Previous Year Question Paper 2018

SECTION A

1. What is the value of $(\cos^2 67° - \sin^2 23°)$?

Ans. $\cos^2 67^0 - \sin^2 23^0$

as $\cos(90^0 - \theta) = \sin\theta$

Let $\theta = 23^0$

$\cos(90^0 - 23^0) = \sin 23^0$

$\cos 67^0 = \sin 23^0$

$\therefore \cos^2 67^0 = \sin^2 23^0$

$\therefore \cos^2 67^0 - \sin^2 23^0 = 0$

2. In an AP, if the common difference $(d) = -4$, and the seventh term (a_7) is 4, then find the first term.

Ans. $a_7 = 4$

$a + 6d = 4$ $(as\ a_n = a + (n-1)d)$

but $d = -4$

$a + 6(-4) = 4$

$a + (-24) = 4$

$a = 4 + 24 = 28$

Therefore first term $a = 28$

3. Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{\text{ar}\,\Delta ABC}{\text{ar}\,\Delta PQR}$.

Ans. $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$ (Ratio of area of similar triangle is equal to square of their praportional sides)

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

4. What is the HCF of smallest prime number and the smallest composite number ?

Ans. Smallest prime number is 2.

Smallest composite number is 4

Therefore HCF is 2.

5. Find the distance of a point $P(x, y)$ from the origin.

Ans. Using distance formual

$$\ell(OP) = \sqrt{(x-0)^2 + (y-0)^2}$$

$$\ell(OP) = \sqrt{x^2 + y^2}$$

6. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.

Ans. $\because$ x = 3 is one of the root of $x^2 - 2kx - 6 = 0$

$$(3)^2 - 2k(3) - 6 = 0$$

$$9 - 6k - 6 = 0$$

$$3 - 6k = 0$$

$$3 = 6k$$

$$k = \frac{3}{6} = \frac{1}{2}$$

SECTION B

7. Two different dice are tossed together. Find the probability :

(i) of getting a doublet

(ii) of getting a sum 10, of the numbers on the two dice.

Ans. Sample space $= S = \{(1,1)(1,2).......,(6,6)\}$

$n(s) = 36$

i) A = getting a doublet

A = {(1, 1), (2, 2) ...., (6, 6)}

$n(A) = 6$

$$\therefore \; P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

ii) B = getting sum of numbers as 10.
B = {(6, 4), (4, 6), (5, 5)}

$n(B) = 3$

$$\therefore \; P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

8.Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m.

Ans. Suppose the point P(4, m) divides the line segment joining the points A(2, 3) and B(6, -3) in the ratio K : 1.

A —— K —— P —— 1 —— B
(2, 3) (4, m) (6, -3)

Co-ordinates of point $P \equiv \left(\frac{6K+2}{K+1}, \frac{-3K+3}{K+1} \right)$

But the co-ordinates of point P are given as (4, m)

$$\frac{6K+2}{K+1} \Rightarrow 4 \qquad ......(1) \text{ and}$$

$$\frac{-3K+3}{K+1} = m \qquad ......(2)$$

$6K + 2 = 4K + 4$

$2K = 2$

$K = 1$

Putting K = 1 in eq. (2)

$$\frac{-3(1)+3}{1+1} = m$$

$\therefore \; m = 0$

Ratio is 1 : 1 and m = 0

i.e. P is the mid point of AB

9. An integer is chosen at random between 1 and 100. Find the probability that it is :

(i) divisible by 8

(ii) not divisible by 8

Ans. An integer is chosen at random from 1 to 100

Therefore n(S) = 100

(i) Let A be the event that number chosen is divisible by 8

$\therefore \; A = \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96\}$

$\therefore \ n(A) = 12$

Now, P (that number is divisible by 8) $= P(A) = \frac{n(A)}{n(S)}$

$= \frac{12}{100} = \frac{6}{50} = \frac{3}{25}$

$\boxed{P(A) = \frac{3}{25}}$

(ii) Let 'A' be the event that number is not divisible by 8.

$\therefore \ P(A') = 1 - P(A)$

$= 1 - \frac{3}{25}$ $\boxed{P(A') = \frac{22}{25}}$

10. In figure.1, ABCD is a rectangle. Find the values of x and y.

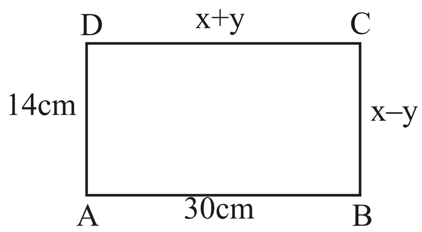


Figure 1

Ans. Since it is a rectangle

$\ell(AB) = \ell(CD)$

$x + y = 30$ ...(i)

$\ell(AD) = \ell(BC)$

$x - y = 14$ ...(ii)

Adding (1) and (2), we get

$2x = 44$

$x = 22$

Putting $x = 22$ in equation (i)

$22 - y = 14 \Rightarrow 22 - 14 = y$

$\therefore \ y = 8$

$\therefore \ x = 22$ and $y = 8$

11. Find the sum of first 8 multiples of 3.

Ans. First 8 multiples of 3 are

3, 6, 9, 12, 15, 18, 21, 24

The above sequence is an A.P.

