Previous Year Question Paper 2020

- Please check that this question paper contains **23** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **40** questions.
- **Please write down the Serial Number of the question in the answer-book before attempting it.**
- 15 minutes of time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer script during this period.

MATHEMATICS (Standard) - Theory

Time Allowed: **3** hours | Maximum Marks: **80**

General Instructions:

Read the following instructions very carefully and strictly follow them :

1. Please check that this question paper contains **23** printed pages.
2. The question paper comprises of **four** sections – A, B, C and D. This question paper carries **40** questions. **All** questions are compulsory.
3. **Section A:** Question numbers **1** to **20** comprises of **20** questions of **one** mark each.
4. **Section B:** Question numbers 21 to **26** comprises of **6** questions of **two** mark each.

5. **Section C:** Question numbers **27** to **34** comprises of **8** questions of **three** mark each.

6. **Section D:** Question numbers **35** to **40** comprises of **6** questions of **four** mark each.

7. There is no overall choice in the paper. However, internal choice is provided in 2 questions of one mark, 2 questions of one mark, 2 questions of two marks, 3 questions of three marks and three questions of four marks. You have to attempt only one of the choices in such questions.

8. In addition to this, separate instructions are given with each section and question, wherever necessary.

9. Use of calculators is **not** permitted.

SECTION-A

1. The value(s) of k for which the quadratic equation $2x^2+kx+2=0$ has equal roots, is **1 Mark**

(A) 4

(B) ± 4

(C) -4

(D) 0

Ans: Given quadratic equation is $2x^2+kx+2=0$.

For equal roots, D =0

$\Rightarrow b^2-4ac=0$

Here, a=2, b=k, and c=2

$\Rightarrow k^2-4\times 2\times 2=0$

$\Rightarrow k^2-16=0$

$k^2=16$

$\therefore k=\pm 4$

Therefore, the values of k is ± 4.

Hence, option (B) is correct.

2. Which of the following is not an A.P.? **1 Mark**

(A) -1.2,0.8, 2.8,…

(B) 3. 3+$\sqrt{2}$. 3+2$\sqrt{2}$. 3+3$\sqrt{2}$

(C) $\frac{4}{3},\frac{7}{3},\frac{9}{3},\frac{12}{3},......$

(D) $\frac{-1}{5},\frac{-2}{5},\frac{-3}{5},......$

Ans: (A) We have,

$\Rightarrow a_2-a_1=a_3-a_2=2$

Hence, it is an A.P.

(B) We have,

$\Rightarrow a_2-a_1=a_3-a_2=\sqrt{2}$

Hence, it is an A.P.

(C) We have, $a_2-a_1=1$ and $a_3-a_2=\frac{2}{3}$.

So, $a_2-a_1 \neq a_3-a_2$.

Hence, it is not an A.P.

(D) We have,

$a_2-a_1=a_3-a_2=\frac{-1}{5}$

Hence, it is an A.P.

Therefore, option (C) is correct.

3. The radius of a sphere (in cm) whose volume is $12\pi cm^3$, is **1 Mark**

(A) 3

(B) $3\sqrt{3}$

(C) $3^{\frac{2}{3}}$

(D) $3^{\frac{1}{3}}$

Ans: Volume of a sphere $=\frac{4}{3}\pi r^3$

Given, Volume of the sphere $=12\pi r\ cm^3$

On equating we get,

$\Rightarrow \frac{4}{3}\pi r^3=12\pi r\ cm^3$

$\Rightarrow r^3=\frac{12\pi\times3}{4\pi}cm^3$

$\Rightarrow r^3=9\ cm^3$

$\Rightarrow r^3=(3)^2\ cm^3$

$\Rightarrow r=(3)^{\frac{2}{3}}\ cm$

Therefore, radius of the sphere is $(3)^{\frac{2}{3}}$ cm.

Hence, option (C) is correct.

4. The distance between the points (m, -n) and (-m, n) is **1 Mark**

(A) $\sqrt{m^2+n^2}$

(B) m+n

(C) $2\sqrt{m^2+n^2}$

(D) $\sqrt{2m^2+2n^2}$

Ans: Let the points be A(m, -n) and B(-m, n).

From distance formula we get,

$\Rightarrow AB=\sqrt{(-m-m)^2+(n-(-n))^2}$

$\Rightarrow AB=\sqrt{(-2m)^2+(2n)^2}$

$\Rightarrow AB=\sqrt{4m^2+4n^2}$

$\Rightarrow AB=2\sqrt{m^2+n^2}$

Therefore, the distance between the points (m, -n) and (-m, n) is $2\sqrt{m^2+n^2}$.

Hence, option (C) is correct.

5. In Figuire-1, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR=90^{\circ}$, then length of PQ is **1 Mark**

(A) 3 cm

(B) 4 cm

(C) 2 cm

(D) $2\sqrt{2}$ cm

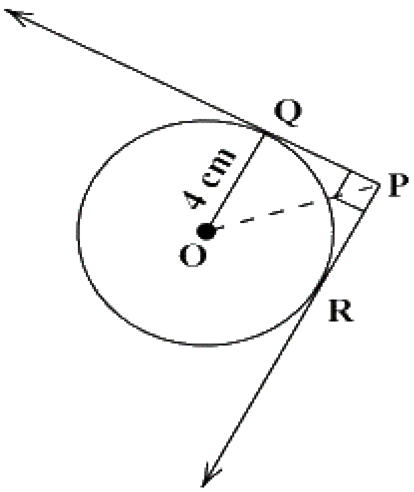


Figure-1

Ans: Given, $\angle QPR=90^{\circ}$

Since, the line from the centre of the circle bisects the angle between the tangents.

Therefore, $\angle OPQ=45^{\circ}$.

In ΔPOQ, we have

$$\Rightarrow \tan 45^{\circ}=\frac{OQ}{PQ}$$

$\Rightarrow 1=\frac{OQ}{PQ}$

$\Rightarrow PQ=OQ$

$\Rightarrow PQ=4\,cm$

Therefore, PQ is 4cm.

Hence, option (B) is correct.

6. On dividing a polynomial p(x) by x^2-4, quotient and remainder are found to be x and 3 respectively. The polynomial p(x) is **1 Mark**

(A) $3x^2+x-12$

(B) x^3-4x+3

(C) x^2+3x-4

(D) x^3-4x-3

Ans: As dividend = (divisor × quotient)+remainder

$\Rightarrow p(x)\left[\left(x^2-4\right)\times x\right]+3$

$\Rightarrow p(x)=x^2-4x+3$

Therefore, polynomial p(x) is x^2-4x+3.

