Previous Year Question Paper 2019

GENERAL INSTRUCTIONS :

(i) All questions are compulsory.
(ii) This question paper consists of 30 questions divided into four ***sections - A, B, C*** *and* ***D****.*
(iii) ***Section A*** *contains 6 questions of 1 mark each.*
Section B *contains 6 questions of 2 marks each.*
Section C *contains 10 questions of 3 marks each.*
Section D *contains 8 questions of 4 marks each.*
(iv) There is no overall choice. However, an internal choice has been provided in two questions of 1 mark, two questions of 2 marks, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculator is not permitted.

Section A

Question numbers 1 to 6 carry 1 mark each.

1. Find the coordinates of a point A, where AB is diameter of a circle whose centre is $(2,-3)$ and B is the point $(1,4)$.

Solution: Let the centre be O and coordinates of point A be (x, y)

$\frac{x+1}{2}=2$ [By Mid-point formula]

Implies that

$x=3$

$\frac{y+4}{2}=-3$

$\therefore$ Coordinates of $A=(3,-10)$

2. For what values of k, the roots of the equation $x^2+4x+k=0$ are real?

OR

Find the value of k for which the roots of the equation $3x^2-10x+k=0$ are reciprocal of each other.

Solution:

$x^2+4x+k=0$

$\therefore$ Roots of given equation are real,

$D\geq 0$

Implies that

$(4)^2 - 4 \times k \geq 0$

Implies that

$-4k \geq -16$

Implies that

$k \leq 4$

$\therefore$ k has all real values ≤ 4

OR

$3x^2 - 10x + k = 0$

$\because$ Roots of given equation are reciprocal of each other.

Let the roots be α and $\frac{1}{\alpha}$

Product of roots $= \frac{\text{c}}{\text{a}}$

Implies that

$\alpha, \frac{1}{\alpha} = \frac{k}{3}$

$\therefore$ $k = 3$.

3. Find A if $\tan\ 2A = \cot(A - 24°)$

OR

Find the value of $(\sin^2 33° + \sin^2 57°)$

Solution:

Given:

$\tan 2A = \cot(A - 24°)$

Implies that $\tan 2A = \tan[90° - (A - 24°)]$

Implies that $\tan 2A = \tan[90° - A + 24°]$

Implies that $\tan 2A = \tan[114° - A]$

Implies that $2A = 114° - A$

Implies that $3A = 114°$

Implies that $A = \frac{114°}{3}$

Implies that $A = 38°$

OR

Given:

$\sin^2 33° + \sin^2 57°$

$= \sin^2 33° + [\cos(90° - 57°)]^2$

$= \sin^2 33° + \cos^2 33°$

$= 1$

4. How many two digits numbers are divisible by 3?

Solution:

Two digits numbers divisible by 3 are

$12, 15, 18, \ldots, 99.$

$a = 12, d = 15 - 12 = 3$

Implies that

$T_n = 99$

Implies that

$a + (n-1)d = 99$

Implies that

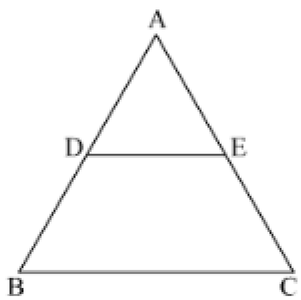
$12 + (n-1)3 = 99$

Implies that

$n = 30$

Number of two digit numbers divisible by 3 are 30.

5.In Fig. 1, $DE \parallel BC, AD = 1\ cm$ and $BD = 2\ cm$. What is the ratio of the ar (ΔABC) to the ar (ΔADE)?



Solution:

$DE \parallel BC$

$\Delta ADE \sim \Delta ABC$ [By AA similarity]

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{AB^2}{AD^2}$ [By area similarity theorem]

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{3^2}{1^2}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{9}{1}$

6. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Sol. Rational number lying between $\sqrt{2}$ and $\sqrt{3}$ is $1.5 = \frac{15}{10} = \frac{3}{2}$

$\left[\because \sqrt{2} \sim 1.414 \text{ and } \sqrt{3} \sim 1.732\right]$

Section B

7. Find the HCF of 1260 and 7344 using Euclid's algorithm.

OR

Show that every positive odd integer is of the form $(4q+1)$ or $(4q+3)$, where q is some integer.

Since 7344 > 1260

$7344 = 1260 \times 5 + 1044$

Since remainder $\neq 0$

$1260 = 1044 \times 1 + 216$

$1044 = 216 \times 4 + 180$

$216 = 180 \times 1 + 36$

$180 = 36 \times 5 + 0$

The remainder has now become zero.

$\therefore$ HCF of 1260 and 7344 is 36.

OR

Let a be positive odd integer

Using division algorithm on a and b = 4

$a = 4q + r$ Since $0 \leq r < 4$, the possible remainders are 0, 1, 2 and 3

$\therefore$ a can be 4q or 4q + 1 or 4q + 2 or 4q + 3, where q is the quotient

Since a is odd, a cannot be 4q and 4q + 2

$\therefore$ Any odd integer is of the form 4q + 1 or 4q + 3, where q is some integer.

8. Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

OR

If S_n, the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the nth term.

Solution:

Given AP is

3, 15, 27, 39 ....

where a = 3, d = 15 – 3 = 12

Let the nth term be 120 more than its 21st term.

