sqrt{3(14x+x^2)} = 28 + (12+2x) + (16+2x)$

$\Rightarrow 4\sqrt{3(14x+x^2)} = 56 + 4x$

$\Rightarrow \sqrt{3(14x+x^2)} = 14 + x$

$\Rightarrow 3(14x+x^2) = (14+x)^2 \qquad \Rightarrow 42x + 3x^2 = 196 + x^2 + 28x$

$\Rightarrow 2x^2 + 14x - 196 = 0 \qquad \Rightarrow x^2 + 7x - 98 = 0$

$\Rightarrow x^2 + 14x - 7x - 98 = 0 \qquad \Rightarrow x(x+14) - 7(x+14) = 0$

$\Rightarrow (x+14)(x-7) = 0$

$\Rightarrow x + 14 = 0$ or $x - 7 = 0$

Therefore, $x = -14$ or 7

But the side of triangle can't be negative, $x \neq -14$. Therefore, $x = 7$

Hence, AB = x + 8 = 7 + 8 = 15 cm and CA = 6 + x = 6 + 7 = 13 cm

Question 13:

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer 13:

Quadrilateral ABCD is circumscribing a circle with centre O, touching at points P, Q, R and S.

Join the points P, Q, R and S from the centre O.

In ΔOAP and ΔOAS,

OP = OS [Radii of same circle]

AP = AS [Tangents drawn from point A]

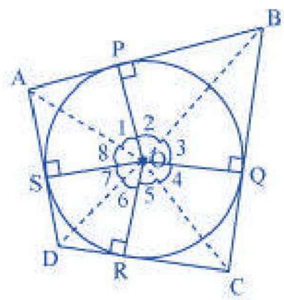
AO = AO [Common]

ΔOPA ≅ ΔOCA [SSS Congruency rule]

Hence, ∠POA = ∠SOA or ∠1 = ∠8

Similarly, ∠2 = ∠3, ∠4 = ∠5 and ∠6 = ∠7

Sum of all angles at point O is 360°. Therefore



∠1+ ∠2 + ∠3 + ∠4 + ∠5 + ∠6 + ∠7 + ∠8 = 360°

⇒ (∠1 + ∠8) + (∠2 + ∠3) + (∠4 + ∠5) + (∠6 + ∠7) = 360°

⇒ 2∠1 + 2∠2 + 2∠5 + 2∠6 = 360°

⇒ 2(∠1 + ∠2) + 2(∠5 + ∠6) = 360°

⇒ (∠1 + ∠2) + (∠5 + ∠6) = 180°

⇒ ∠AOB + ∠COD = 180°

Similarly, we can prove ∠BOC + ∠DOA = 180°

Hence, the opposite sides subtend supplementary angles at the centre.