7. In Figure-2, DE || BC. If $\frac{AD}{DB}=\frac{3}{2}$ and AE = 2.7 cm, then EC is equal to **1 Mark**

(A) 2.0 cm

(B) 1.8 cm

(C) 4.0 cm

(D) 2.7 cm

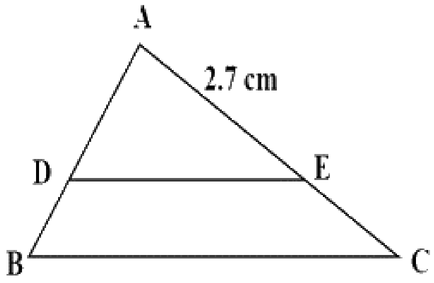


Figure-2

Ans: Given, DE || BC.

Therefore, $\frac{AD}{DB}=\frac{AE}{EC}$

$\Rightarrow \frac{3}{2}=\frac{2.7\text{ cm}}{EC}$

$\Rightarrow EC=\frac{2\times 2.7\text{ cm}}{3}$

$\Rightarrow$ EC = 1.8 cm

Therefore, EC is equal to 1.8 cm.

Hence, option (B) is correct.

8. The point on the x-axis which is equidistant from (-4, 0) and (10, 0) is

1 Mark

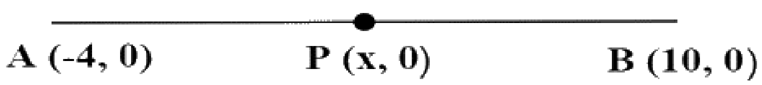
(A) (7, 0)

(B) (5, 0)

(C) (0, 0)

(D) (3, 0)

Ans: Let the point on the x-axis is P(x, 0) which is equidistant from A(-4, 0) and B(10, 0).



We have, AP = PB

Using distance formula, we get

$\Rightarrow \sqrt{\left(x-(-4)\right)^2+(0-0)^2}=\sqrt{(10-x)^2+(0-0)^2}$

Squaring both the sides,

$\Rightarrow (x+4)^2=(10-x)^2$

$\Rightarrow x^2+8x+16=100-20x+x^2$

$\Rightarrow 8x+20x=100-16$

$\Rightarrow 28x=84$

$\Rightarrow x=3$

Therefore, (3, 0) is equidistant from (-4, 0) and (10, 0).

Hence, option (D) is correct.

Or

The centre of a circle whose end points of a diameter are (-6, 3) and (6, 4) is **1 Mark**

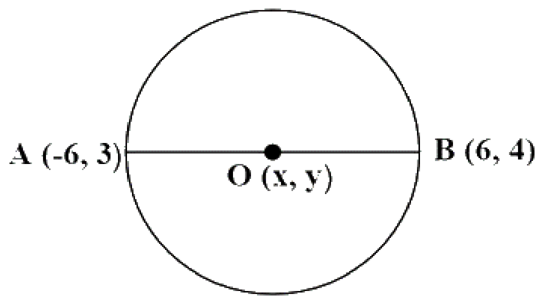
(A) (8, -1)

(B) (4, 7)

(C) $\left(0,\frac{7}{2}\right)$

(D) $\left(4,\frac{7}{2}\right)$

Ans: Let centre be O(x, y) and end points of the diameter be A(-6, 3) and B(6, 4).



Since, centre is the midpoint of diameter. So,

$\Rightarrow x=\frac{-6+6}{2}$ and $y=\frac{3+4}{2}$

$\Rightarrow x=0 \text{ and } y=\frac{7}{2}$

Therefore, centre of the circle is $\left(0,\frac{7}{2}\right)$.

Hence, option (C) is correct.

9. The pair of linear equations **1 Mark**

$\mathbf{\frac{3x}{2}+\frac{5y}{3}=7 \text{ and } 9x+10y=14}$

(A) consistent

(B) inconsistent

(C) consistent with one solution

(D) consistent with many solutions

Ans: Given $\frac{3x}{2}+\frac{5y}{3}=7$ and $9x+10y=14$

Here, $a_1=\frac{3}{2}, b_1=\frac{5}{3}, c_1=7, a_1=9, b_1=10$ and $c_1=14$.

$\Rightarrow \frac{a_1}{a_2}=\frac{\frac{3}{2}}{9}, \frac{b_1}{b_2}=\frac{\frac{5}{3}}{10}$ and $\frac{c_1}{c_2}=\frac{7}{14}$

$\Rightarrow \frac{a_1}{a_2}=\frac{1}{9}, \frac{b_1}{b_2}=\frac{1}{6}$ and $\frac{c_1}{c_2}=\frac{1}{2}$

$\therefore \frac{a_1}{a_2}=\frac{b_1}{b_1}1\frac{c_1}{c_1}$

Therefore, these lines equation intersect each other at one point and only have one possible solution.

Hence, the pair of linear equation is inconsistent.

Hence, option (B) is correct.

10. In Figure-3, PQ is tangent to the circle with centre at O, at the point B.

If $\angle AOB=100^o$, then $\angle ABP$ is equal to **1 Mark**

(A) 50o

(B) 40o

(C) 60o

(D) 80o

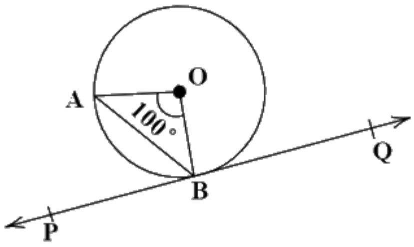


Figure-3

Ans: In ΔAOB, AO=OB

$\therefore \angle OAB=\angle OBA=40^o$

Since PQ is tangent at the point B, $\angle OBP=90^o$

$\Rightarrow \angle OBP = \angle OBA + \angle ABP$

$\Rightarrow 90^o = 40^o + \angle ABP$

$\Rightarrow \angle ABP=90^o-40^o$

$\Rightarrow ABP=50^o$

Therefore, $\angle ABP$ is equal to 50^o.

Hence, option (A) is correct.

Fill in the blanks in question numbers 11 to 15.

11. Simplest form of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is _________. **1 Mark**

Ans: $\frac{1+\tan^2 A}{1+\cot^2 A}$ can be simplified as

$$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$\Rightarrow \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$\Rightarrow \tan^2 A$$

12. If the probability of an event E happening is 0.023, then $P(\overline{E})=$ _________. 1 Mark

Ans: Given, $P(E)=0.023$.