$t_n = t_{21} + 120$

$\Rightarrow 3 + (n-1)12 = 3 + 20 \times 12 + 120$

$\Rightarrow (n-1) \times 12 = 363 - 3$

$\Rightarrow (n-1) = \frac{360}{12}$

$\therefore n = 31$

Hence, the required term is

$t_{31} = 3 + 30 \times 12$

$= 363$

OR

$S_n = 3n^2 - 4n$

Let S_{n-1} be sum of $(n-1)$ terms

$t_n = S_n - S_{n-1}$

$= (3n^2 - 4n) - \left[3(n-1)^2 - 4(n-1)\right]$

$= (3n^2 - 4n) - \left[3n^2 - 6n + 3 - 4n + 4\right]$

$= 3n^2 - 4n - 3n^2 + 10n - 7$

$\therefore t_n = 6n - 7$

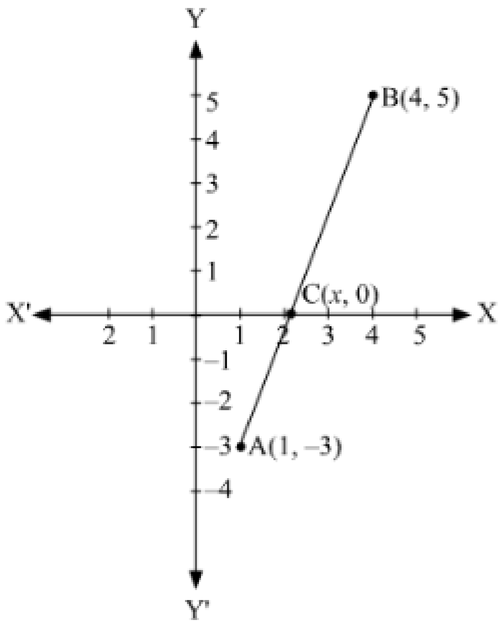
Therefore, required nth term $= 6n - 7$

9. Find the ratio in which the segment joining the points (1, –3) and (4, 5) is divided by x-axis? Also find the coordinates of this point on x-axis.

Solution:

Let $\mathrm{C}(x,0)$ divides the line segment joining the points $\mathrm{A}(1,-3)$ and $\mathrm{B}(4,5)$ in $k:1$ ratio.

By section formula,



$(x,y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

Implies that

$(x,0) = \left(\frac{4k + 1\times1}{k+1}, \frac{5k + 1\times(-3)}{k+1}\right)$

Implies that

$(x,0) = \left(\frac{4k+1}{k+1}, \frac{5k-3}{k+1}\right)$

Implies that

$\frac{5k-3}{k+1} = 0$

Implies that

$5k - 3 = 0$

Implies that

$5k=3$

$k=\frac{3}{5}$

and $x=\frac{4k+1}{k+1}=\frac{4\times\frac{3}{5}+1}{\frac{3}{5}+1}$

$\Rightarrow x=\frac{\frac{12+5}{5}}{\frac{3+5}{5}}$

$\Rightarrow x=\frac{17}{8}$

Therefore, coordinates of point P are $\left(\frac{17}{8},0\right)$.

10. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

Solution:

Total possible outcomes are (HHH), (HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT) i.e., 8. [½]
The favourable outcomes to the event E 'Same result in all the tosses' are TTT, HHH. Therefore, the number of favourable outcomes = 2

$\because \mathrm{P}(\mathrm{E})=\frac{2}{8}=\frac{1}{4}$

Hence, probability of losing the game $=1-\mathrm{P}(\mathrm{E})$

$=1-\frac{1}{4}=\frac{3}{4}$

11. A die is thrown once. Find the probability of getting a number which (i) is a prime number (ii) lies between 2 and 6.

Solution:

Total outcomes = 1, 2, 3, 4, 5, 6 Prime numbers = 2, 3, 5 Numbers lie between 2 and 6 = 3, 4, 5

(i) P (Prime Numbers) $=\frac{3}{6}=\frac{1}{2}$

(ii) P (Numbers lie between 2 and 6) $=\frac{3}{6}=\frac{1}{2}$

12. Find c if the system of equations $cx+3y+(3-c)=0, 12x+cy-c=0$ **has infinitely many solutions?**

Solution:

$cx+3y+(3-c)=0$ ....(i)

$12x+cy-c=0$ ....(ii)

For infinitely many solutions $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$\Rightarrow \frac{c}{12}=\frac{3}{c}=\frac{3-c}{-c}$

$\Rightarrow \frac{c}{12}=\frac{3}{c}$ or

$\Rightarrow c^2=36$

$\Rightarrow c=\pm 6$

$\frac{3}{c}=\frac{3-c}{-c}$

$\Rightarrow c(c-6)=0$

$\Rightarrow c=0,6$

Hence the value of $c=6$.

Section C

Question numbers 13 to 22 carry 3 marks each.

13. Prove that $\sqrt{2}$ is an irrational number.

Solution:

Let $\sqrt{2}$ be rational.

$\therefore \sqrt{2}=\frac{p}{q}$ where p and q are co-prime integers and , q ≠0

Implies that $\sqrt{2}q=p$

$2q^2=p^2.......(\text{i})$

$\Rightarrow 2 \text{ divides } p^2$

$\Rightarrow 2 \text{ divides } p.......(\text{A})$

Let $p=2c$ for some integer c

$p^2=4c^2$

$\Rightarrow 2q^2=4c^2$

$\Rightarrow q^2=2c^2$

$\Rightarrow 2 \text{ divides } q^2$

$\Rightarrow 2 \text{ divides } q........(\text{B})$

From (A) and (B), we get

$\therefore$ 2 is common factor of both p and q. But this contradicts the fact that p and q have no common factor other than 1

$\therefore$ Our supposition is wrong Hence, $\sqrt{2}$ is an irrational number.