$a = 3$, $d = 3$ and last term $l = 24$

$$S_n = \frac{n}{2}(a + l) = \frac{8}{2}[3 + 24] = 4(27)$$

$$S_n = 108$$

12. Given that $\sqrt{2}$ is irrational, prove that $\left(5 + 3\sqrt{2}\right)$ is an irrational number.

Ans. Let us assume that $\left(5 + 3\sqrt{2}\right)$ is rational. Then there exist co-prime positive integers a and b such that

$$5 + 3\sqrt{2} = \frac{a}{b}$$

$$3\sqrt{2} = \frac{a}{b} - 5$$

$$\sqrt{2} = \frac{a - 5b}{3b}$$

$\Rightarrow \sqrt{2}$ is rational. [$\because$ a, b are integers , $\because \frac{a-5b}{3b}$ is rational].

This contradicts the fact that $\sqrt{2}$ is irrational.

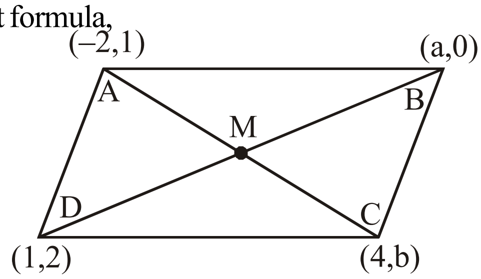
So our assumption is incorrect.

Hence, $\left(5 + 3\sqrt{2}\right)$ is an irrational number.

SECTION C

13. If $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$ are the vertices of a parallelogram $ABCD$, find the values of a and b. Hence find the lengths of its sides.

Ans. M is midpoint of AC and BD using midpoint formula,



$$\left(\frac{-2+4}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{2+0}{2}\right)$$

$$\left(\frac{2}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{2}{2}\right)$$

$\therefore\ \frac{2}{2}=\frac{a+1}{2} \Rightarrow a+1=2 \Rightarrow a=1$

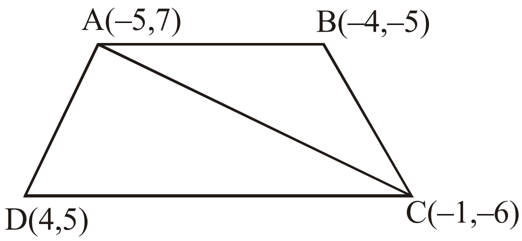
and $\frac{1+b}{2}=\frac{2}{2} \Rightarrow 1+b=2 \Rightarrow b=1$

OR

If $A(-5, 7)$, $B(-4, -5)$, $C(-1, -6)$ and $D(4, 5)$ are the vertices of quadrilateral, find the area of the quadrilateral *ABCD*.

Ans. $A(\Delta ABC)=\frac{1}{2}\left(x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right)$

If $A=\left(x_1, y_1\right), B=\left(x_2, y_2\right), C=\left(x_3, y_3\right)$ are vertices of ΔABC.



$A(\square ABCD)=A(\Delta ABC)+A(\Delta ADC)$ ....(i)

$A(\square ABC)=\frac{1}{2}[-5(-5+6)-4(-6-7)-1(7+5)]$

$=\frac{1}{2}[-5+52-12]$

$=\frac{1}{2}[35]$

$=\frac{35}{2}$ Sq.units

$A(\Delta ADC)=\frac{1}{2}[-5(5+6)+4(-6-7)-1(7-5)]$

$=\frac{1}{2}[-55-52-2]$

$=\frac{-109}{2}$

$\because$ Area cannot be negative.