As, $P(E)+P(\overline{E})=1$

$$\Rightarrow 0.023+P(\overline{E})=1$$

$$\Rightarrow P(\overline{E})=1-0.023$$

$$\Rightarrow P(\overline{E})=0.977$$

Therefore. $P(\overline{E})=0.977$

13. All concentric circles _________ to each other. 1 Mark

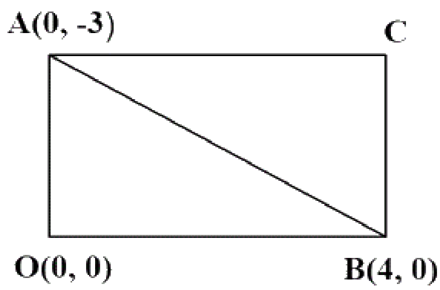
Ans: All concentric circles are similar to each other.

14. The probability of an event that is sure to happen, is _________. 1 Mark

Ans: The probability of an event that is sure to happen is 1.

15. AOBC is a rectangle whose three vertices are A(0, -3), O(0. 0) and B(4, 0). The length of its diagonal is _________. 1 Mark

Ans:



Length of diagonal AB $=\sqrt{(4-0)^2+(0-(-3))^2}$

$=\sqrt{(25)^2+(3)^2}$

$=\sqrt{25}$

$= 5$

Therefore, length of diagonal is 5 units.

Answer the following question numbers 16 to 20.

16. Write the value of $\sin^2 30^\circ+\cos^2 30^\circ$. **1 Mark**

Ans: As, $\sin^2\theta+\cos^2\theta=1$

$\therefore \sin^2 30^\circ+\cos^2 30^\circ=1$

17. Form a quadratic polynomial, the sum and product of whose zeroes are (-3) and 2 respectively. **1 Mark**

Ans: Given, sum of zeroes =-3 and product of zeroes =2

The quadratic equation is given by

x2-(sum of zeroes)x+(product of zeroes) = 0

x2-(-3)x+(2)=0

x2+3x+2=0

Or

Can (x2-1) be a remainder while dividing x^4-3x+5x-9 by (x^3+3)?

Justify your answer with reasons. **1 Mark**

Ans: On dividing x^4-3x+5x-9 by (x^3+3) we get (5x+9) as a remainder.

$$\begin{array}{r|l} & x^2-6 \\ \hline x^2+3 & x^4-3x^2+5x-9 \\ & x^4+3x^2 \\ \hline & -6x^2+5x-9 \\ & -6x^2-18 \\ \hline & 5x+9 \end{array}$$

Therefore, $\left(x^2-1\right)$ can't be a remainder while dividing x^4-3x^2+5x-9 by $\left(x^2+3\right)$

.

18. Find the sum of the first 100 natural numbers. **1 Mark**

Ans: Let, the sum be $s_{100}=1+2+3+...+100$

Here, $a=1$, $d=1$, $n=100$ and $l=100$

As, $s_n=\frac{n}{2}\left(a+l\right)$

$\Rightarrow s_{100}=\frac{100}{2}\left(1+100\right)$

$\Rightarrow s_{100}=5050$

Therefore, the sum of the first 100 natural numbers is 5050.

19. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other. **1 Mark**

Ans: Let the other number be x.

Product of number $=\text{LCM}\times\text{HCF}$

$\Rightarrow 26\times x=182\times 13$

$\Rightarrow x=91$

Therefore, other number is 91.

20. In Figure-4, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30o. Find the height of the tower. **1 Mark**

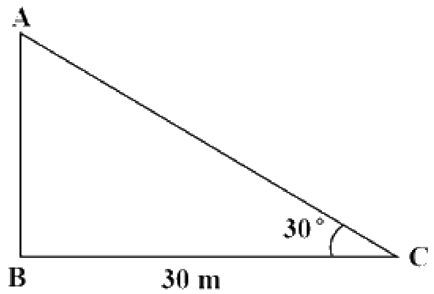


Figure-4

Ans: Let the tower be AB.

Since the tower is vertical, therefore $\angle ABC = 90^{\circ}$.

In ΔABC, $tab30^{\circ} = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30}$

$\Rightarrow AB = 10\sqrt{3}$

Therefore, height of the tower is $10\sqrt{3}$.

SECTION-B

Question numbers 21 to 26 carry 2 marks each.

21. A cone and a cylinder have the same radii but the height of the cone is 3 times that of the cylinder. Find the ratio of their volumes. 2 Marks

Ans: Let radius of cone = radius of cylinder =r and height of cylinder be h, then height of cone will become 3h.

Volume of a cone $= \frac{1}{3}\pi r^2 H$

$\Rightarrow$ Volume of given cone $= \frac{1}{3}\pi r^2 (3h)$

$= \pi r^2 h$

Volume of a cylinder = $\pi r^2 H$

$\Rightarrow$ Volume of given cylinder = $\pi r^2 h$

Therefore, $\frac{\text{Volume of the cone}}{\text{Volume of the cylinder}} = \frac{\pi r^2 h}{\pi r^2 h}$

$\Rightarrow \frac{\text{Volume of the cone}}{\text{Volume of the cylinder}} = \frac{1}{1}$

Therefore, ratio of volume of cone to volume of cylinder is 1:1.

22. In Figure-5, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB+CD=BC+AD$**.** **2 Marks**

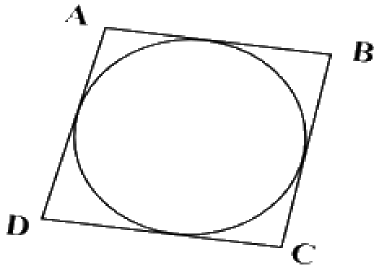


Figure-5

Ans: As we know, length of tangents drawn from an external point are equal. Therefore, we can write

$\Rightarrow AP=AS$ ---(1)

$\Rightarrow BP=BQ$ ---(2)

$\Rightarrow CR=CQ$ ---(3)

$\Rightarrow BR=DS$ ---(4)

On adding equation (1), (2), (3) and (4), we get

$\Rightarrow AP+BP+CR+DR=AS+BQ+CQ+DS$

$\Rightarrow$ $(AP+BP)+(CR+DR)=(AS+DS)+(BQ+CQ)$

$\Rightarrow AB+CD=AD+BC$

Hence, proved that $AB+CD=AD+BC$

Or

In Figure-6, find the perimeter of ΔABC**, if** $AP=12$ **cm.** **2 Marks**

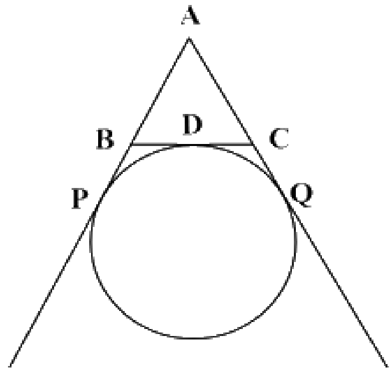


Figure-6

Ans: As we know, tangents drawn from an external point are equal.