14. Find the value of k such that the polynomial $x^2-(k+6)x+2(2k-1)$ has sum of its zeros equal to half of their product.

Solution:

For given polynomial $x^2-(k+6)x+2(2k-1)$

Here

$\text{a}=1, b=-(k=6), c=2(2k-1)$

Given that:

$\because$ Sum of zeroes $= \frac{1}{2}$ (product of zeroes)

$\Rightarrow \frac{-[-(k+6)]}{1} = \frac{1}{2} \times \frac{2(2k-1)}{1}$

$\Rightarrow k+6 = 2k-1$

$\Rightarrow 6+1 = 2k-k$

$\Rightarrow k = 7$

Therefore, the value of $k = 7$

15. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

OR

A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction.

Solution:

Let the sum of ages of two sons be x years

Age of man $= 3x$ years

After 5 years age of the man $= (3x+5)$ years

Sum of ages of two sons $= (x+10)$ years

Given, $(3x+5) = 2(x+10)$

$\Rightarrow (3x+5) = 2x+20$

$\Rightarrow x = 15$

Hence $3x = 3(15) = 45$

Thus the age of the man(father) is 45 years.

OR

Let's assume the fraction be $\frac{x}{y}$

First condition:

$\frac{x-2}{y} = \frac{1}{3}$

$\Rightarrow 3x-6 = y$

$\Rightarrow 3x-y = 6 \ldots\ldots\ldots\ldots(1)$

Second condition:

$\frac{x}{y-1} = \frac{1}{2}$

$\Rightarrow 2x = y-1$

$\Rightarrow 2x-y = -1 \ldots\ldots\ldots\ldots(2)$

Using eliminated method:

Multiplying (2) by -1 and then adding (1) and (2)

$\Rightarrow 3x - y = 6$

$\Rightarrow -2x + y = 1$

$\Rightarrow x = 7$

Now,from (1),

$\Rightarrow 3x - y = 6$

$\Rightarrow 3(7) - y = 6$

$\Rightarrow 21 - y = 6$

$\Rightarrow y = 15$

Hence the required fraction is $\frac{7}{15}$.

16. Find the point on y-axis which is equidistant from the points (5, – 2) and (– 3, 2).

OR

The line segment joining the points A(2, 1) and B(5, – 8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by 2x – y + k = 0, find the value of k.

Solution:

Since the point is on y-axis so,X-coordinate is zero

Let the point be (0, y)

It's distance from A(5, –2) and B(– 3, 2) are equal

$\therefore \sqrt{(0-5)^2 + (y+2)^2} = \sqrt{(0+3)^2 + (y-2)^2}$

$\Rightarrow 25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4 \quad [\text{squaring both sides}]$

$\Rightarrow 4y + 29 = -4y + 13$

$\Rightarrow 4y + 4y = 13 - 29$

$\Rightarrow 8y = -16 \therefore y = \frac{-16}{8} = -2$

Thus, the point is $(0, -2)$.

OR

As line segment AB is trisected by the points P and Q. Therefore,

Case I: When AP : PB = 1 : 2.

Then, coordinates of P are

$\left\{ \frac{1 \times 5 + 2 \times 2}{1+2}, \frac{1 \times (-8) + 1 \times 2}{1+2} \right\}$

Implies that

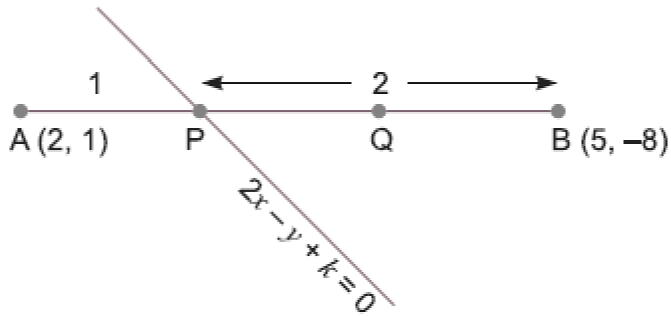
$P(3, -2)$

Since the point $P(3, -2)$ lies on the line

$2x - y + k = 0$

$\Rightarrow 2 \times 3 - (-2) + k = 0$

$\Rightarrow k = -8$



17. Prove that : $(\sin\theta + \cos ec\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

OR

Prove that: $(1 + \cot A - \cos ecA)(1 + \tan A + \sec A) = 2.$

Solution:

$\text{L.H.S}: (\sin\theta + \cos ec\theta)^2 + (\cos\theta + \sec\theta)^2$

$= sin^2\theta + cosec^2\theta + 2 + cos^2\theta + sec^2\theta + 2 \qquad \left[\because \sin\theta = \frac{1}{\cos ec\theta} \text{ and } \cos\theta = \frac{1}{\sec\theta}\right]$

$= \sin^2\theta + \cos^2\theta + 1 + \cot^2\theta + 1 + \tan^2\theta + 4 \qquad \left[\because \cos ec^2\theta + 1 + \cot^2\theta \text{ and } \sec^2\theta = 1 + \tan^2\theta\right]$