Therefore, BD=BP , CD=CQ and AP=Q.

Perimeter of ΔABC = AB+BC+CA
=AB+BD+CD+AC
=AP+ AQ
=2Ap
=2×12
=24

Therefore, perimeter of ΔABC is 24 cm.

23.Find the mode of the following distribution:

Marks:	**0-12**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**
Number of students	**4**	**6**	**7**	**12**	**5**	**6**

2 Marks

Ans: Here, modal class is 30-40.

$$\text{Mode}=l+\frac{f_1-f_0}{2f_1-f_0-f_2}\times h$$

$$\Rightarrow \text{Mode}=30+\frac{12-7}{2\times(12)-7-5}\times 10$$

$$\Rightarrow \text{Mode}=34.16$$

Therefore, mode is 34. 16.

24. In Figure-7, if PQ || BC and PR || CD, prove that $\frac{QB}{AQ}=\frac{DR}{AR}$. 2 Marks

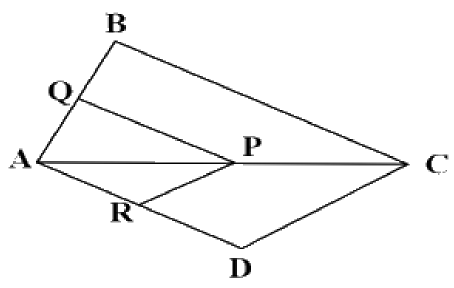


Figure-7

Ans: Since, PQ || BC in ΔABC

By basic proportionality theorem, we get

$$\Rightarrow \frac{AQ}{AB}=\frac{AP}{AC}---(1)$$

In ΔACD, PR || CD

By basic proportionality theorem, we get

$$\Rightarrow \frac{AP}{AC}=\frac{AR}{AD}---(2)$$

From (1) and (2), we get

$$\Rightarrow \frac{AB}{AQ}=\frac{AD}{AR}$$

$$\Rightarrow \frac{AQ+QB}{AQ}=\frac{AR+RD}{AR}$$

$$\Rightarrow 1+\frac{QB}{AQ}=1+\frac{RD}{AR}$$

$$\Rightarrow \frac{QB}{AQ}=\frac{RD}{AR}$$

Hence, proved that $\frac{QB}{AQ}=\frac{RD}{AR}$.

25. Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number. 2 Marks

Ans: Let $5+2\sqrt{7}$ is a rational number.

So, we can write

$$\Rightarrow 5+2\sqrt{7}=\frac{P}{q}$$

$$\Rightarrow \sqrt{7}=\frac{P}{2q}=\frac{5}{2}$$

p, q, 5 and 2 are integers. So, $\frac{P}{2q} - \frac{5}{2}$ is a rational number.

Therefore, $\sqrt{7}$ is also a rational number.

But $\sqrt{7}$ is given to be an irrational number.

This is a contradiction which raised due to our assumption that $5+2\sqrt{7}$ is a rational number.

Therefore, $5+2\sqrt{7}$ is an irrational number.

Or

Check whether 12^n can end with the digit 0 for any natural number n.
2 Marks

Ans: We can write, $12^n = \left(2^n \times 3\right)^n$.

If a number ends with 0 then it is divisible by 5. But, prime factorisation of 12^n does not contains 5.

Therefore, 12^n can't end with the digit 0 for any natural number n.

26. If A, B and C are interior angles of a ΔABC, then show that $\cos\left(\frac{B+C}{2}\right) = \sin\left(\frac{A}{2}\right)$. **2 Marks**

Ans: In $\Delta ABC, \angle A + \angle B + \angle C = 180^o$ or A+B+C=180o.

B+C=180o-A

$$\text{L.H.S.} = \cos\left(\frac{B+C}{2}\right)$$

$$= \cos\left(\frac{180^o - A}{2}\right)$$

$$= \cos\left(90^o - \frac{A}{2}\right)$$

$$= \sin\left(\frac{A}{2}\right)$$

Therefore, $\cos\left(\frac{B+C}{2}\right)=\sin\left(\frac{A}{2}\right)$.

SECTION-C

Question number 27 to 34 carry 3 marks each.

27. Prove that: $\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=2$ **3 Marks**

Ans: On simplification we get,

$$\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=\left(\left(\sin^2\theta\right)^2-\left(\cos^2\theta\right)^2+1\right)\operatorname{cosec}^2\theta$$

$$\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=\left[\left(\sin^2\theta+\cos^2\theta\right)\left(\sin^2\theta+\cos^2\theta\right)+1\right]\operatorname{cosec}^2\theta$$

As $\sin^2\theta+\cos^2\theta=1$, we get

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=\left[\left(\sin^2\theta-\cos^2\theta\right)+1\right]\operatorname{cosec}^2\theta$$

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=\left[1-\cos^2\theta+\sin^2\theta\right]\operatorname{cosec}^2\theta$$

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2=\left[\sin^2\theta+\sin^2\theta\right]\operatorname{cosec}^2\theta$$

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=2\sin^2\theta\operatorname{cosec}^2\theta$$

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=2\sin^2\theta\times\frac{1}{\sin^2\theta}$$

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=2$$

Hence, proved that $\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=2$.

28. Find the sum: (-5)+(-8)+(-11)+…+(-230) **3 Marks**

Ans: a1**=-5**, a2=-8, a3=-11

$\Rightarrow a_2-a_1=a_3-a_2=-3$

It is an A.P., in which first term is -5, common difference is -8 and last term is -230.

$\therefore a_1=-5d,\ d=-3$ and $l=-230$

As $l=a_1+(n-1)d$ where n is number of terms in the A.P.

From this we get,

$\Rightarrow$-230=-5+(n-1)(-3)

$\Rightarrow$-230+5-3=3n+3

$\Rightarrow$-230+3-3=-3n

$\Rightarrow$3n=228

$\Rightarrow$n=76

As we know, sum of the series $(s_n)=\frac{n}{2}(a+l)$

$\Rightarrow(s_n)=\frac{76}{2}(-5-230)$

$\Rightarrow(s_n)=\frac{76}{2}\times(-235)$

$\Rightarrow(s_n)=-8930$

Therefore, the sum of he given series is -8930.

29. Construct a ΔABC with sides BC=6 cm, AB=5 cm and $\angle$ABC=60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC. 3 Marks

Ans: Construction of ΔABC:

(1) Draw a line segment AB of length 5 cm.

(2) Draw a line segment BC by making an angle of 60o from point B.

(3) Join A and C to get the required ΔABC.

Now, construction of ΔA'BC' whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC are as follows:

(1) Draw a ΔABC with sides BC= 6cm , AB = 5 cm and $\angle$ABC=60°.

(2) On opposite side of vertex A, draw a ray BX making an acute angle with

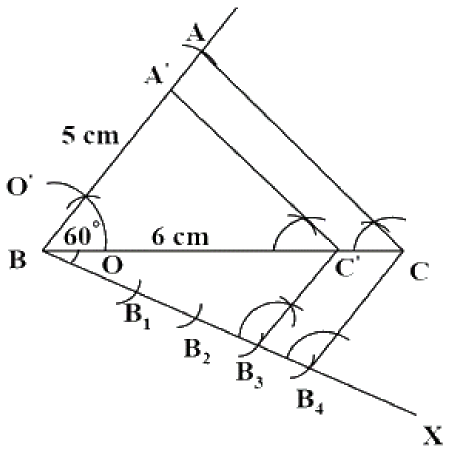
BC.

(3) On the line segment BX, locate four points B1,B2,B3 and B4.

(4) Join B4 and C. Draw a line through B3 parallel to B4C intersecting BC at C

(5) Draw a line parallel to AC through C intersecting AB at A.

Hence, we obtained the required $\Delta A'BC'$.



Or

Draw a circle of radius 3.5 cm . Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents to the circle from that point. 3 Marks

Ans: Construction:

(1) Draw a line OP=6 cm.

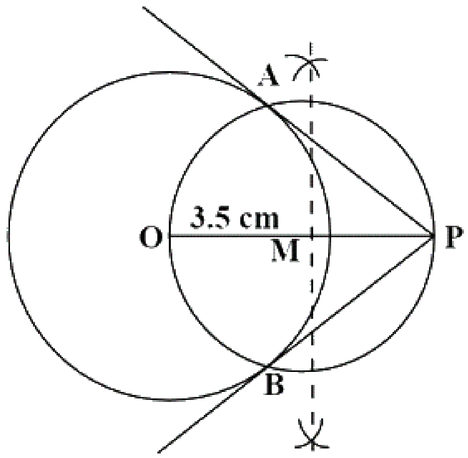
(2) Draw a circle of radius 3.5 cm with O as centre.

(3) Draw a perpendicular bisector of OP that cuts OP at M.

(4) With M as a centre and MP (or OP) as radius, draw a circle which intersects the first circle at A and B.

(5) Join PA and PB.

PA and PB are the required tangents.



30. In Figure-8, ABCD is a parallelogram. A semicircle with a centre O and the diameter AB has been drawn and it passes through D. If AB=12 cm and $OD \perp AB$, then find the area of the shaded region. (Use π=3.14)

3 Marks

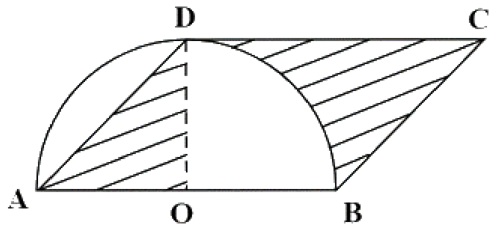


Figure-8

Ans: Given, Diameter = AB = 12 cm

AO=OB=OD=6 cm

$\Rightarrow$ area of OBD = $\frac{1}{4}\times\pi\times(6)^2$

$=28.26\,cm^2$

Area of parallelogram ABCD = base× height

= 12×6

=72 cm2

Area of shaded region = area of ABCD – area of OBD

=(72-28.26) cm2

= 43.74 cm2

Therefore, area of shaded region is 43.74 cm2.

31. Read the following passage and answer the questions given at the end:

Diwali Fair A game in a booth at a Diwali Fair involves using a spinner first. Then, if the spinner stops on an even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bag are represented in Figure-9.Prizes are given, when a black marble is picked. Shweta plays the game once.

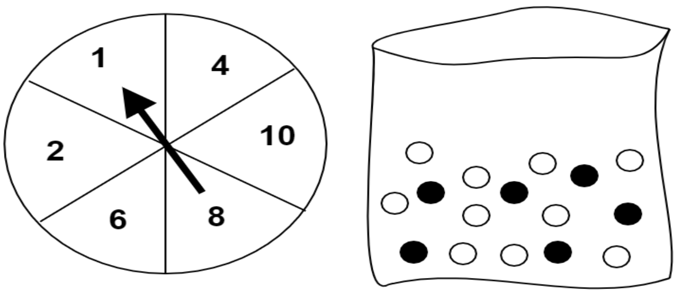


Figure 9

(i) What is the probability that she will be allowed to pick a marble from the bag?

(ii) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black? 3 Marks

Ans: The player is allowed to pick a marble from a bag if the spinner stops on an even number.

So, the favourable outcomes are 2, 4, 6, 8 and 10.

Number of favourable outcomes =5

Total number of outcomes =6

(i) The probability that she will be allowed to pick a marble from the bag

$$=\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$=\frac{5}{6}$$

(ii) The bag contains 20 balls out of which 6 are black and prizes are given, when a black marble is picked.

Number of favourable outcomes =6

Total number of outcomes =20

The probability of getting a prize $=\dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$=\dfrac{6}{20}$

$=\dfrac{3}{10}$

32. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

3 Marks

Ans: Let the numerator be x and denominator be y. So, fraction is $\frac{x}{y}$.