$= \sin^2\theta + \cos^2\theta + 1 + \cot^2\theta + 1 + \tan^2\theta + 4 \qquad \left[\because \cos ec^2\theta + 1 + \cot^2\theta \; and \; \sec^2\theta = 1 + \tan^2\theta\right]$

$= 1 + 1 + 1 + 4 + \tan^2\theta + \cot^2\theta \qquad \left[\because \cos^2\theta + \sin^2\theta = 1\right]$

$= 7 + \tan^2\theta + \cot^2\theta$

L.H.S=R.H.S

OR

$\text{L.H.S}: \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$

$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$

$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cdot \cos A}$

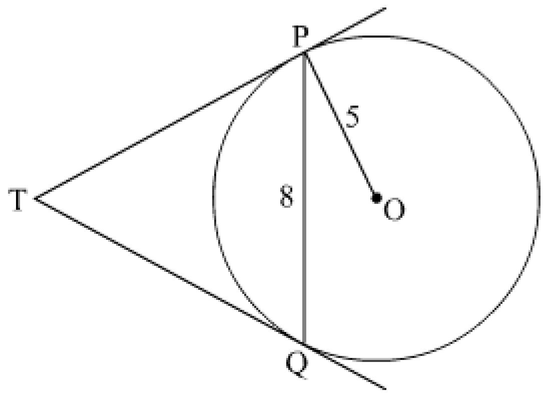
$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cdot \cos A - 1}{\sin A \cdot \cos A}$

$= \frac{1 + 2\sin A \cdot \cos A - 1}{\sin A \cdot \cos A}$

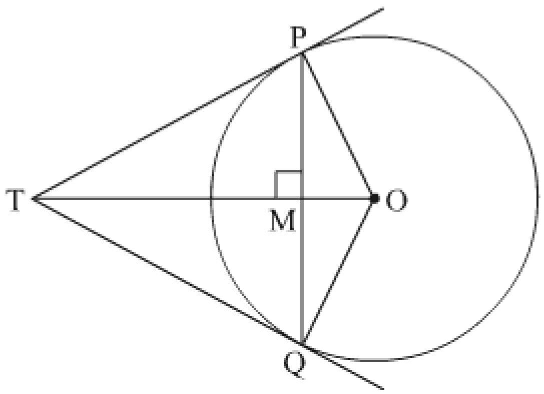
$= 2$

Hence, L.H.S=R.H.S.

18.In Fig. , PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.



Solution:



Given radius, $OP = OQ = 5\text{ cm}$

Length of chord, $PQ = 4\text{ cm}$

$OT \perp PQ$,

$\therefore\ PM = MQ = 4\text{ cm}$ [Perpendicular draw from the centre of the circle to a chord bisect the chord]

In right ΔOPM,

$OP^2 = PM^2 + OM^2$

$\Rightarrow 5^2 = 4^2 + OM^2$

$\Rightarrow OM^2 = 25 - 16 = 9$

Hence $OM = 3\text{cm}$

In right ΔPTM,

$PT^2 = TM^2 + PM^2 \rightarrow (1)$

$\angle OPT = 90^o$ [Radius is perpendicular to tangent at point of contact]

In right ΔOPT,

$OT^2 = PT^2 + OP^2 \rightarrow (2)$

From equations (1) and (2), we get

$OT^2 = \left(TM^2 + PM^2\right) + OP^2$

$\Rightarrow \left(TM + OM\right)^2 = \left(TM^2 + PM^2\right) + OP^2$

$\Rightarrow TM^2 + OM^2 + 2\times TM\times OM = TM^2 + PM^2 + OP^2$

$\Rightarrow OM^2 + 2\times TM\times OM = PM^2 + OP^2$

$\Rightarrow 3^2 + 2\times TM\times 3 = 4^2 + 5^2$

$\Rightarrow 9 + 6TM = 16 + 25$

$\Rightarrow 6TM = 32$

$\Rightarrow TM = \frac{32}{6} = \frac{16}{3}$

Equation (1) becomes,

$PT^2 = TM^2 + PM^2$

$= \left(\frac{16}{3}\right)^2 + 4^2$

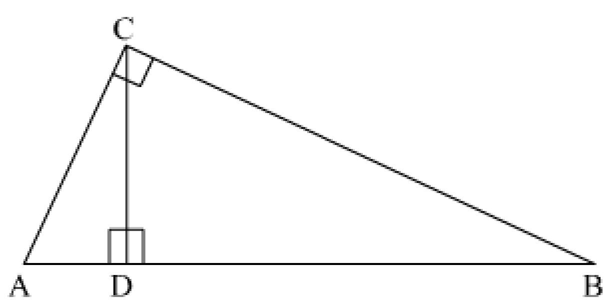
$= \left(\frac{256}{9}\right) + 16 = \frac{(256+144)}{9}$

$= \left(\frac{400}{9}\right) = \left(\frac{20}{3}\right)^2$

Hence $PT = \frac{20}{3}$

Thus, the length of tangent PT is $\frac{20}{3}$ cm.