Given, when 1 is subtracted from the numerator the fraction becomes $\frac{1}{3}$ i.e.,

$\frac{x-1}{y}=\frac{1}{3}$

$\Rightarrow 3(x-1)y$

$\Rightarrow 3x-3=y$

$\Rightarrow 3x-y=3$ ---(1)

Also given, the fraction becomes $\frac{1}{4}$ when 8 is added to its denominator i.e.,

$\frac{x}{y+8}=\frac{1}{4}$

$\Rightarrow 4x=y+8$

$\Rightarrow y=4x-8$---(1)

Putting (2) in (1), we get

3x+4x+8=3

-x=-5

$x=5$

Putting the value of x in (1), we get

$y=4\times5-8$

$y=4\times5-8$

$y=12$

Therefore, the fraction is $\frac{x}{y}$ i.e., $\frac{5}{12}$.

Or

The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages. 3 Marks

Ans: Let the son's age be x.

Given, age of the father is three years more than three times the age of his son.

Therefore, father's age is $3x+3$.

After three years,

Age of the son $=x+3$

Age of the father $=3x+3+3$ ---(1)

But, according to question, after three years the father's age will be 10 years more than twice the age of the son.

Age of the father $10+2(x+3)$ ---(2)

From (1) and (2), we get

$\Rightarrow 3x+6=10+2(x+3)$

$\Rightarrow 3x+6=10+2x+6$

$\Rightarrow x=10$

So, the present age of son is 10 years.

Present age of father $=3\times10+3$

$=33$ years

Therefore, the present age of son is 10 years and father is 33 years.

33. Find the ratio in which the y-axis divides the line segment joining the points (6, -4) and (-2, 7). Also find the point of intersection. 3 Marks

Ans: Let the y-axis divides the line segment joining the points (6, -4) and (-2, -7) in the ratio k:1 and the point be (0, y).

From section formula we know that if a point (x, y) divides the line joining the points (x1, y1) and (x2, y2) in the ratio m : n, then $\left(x,y\right)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$

Using this, we get

$\Rightarrow 0=\frac{k\times\left(-2\right)+1\times 6}{k+1}$

$\Rightarrow -2k+6=0$

$\Rightarrow 2k=6$

$\Rightarrow k=3$

Therefore, the y-axis divides the line segment joining the points (6, -4) and (-2, -7) in 3:1.

Now,

Coordinate of point of intersection $=\left(0,\frac{3\times\left(-7\right)+1\times\left(-4\right)}{3+1}\right)$

$=\left(0,\frac{-25}{4}\right)$

Therefore, point of intersection is $\left(0,\frac{-25}{4}\right)$.

Or

Show that the points (7, 10), (-2, 5) and (3, -4) are vertices of an isosceles right triangle. 3 Marks

Ans: Let A(7, 10), B(-2, 5) and C(3, -4).

Using distance formula,

$\Rightarrow AB=\sqrt{\left(-2-7\right)^2+\left(5-10\right)^2}$

$=\sqrt{\left(-9\right)^2+\left(-5\right)^2}$

$=\sqrt{106}$

$\Rightarrow BC=\sqrt{(3+2)^2+(-4-5)^2}$

$=\sqrt{(5)^2+(-9)^2}$

$=\sqrt{106}$

$\Rightarrow AC=\sqrt{(3-7)^2+(-4-10)^2}$

$=\sqrt{(-4)^2+(-14)^2}$

$=\sqrt{212}$

We can see that AB = BC.

Also,

$\Rightarrow AB^2+BC^2=106+106$

= 212

= AC2

Therefore, by Pythagoras theorem ΔABC is a right-angled triangle.

Hence, the points (7, 10) (-2, 5) and (3, -4) are vertices of an isosceles right triangle.

34. Use Euclid Division Lemma to show that the square of any positive integer is either of the form 3q or 3q+1 for some integer q. 3 Marks

Ans: We know from Euclid Division Lemma that if a and b are two positive integers then a=bm+r where $0\leq 0r<b$.

Now, let the positive integer be a and b=3.

r is an integer greater than or equal to zero and less than 3. Therefore, r can be either 0,1 or 2.

For r=0, the equation becomes

$\Rightarrow a=3m+0$

$\Rightarrow a=3m$

Squaring both the sides,

$\Rightarrow a^2=(3m)^2$

$\Rightarrow a^2=3(3m)^2$

Let $3m^2=q$.

$\Rightarrow a^2=3q$

For r=1, the equation becomes

$\Rightarrow a=3m+1$

Squaring both the sides,

$\Rightarrow a^2=(3m+1)^2$

$\Rightarrow a^2=9m^2+6m+1$

$\Rightarrow a^2=3(3m^2+2m)+1$

Let $q=3m^2+2m$.

$\Rightarrow a^2=3q+1$

For r=2, the equation becomes

$\Rightarrow a=3m+2$

Squaring both the sides,

$\Rightarrow a^2=(3m+2)^2$

$\Rightarrow a^2=9m^2+12m+4$

$\Rightarrow a^2=9m^2+12m+3+1$

$\Rightarrow a^2=3(3m^2+4m+1)+1$

Let $q=3m^2+4m+1$.

$\Rightarrow a^2=3q+1$

Hence proved that square of any positive integer is either of the form 3q or 3q+1 for some integer q.

SECTION-D

Question numbers 35 to 40 carry 4 marks each.

35. Sum of the areas of two squares is 544m2. If the difference of their perimeter is 32m, find the sides of the two squares. 4 Marks

Ans: Let the sides of two squares be x and y where x>y.

Perimeter of first square =4x

Perimeter of second square =4y

Given, the difference of their perimeter is 32m.

$\Rightarrow$4x-4y=32

$\Rightarrow$x-y=8

$\Rightarrow$x=5+y

Area of first square =x2

Area of second square =y2

Given, sum of the areas of two squares is 544 m2.

$\Rightarrow x^2+y^2=544$

Putting x=8+y, we get

$\Rightarrow (y+8)^2+y^2=544$

$\Rightarrow y^2+16y+64+y^2=544$

$\Rightarrow 2y^2+16y-480=0$

$\Rightarrow y^2+8y-240=0$

$\Rightarrow y^2+20y-12y-240-0$

$\Rightarrow y(y+20)-12(y+20)=0$

$\Rightarrow (y+20)(y-12)=0$

$\Rightarrow (y=-20)$ or $(y=12)$

Length cannot be negative. So, $y \neq -20$.

$\therefore$ y=12m

As, x=8+y

x=8+12

x=20

$\therefore x=20$

Hence, the sides of two squares are 20m and 12m.

Or

A motorboat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. 4 Marks

Ans: Let the speed of the stream be x km/h.

Given, speed of boat in still water is 18 km/h$^{18\text{ km/h}}$.

Speed of boat in downstream = speed of boat in still water + speed of the stream

$=(18+x)$ km/h

Speed of boat in upstream = speed of boat in still water - speed of the stream

$=(18-x)$ km/h

As given that motorboat takes 1 hour more to go upstream than to return downstream to the same spot.

So, we can write

$\Rightarrow$ Time taken for upstream = time taken for downstream +1

As $\text{Time}=\frac{\text{Distance}}{\text{Speed}}$, we can write

$$\Rightarrow \frac{\text{Distance covered in upstream}}{\text{Speed of upstream}}=\frac{\text{Distance covered in downstream}}{\text{Speed of downstream}}+1$$

$$\Rightarrow \frac{24}{18-x}=\frac{24}{18+x}+1$$

$$\Rightarrow 24(18+x)=24(18-x)+18(18-x)(18+x)$$

On simplifying we get,

$$\Rightarrow (24\times18)+24x=(24\times18)-24x+((18\times18)-s^2)$$

$$\Rightarrow x^2+48x-324=0$$

$$\Rightarrow x^2+54x-6x-324=0$$

$$\Rightarrow x(x+54)-6(x+54)=0$$

$$\Rightarrow (x+54)(x-6)=0$$

$\Rightarrow (x=-54)$ or $(x=6)$

Speed of stream cannot be negative. So, $x \neq -54$.

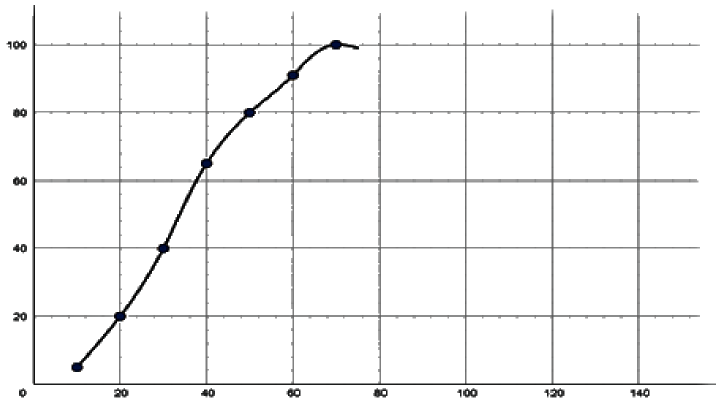
Therefore, speed of stream is 6 km/h.

36. For the following data, draw a 'less than' ogive and hence find the median of the distribution.

Age (in years)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of persons:	5	15	20	25	15	11	9

4 Marks

Ans: Plotting age (in years) on the x-axis and cumulative frequency on y-axis**.**



Age	Numbers of persons
Less than 10	5
Less than 20	20
Less than 30	40
Less than 40	65
Less than 50	80
Less than 60	91
Less than 70	100

Here, we have N=100

$\Rightarrow \frac{N}{2}=50$

From the curve we get x-ordinate as 33.5 when ordinate is 50.

Therefore, the median of the given distribution is 33.5.

Or

The distribution given below shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean and the median of the number of wickets taken.

Number of wickets:	**20-60**	**60-100**	**100-140**	**140-180**	**180-220**	**220-260**
Number of bowlers:	**7**	**5**	**16**	**12**	**2**	**3**

4 Marks

Ans: Assuming a=120 and h=40.

Number of wickets:	**Number of bowlers (f)**	**xi**	$u_i=\frac{(x_i-a)}{h}$	f_iu_i	**cf**
20-60	7	40	-2	-14	7
60-100	5	80	-1	-5	12
100-140	16	120	0	0	28
140-180	12	160	1	12	40
180-220	2	200	2	4	42
220-260	3	240	3	9	45

We get, $\sum f_i=40$ and $\sum f_iu_i=6$.

$$\text{Mean} = a+\frac{\sum f_iu_i}{\sum f_i}\times h$$

$$=120+\frac{6\times 40}{45}$$

$=125.33$

We have, N=45

$$\Rightarrow \frac{N}{2}=22.5$$

Therefore, Median class = 100-140, Cumulative frequency = 28, i = 100, cf=12, f=16 and h=40.

$$\text{Median}=l+\left(\frac{\frac{N}{2}\text{-cf}}{f}\right)\times h$$

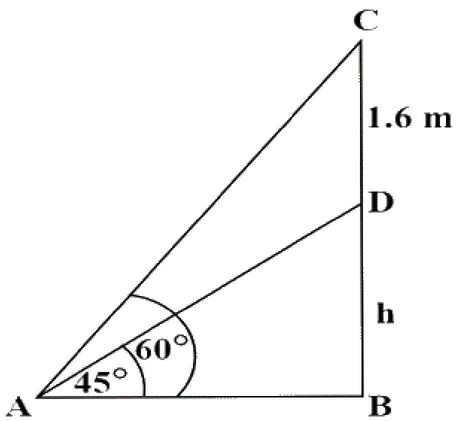
$$\Rightarrow \text{Median}=100+\left(\frac{22.5-12}{16}\right)\times 40$$

= 126.25

Therefore, the mean is 125.33 and the median is 126.25 of the number of wickets taken.

37. A statue 1.6 m tall, stand on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60o and from the same point the angle of elevation of the top of the pedestal is 45o. Find the height of the pedestal. (Use $\sqrt{3}$=1.73) 4 Marks

Ans: Let the height of pedestal be h metres.



In Δ ABD, we have

$$\Rightarrow \tan 45^\circ=\frac{BD}{AB}$$

$$\Rightarrow 1=\frac{h}{AB}$$

$$\Rightarrow AB=h\text{---}(1)$$

In ΔABC, we have

$$\Rightarrow \tan 60^{\circ}=\frac{BC}{AB}$$

$$\Rightarrow \sqrt{3}=\frac{BD+DC}{AB}$$

$$\Rightarrow \sqrt{3}=\frac{h+1.6}{AB}$$

$$\Rightarrow AB=\frac{h+1.6}{\sqrt{3}} \text{---}(2)$$

From (1) and (2), we get

$$\Rightarrow h=\frac{h+1.6}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}h=h+1.6$$

$$\Rightarrow \left(\sqrt{3}-1\right)h=1.6$$

$$\Rightarrow h=\frac{1.6}{\left(\sqrt{3}-1\right)}$$

$$\Rightarrow h=\frac{1.6}{(1.73-1)}$$

$$\Rightarrow h=2.19$$

Therefore, the height of the pedestal is 2.19 m.