19.In Fig. 3, ∠ACB = 90° and CD ⊥ AB, prove that CD2 = BD × AD. [3] A B C D Fig. 3



OR

If P and Q are the points on side CA and CB respectively of Δ ABC, right angled at C, prove that (AQ2 + BP2) = (AB2 + PQ2)

Solution:

Given that : $CD \perp AB$

$\angle ACB = 90^\circ$

To Prove : $CD^2 = BD\times AD$

Using Pythagoras Theorem in ΔACD

$AC^2 = AD^2 + CD^2 ......(1)$

Using Pythagoras Theorem in ΔCDB

$CB^2 = BD^2 + CD^2 ......(2)$

Similarly in ΔABC,

$AB^2 = AC^2 + BC^2 .....(3)$

As AB = AD + DB

Since, AB=AD+BD.....(4)

Squaring both sides of equation (4), we get

$(AB)^2 = (AD+BD)^2$

Since, $AB^2 = AD^2 + BD^2 + 2 \times BD \times AD$

From equation (3) we get

$AC^2 + BC^2 = AD^2 + BD^2 + 2 \times BD \times AD$

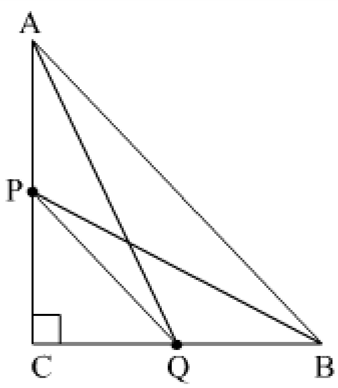
Substituting the value of AC^2 from equation (1) and the value of BC^2 from equation (2), we get

$AD^2 + CD^2 + BD^2 + CD^2 = AD^2 + BD^2 + 2 \times BD \times AD$

Since, $2\ CD^2 = 2 \times BD \times AD$

Hence, $CD^2 = BD \times AD$

OR



Using the Pythagoras theorem in ΔABC,
ΔACQ, ΔBPC, ΔPCQ, we get

$AB^2 = AC^2 + BC^2 .....(1)$

$AQ^2 = AC^2 + CQ^2 ..... (2)$

$BP^2 = PC^2 + BC^2 ..... (3)$

$PQ^2 = PC^2 + CO^2 ..... (4)$

Adding the equations (2) and (3) we get

$AQ^2 + BP^2 = AC^2 + CQ^2 + PC^2 + BC^2$

$= (AC^2 + BC^2) + (CQ^2 + PC^2)$

$= AB^2 + PQ^2$

As L.H.S $= AQ^2 + BP^2$

$= AB^2 + PQ^2$ = R.H.S

20. Find the area of the shaded region in Fig. 4, if ABCD is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take π = 3.14)

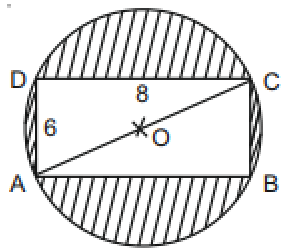


Fig. 4

Solution:

Here, diagonal AC also represents the diameter of the circle.

Using Pythagoras theorem:

$AC = \sqrt{AB^2 + BC^2}$

$AC = \sqrt{8^2 + 6^2}$

$AC = \sqrt{64 + 36}$

$AC = \sqrt{100}$

$AC = 10$

$\therefore$ Radius of the circle, $\text{OC} = \frac{\text{AC}}{2} = 5\text{cm}$

Area of the shaded region = Area of the circle – Area of rectangle

$= \pi r^2 - AB \times BC$

$= \pi (OC)^2 - AB \times BC$

$= 3.14 \times 5^2 - 8 \times 6$

$= 78.5 - 48$

$= 30.5$

Therefore,the area of shaded region is 30.5cm^2.

21. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed?

Solution: Width of the canal = 6 m

Depth of the canal = 1.5 m

Length of the water column formed in $\frac{1}{2}$ hr

= 5 km or 5000 m

$\therefore$ Volume of water flowing in $\frac{1}{2}$ hr

= Volume of cuboid of length 5000 m, width 6 m and depth 1.5 m.

= 5000 × 6 × 1.5 = 45000 m^3

On comparing the volumes,

Volume of water in field = Volume of water coming out from canal in 30 minutes.

Irrigated area × standing water = 45000.

Irrigated Area $\frac{45000}{\frac{8}{100}}$ $\quad [\because 1\text{ m}=100\text{ cm}]$

$\frac{45000\times 100}{8}=5,62,500\text{m}^3$

22. Find the mode of the following frequency distribution.

Class	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**
Frequency	**8**	**10**	**10**	**16**	**12**	**6**	**7**

Solution:

Class	Frequency
0-10	8
10-20	10
20-30	$10\to f_0$
30-40	$16\to f_1$
40-50	$12\to f_2$
50-60	6
60-70	7

Here, 30 - 40 is the modal class, and I = 30, h = 10

$$\therefore \text{Mode}=I+\left(\frac{f_0-f_1}{2f_1-f_0-f_2}\right)\times h$$

$$=30+\left(\frac{16-10}{2\times 16-10-12}\right)\times 10$$

$$=30+\frac{16}{10}\times 10=30+6=36$$

Section D

23. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

OR

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream.

Determine the speed of the stream and that of the boat in still water.

Solution:

Let the time in which tap with longer and smaller diameter can fill the tank separately be x hours and y hours respectively.