38. Obtain other zeroes of the polynomial

$p(x)=2x^4-x^3-11x^2+5x+5$ if two zeroes are $\sqrt{5}$ and $-\sqrt{5}$. **4 Marks**

Ans: Two zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

So, we can write, $x=\sqrt{5}$ and $x=-\sqrt{5}$.

We get, $x-\sqrt{5}$ and $x+\sqrt{5}=0$.

Multiplying both the factors we get,

$\Rightarrow\left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)=0$

$\Rightarrow x^2-5=0$

x^2-5 is a factor of $p\left(x\right)=2x^4-x^3-11x^2+5x+5$.

Dividing $2x^4-x^3-11x^2+5x+5$ by x^2-5, we get the quotient as $2x^2-x-1$.

On factorising $2x^2-x-1$, we get

$\Rightarrow 2x^2-2x+x-1=0$

$\Rightarrow 2x\left(x-1\right)+1\left(x-1\right)=0$

$\Rightarrow\left(x-1\right)\left(2x+1\right)=0$

$\Rightarrow x=1,\ x=-\frac{1}{2}$

Therefore, other two zeroes of the polynomial are 1 and $-\frac{1}{2}$.

Or

What minimum must be added to $2x^3-3x^2+6x+7$ so that the resulting polynomial will be divisible by x^2-4x+8? 4 Marks

Ans:

$$
\begin{array}{r|l}
 & 2x+5 \\
\hline
x^2-4x+8 & 2x^3-3x^3+6x+7 \\
 & \underline{2x^3-8x+16x} \\
 & 5x^2-10x+7 \\
 & \underline{5x^2-20x+40} \\
 & 10x-33
\end{array}
$$

For $2x^3-3x^3+6x+7$ to be divisible by x^2-4x+8, remainder should be zero when we divide $2x^3-3x^3+6x+7$ by x^2-4x+8.

Dividing $2x^3-3x^3+6x+7$ by x^2-4x+8, we get the remainder as 10x-33.

Therefore, we have to add –(10x-33) i.e., 33-10x so that the resulting polynomial will be divisible by x2-4x+8.

39. In a cylindrical vessel of radius 10 cm, containing some water, 9000

small spherical balls are dropped which are completely immersed in water which raises the water level. If each spherical ball is of radius 0.5 cm, then find the rise in the level of water in the vessel. 4 Marks

Ans: Given, Radius of spherical balls =0.5 cm

Radius of cylindrical vessel =10 cm

As we know, Volume of a sphere $=\frac{4}{3}\times\pi\times(\text{radius})^3$

$\Rightarrow$Volume of 9000 spherical balls $=9000\times\frac{4}{3}\times\pi\times(\text{radius of spherical ball})^3$

$\Rightarrow$Volume of 9000 spherical balls $=9000\times\frac{4}{3}\times\pi\times(0.5)^3$

$\Rightarrow$Volume of cylinder $=\pi\times(\text{radius of cylinder})^2\times(\text{rise in the level})$

Now, Volume of cylinder = Volume of 9000 spherical balls

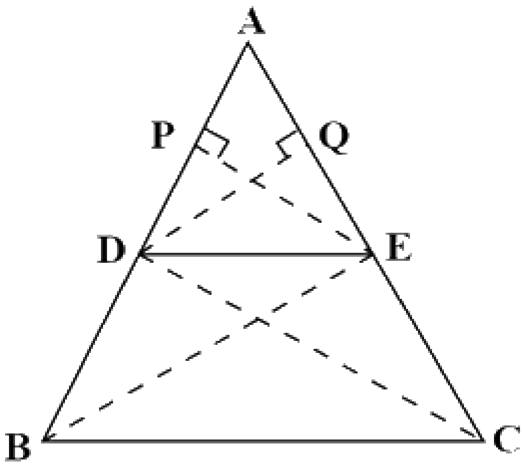
$\Rightarrow \pi\times(10)^2\times(\text{rise in the level})=9000\times\frac{4}{3}\times\pi\times(0.5)^3$

$\Rightarrow$Rise in the level = 15 cm

Therefore, the rise in the level of water in the vessel is 15 cm.

40. If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, prove that the other two sides are divided in the same ratio. 4 Marks

Ans: Consider a ΔABC in which DE is drawn parallel to BC which intersects the side AB and AC at D and E respectively.



We have to prove $\frac{AD}{DB}=\frac{AE}{EC}$.

Construction: Join BE and CD and draw $DQ \perp AC$ and $EP \perp AC$.

$\text{Area of } \Delta ADE=\frac{1}{2}\times \text{base}\times \text{height}$

$\Rightarrow \text{Area of } \Delta ADE=\frac{1}{2}\times AD\times EP$

Also,

$\Rightarrow \text{Area of } \Delta ADE=\frac{1}{2}\times AE\times DQ$

Similarly,

$\Rightarrow \text{Area of } \Delta BDE=\frac{1}{2}\times BD\times EP$

$\Rightarrow \text{Area of } \Delta DEC=\frac{1}{2}\times EC\times DQ$

On taking ratio we get,

$$\Rightarrow \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE}=\frac{\frac{1}{2}\times AD\times EP}{\frac{1}{2}\times BD\times EP}$$

$$\Rightarrow \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE}=\frac{AD}{BD}\text{----}(1)$$

Similarly,

$$\Rightarrow \frac{\text{Area of }\Delta\text{ADE}}{\text{Area of }\Delta\text{DEC}}=\frac{\frac{1}{2}\times\text{AE}\times\text{DQ}}{\frac{1}{2}\times\text{EC}\times\text{DQ}}$$

$$\Rightarrow \frac{\text{Area of }\Delta\text{ADE}}{\text{Area of }\Delta\text{DEC}}=\frac{\text{AE}}{\text{EC}}---(2)$$

Also, in ΔBDE and ΔDEC has the same base DE and between the same parallel lines BC and DE.

So, Area of ΔBDE = Area of ΔDEC ----(3)

Therefore, from equation (1), (2), and (3) we get

$$\Rightarrow \frac{\text{AD}}{\text{DB}}=\frac{\text{AE}}{\text{EC}}$$

Hence proved that if a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then the other two sides are divided in the same ratio.