$\frac{1}{x}+\frac{1}{y}=\frac{8}{15}$ ....(i)

And $x=y-2$ ....(ii)

On substituting $x=y-2$ from (ii) in (i), we get

$\frac{1}{y-2}+\frac{1}{y}=\frac{8}{15}$

$\Rightarrow \frac{y+y-2}{y^2-2y}=\frac{8}{15}$

$\Rightarrow 15(2y-2)=8(y^2-2y)$

$\Rightarrow 30y-30=8y^2-16y$

$\Rightarrow 8y^2-46y+30=0$

$\Rightarrow 4y^2-20y-3y+15=0$

$\Rightarrow (4y-3)(y-5)=0$

$\Rightarrow y=\frac{3}{4}, y=5$

Substituting values of y in (ii), we get

$x=\frac{3}{4}-2$

$x=\frac{-5}{4}$

$\therefore x\neq\frac{-5}{4}$

(time cannot be negative)

Hence, the time taken by tap with longer diameter is 3 hours and the time taken by tap with smaller diameter is 5 hours, in order to fill the tank separately

OR

Let the speed of boat is $x\,\text{km/h}$ in still water

And stream $y\,\text{km/h}$

According to question,

$\frac{30}{x-y}+\frac{44}{x+y}=10$

And

$\frac{40}{x-y}+\frac{55}{x+y}=13$

Let $\frac{1}{x-y}=u$ and $\frac{1}{x+y}=v$

$30u+44v=10.....(i)$

$40u+55v=13....(ii)$

On solving equation (i) and (ii) we get,

$u=\frac{1}{5}\Rightarrow x-y=5 \quad \rightarrow \quad \text{(iii)}$

$\text{v}=\frac{1}{11}\Rightarrow x+y=11 \quad \rightarrow \quad \text{(iv)}$

On solving equation (iii) and (iv) we get,

$x=8\,\text{km/h}$

$y=3\text{km/h}$

24. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

Solution:

Given that: $S_4=40$ and $S_{14}=280$

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$S_4=\frac{4}{2}\left[2a+(4-1)d\right]=40$

$\Rightarrow 2a+3d=20.........(i)$

$S_{14}=\frac{14}{2}\left[2a+(14-1)d\right]=280$

$\Rightarrow 2a+13d=40.........(ii)$

$(\text{ii})\text{-}(\text{i}),$

$10d=20\Rightarrow d=2$

Substituting the value of d in (i),we get

$2a+6=20\Rightarrow a=7$

Sum of first n terms,

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$=\frac{n}{2}\left[14+(n-1)2\right]$

$=n(7+n-1)$

$=n(n+6)$

$=n^2+6n$

Therefore, $S_n=n^2+6n$

$\text{L.H.S}=\frac{\sin A-\cos A+1}{\sin A+\cos A-1}$

$=\frac{\tan A-1+\sec A}{\tan A+1-\sec A}$ (Dividing numerator & denominator by $\cos A$)

$=\frac{(\tan A+\sec A)-1}{(\tan A-\sec A)+1}$

$$= \frac{\{(\tan A + \sec A) - 1\}(\tan A - \sec A)}{(\tan A - \sec A) + 1(\tan A - \sec A)}$$

$$= \frac{(\tan^2 A + \sec^2 A) - (\tan A - \sec A)}{\{\tan A - \sec A + 1\}(\tan A - \sec A)}$$

$$= \frac{-1 - \tan A + \sec A}{\{\tan A - \sec A + 1\}(\tan A - \sec A)}$$

$$= \frac{-1}{\tan A - \sec A}$$

$$= \frac{1}{\sec A - \tan A}$$

LHS=RHS

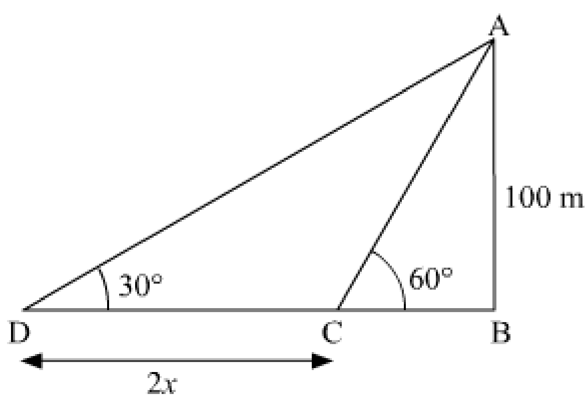
Hence proved.

26. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. $\left[\text{Use} \sqrt{3} = 1.732\right]$

OR

Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Solution:



AB is a lighthouse of height 100m. Let the speed of boat be x metres per minute. And CD is the distance which man travelled to change the angle of elevation.

Therefore,

$\text{CD} = 2x \; [\because \text{Distance=Speed} \times \text{Time}]$

$\tan(60°) = \frac{AB}{BC}$

$\sqrt{3} = \frac{100}{BC}$

$\Rightarrow BC = \frac{100}{\sqrt{3}}$

$\tan(30°) = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BD}$

$BD = 100\sqrt{3}$

CD=BD-BC

$2x = 100\sqrt{3} - \frac{100}{\sqrt{3}}$

$2x = \frac{300-100}{\sqrt{3}}$

$\Rightarrow x = \frac{200}{2\sqrt{3}}$

$\Rightarrow x = \frac{100}{\sqrt{3}}$

Using,

$\sqrt{3} = 1.73$

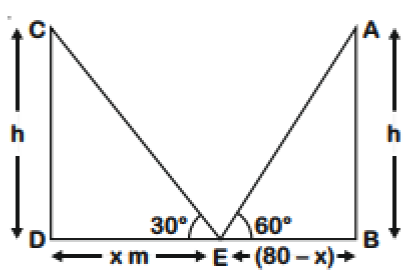
$x = \frac{100}{1.73} \approx 57.80$

Hence,the speed of the boat is 57.80 meters per minute.

OR

Let the poles be AB, CD each of height h meter and E is the point between the poles on the road.

Let ∠AEB = 60°, ∠CED = 30° and DE be x meter.



$\therefore BE = (80 - x)$m

In ΔAEB,

$$\tan 60^\circ = \frac{AB}{BE}$$

$$\Rightarrow \sqrt{3} = \frac{h}{(20-x)}$$

$$\Rightarrow h = \sqrt{3}(80-x)\,\text{m}..........\text{(i)}$$

In ΔCDE,

$$\tan 30^\circ = \frac{CD}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow h = \frac{x}{\sqrt{3}}\,\text{m}........\text{(ii)}$$

From equation (i) and (ii), we get

$$\frac{x}{\sqrt{3}} = \sqrt{3}(80-x)$$

$$\Rightarrow x = 240 - 3x$$

$$\Rightarrow 4x = 240$$

$$\Rightarrow x = 60\ m$$

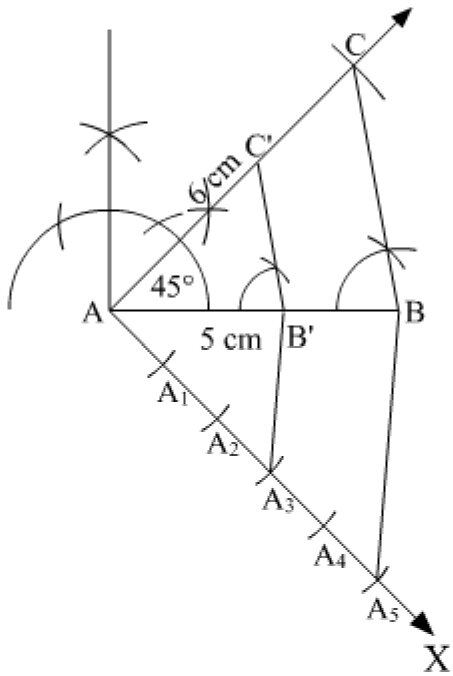
Put value of x in equation (ii), we get

$h = 20\sqrt{3}\text{m}$, DE=60m and BE=20m

Hence, the heights of each pole is $20\sqrt{3}\text{m}$ and distance of the point from the poles are 60 m and 20 m.

27. Construct a ΔABC in which CA = 6 cm, AB = 5 cm and ∠BAC = 45º. Then construct a triangle whose sides are 3 5 of the corresponding sides of ΔABC.

Solution:



Steps of construction:
1. Draw AB = 5 cm. With A as centre, draw $\angle BAC = 45°$. Join BC. $\angle ABC$ is thus formed.
2. Draw AX such that $\angle BAX$ is an acute angle.
3. Cut 5 equal arcs $AA_1, A_1A_2, A_2A_3, A_3A_4$ and A_4A_5.
4. Join A_5 to B and draw a line through A_3 parallel to A_5B which meets AB at B'.

Here, AB' = $\frac{3}{5}$AB

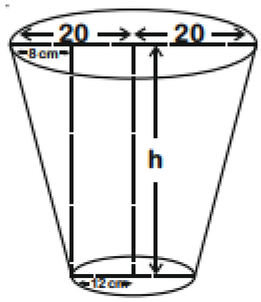
5.Now draw a line through B' parallel to BC which joins AC at C'.

Here, B'C' = $\frac{3}{5}$BC and AC'=$\frac{3}{5}$AC

Thus, AB'C' is the required triangle.

28. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm3. The radii of the top and bottom of circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use π = 3.14)

Solution:



Let the height of the bucket be h cm and slant height be l cm.
Here

$r_1 = 20\ cm$

$r_2 = 12\ cm$

And capacity of bucket = 12308.8 cm3

We know that capacity of bucket $= \frac{\pi h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= 3.14 \times \frac{h}{3}(400 + 144 + 240)$

$= 3.14 \times \frac{h}{3} \times 784$

So we have $= 3.14 \times \frac{h}{3} \times 784 = 12308.8$

$h = \frac{12308.8 \times 3}{3.14 \times 784}$

$= 15\,\text{cm}$

Now, the slant height of the frustum,

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{15^2 + 8^2}$

$= \sqrt{289}$

$= 17\,\text{cm}$

Area of metal sheet used in making it

$= \pi r_2^2 + \pi (r_1 + r_2)^2$

$= 3.174 \times [144 + (20 + 12) \times 17]$

$= 2160.32\,\text{cm}^2$

29. Prove that in a right angle triangle, the square of the hypotenuse is equal the sum of squares of the other two sides.

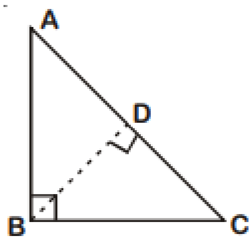
Solution:

Given : A right triangle ABC in which ∠B = 90°

To Prove: $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e. $AC^2 = AB^2 + BC^2$

Construction : From B, draw $BD \perp AC$



In ΔABC and ΔADB

$\angle BAC = \angle DAB \quad [\text{Common}]$

$\angle ABC = \angle ADB \quad [\text{Each } 90^\circ]$

$\therefore \ \Delta ABC \sim \Delta ADB \quad [\text{By AA similarity}]$

$\Rightarrow \dfrac{AB}{AC} = \dfrac{AD}{AB}$

$\Rightarrow AB^2 = AD \times AC \ \ldots(i)$

Similarly, $\Delta ABC \sim \Delta BDC$

$\Rightarrow \dfrac{BC}{DC} = \dfrac{AC}{BC}$

$\Rightarrow BC^2 = AC \times DC \ \ldots(ii)$

On Adding (i) and (ii), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

$\Rightarrow AB^2 + BC^2 = AC(AD + DC)$

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow AC^2 = AB^2 + BC^2$

30. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**Total**
Frequency	f_1	**5**	**9**	**12**	f_2	**3**	**2**	**40**

OR

The marks obtained by 100 students of a class in an examination are given below.

Marks	**Number of students**
0-5	**2**
5-10	**5**
10-15	**6**
15-20	**8**
20-25	**10**
25-30	**25**

30-35	**20**
35-40	**18**
40-45	**4**
45-50	**2**

Draw 'a less than' type cumulative frequency curves (ogive). Hence find median.
Solution:

Class	Frequency	Cumulative Frequency
0-10 10-20 20-30 30-40 40-50 50-60 60-70	f_1 5 9 12 f_2 3 2	f_1 $5+f_1$ $14+f_1$ $26+f_1$ $26+f_1+f_2$ $29+f_1+f_2$ $31+f_1+f_2$
Total=40=n		

$$f_1+5+9+12+f_2+3+2=40$$
$$f_1+f_2=40-31=9 \;...(i)$$

Median=32.5 Given

Since,Median class is 30-40

$$l=30, h=10, cf=14+f_1, f=12$$

$$\text{Median}=l+\left(\frac{\frac{n}{2}-cf}{f}\right)\times h$$

$$32.5=30\left[\frac{\frac{40}{2}-(14+f_1)}{12}\right]\times 10$$

$$2.5=\frac{10}{12(20-14-f_1)}$$
$$3=6-f_1$$
$$f_1=3$$

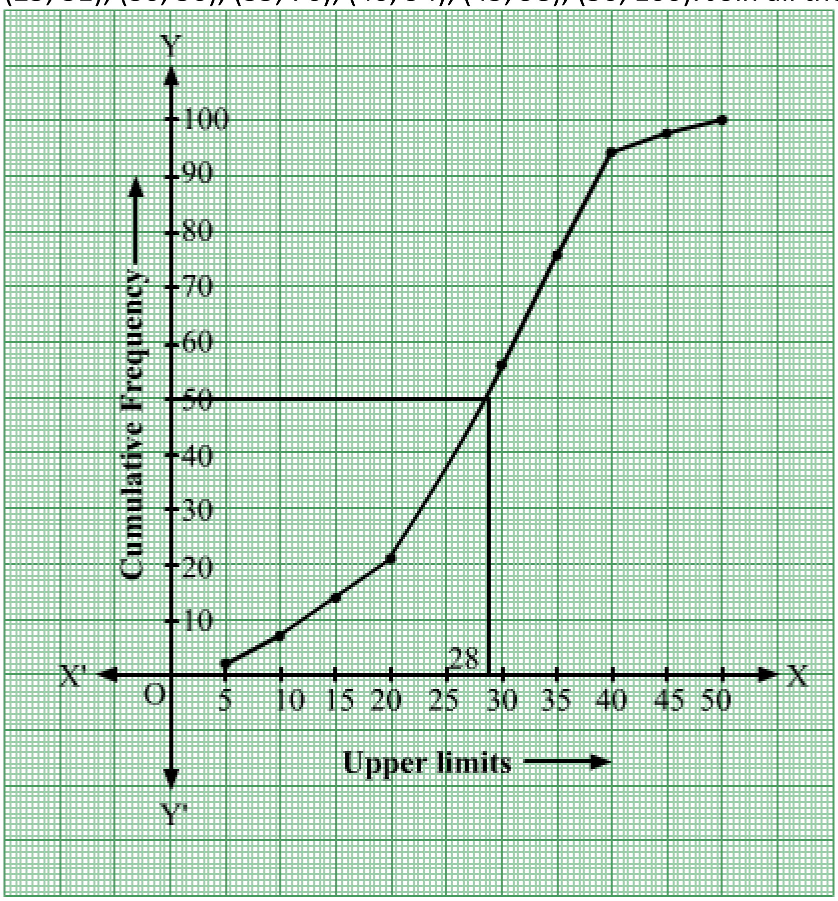
On putting in (i),

$$f_1+f_2=9$$
$$f_2=9-3 \qquad [\because f_1=3]$$
$$=6$$

OR

Marks	Number of students	Marks less than	Cumulative frequency
0-5	2	Less than 5	2
5-10	5	Less than 10	7
10-15	6	Less than 15	13
15-20	2	Less than 20	21
20-25	10	Less than 25	31
25-30	25	Less than 30	56
30-35	20	Less than 35	76
35-40	18	Less than 40	94
40-45	4	Less than 45	98
45-50	2	Less than 50	100

Let us now plot the points corresponding to the ordered pairs (5, 2), (10, 7), (15, 13), (20, 21), (25, 31), (30, 56), (35, 76), (40, 94), (45, 98), (50, 100). Join all the points by a smooth curve.



Locate $\frac{n}{2} = \frac{100}{2} = 50$ on Y-axis

From this point draw a line parallel to X-axis cutting the curve at a point. From this point, draw a perpendicular to X-axis. The point of intersection of perpendicular with the X-axis determines the median of the data. Therefore median = 28.8