$\therefore\ A(\Delta ADC)=\frac{109}{2}$ sq.units

$\therefore\ A(\square ABCD) = \frac{35}{2} + \frac{109}{2} = \frac{144}{2} = 72$ sq. units

14. Find all zeroes of the polynomial $(2x^4 - 9x^3 + 5x^2 + 3x - 1)$ if two of its zeroes are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

Ans. It is given that $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are two zeros of $f(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

$$\{x-(2+\sqrt{3})\}\{x-(2-\sqrt{3})\} = (x-2-\sqrt{3})(x-2+\sqrt{3})$$

$$= (x-2)^2 - (\sqrt{3})^2$$

$$= x^2 - 4x + 1$$

$\therefore\ (x^2 - 4x + 1)$ is a factor of $f(x)$

$$\begin{array}{r|l}
 & 2x^2 - x - 1 \\
x^2 - 4x + 1 & 2x^4 - 9x^3 + 5x^2 + 3x - 1 \\
 & 2x^4 - 8x^3 + 2x^2 \\
 & (-)\quad (+)\quad (-) \\ \hline
 & -x^3 + 3x^2 + 3x - 1 \\
 & -x^3 + 4x^2 - x \\
 & (+)\ (-)\ (+) \\ \hline
 & -x^2 + 4x - 1 \\
 & -x^2 + 4x - 1 \\
 & (+)(-)(+) \\ \hline
 & 0 \\ \hline
\end{array}$$

Let us now divide $f(x)$ by $x^2 - 4x + 1$

We have,

$\therefore\ f(x) = (x^2 - 4x + 1)(2x^2 - x - 1)$

Hence, other two zeros of f(x) are the zeros of the polynomial $2x^2 - x - 1$

We have,

$$2x^2 - x - 1 = 2x^2 - 2x + x - 1$$

$$= 2x(x-1) + 1(x-1)$$

$= (2x+1)(x-1)$

$f(x) = (x-2-\sqrt{3})(x-2+\sqrt{3})(2x+1)(x-1)$

Hence, the other two zeros are $-\frac{1}{2}$ and 1.

15. Find *HCF* and *LCM* of 404 and 96 and verify that $HCF \times LCM$ = Product of the two given numbers.

Ans. Using the factor tree for the prime factorization of 404 and 96, we have

$404 = 2^2 \times 101$ and $96 = 2^5 \times 3$

To find the HCF, we list common prime factors and their smallest exponent in 404 and 96 as under :

Common prime factor = 2, Least exponent = 2

$\therefore$ HCF $= 2^2 = 4$

To find the LCM, we list all prime factors of 404 and 96 and their greatest exponent as follows :

Prime factors of 404 and 96	**Greatest Exponent**
2	5
3	1
101	1

$\therefore$ LCM $= 2^5 \times 3^1 \times 101^1$

$= 2^5 \times 3^1 \times 101^1$

$= 9696$

Now,

HCF × LCM = 9696 × 4 = 38784

Product of two numbers = 404 × 96 = 38784

Therefore HCF × LCM = Product of two numbers.

16. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Ans. Given AP and AQ are two tangents from a point A to a circle C (O, r)

To prove AP = AQ

Construction join OP, OQ and OA

Proof In order to prove that $AP = AQ$, we shall first prove that $\Delta OPA = \Delta OQA$

since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore OP \perp AP$ and $OQ \perp AQ$

$\Rightarrow \angle OPA = \angle OQA = 90^0$ ....(i)

Now, in right triangles OPA and OQA, we have

$OP = OQ$ [Radii of a circle]

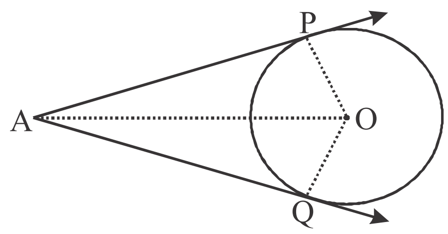
$\angle OPA = \angle OQA$ [from (i)]

and OA = OA

so, by RHS – criterion of congruence, we get

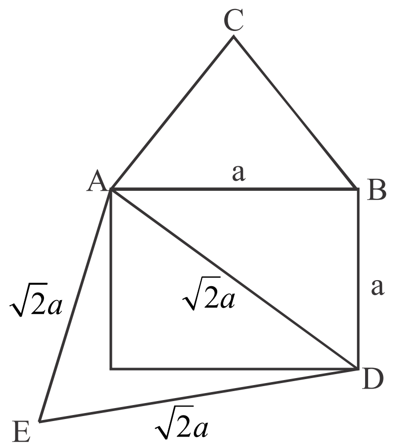
$\Delta OPA \cong \Delta OQA$

$\Rightarrow AP = AQ$



17. Prove that the area of an equilateral traingle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal.

Ans. Let a be the side of square.



$$A(\Delta ABC) = \frac{\sqrt{3}}{4} \times side^2 - \frac{\sqrt{3}}{4} \times a^2 .....(1)$$

using pythagoras theorem

$$AD^2 = AB^2 + BD^2 = a^2 + a^2 = 2a^2$$

$$AD = \sqrt{2}a$$

$$\therefore A(\Delta ADE) = \frac{\sqrt{3}}{4} \times (\sqrt{2}a)^2 = \frac{\sqrt{3}}{4} \times 2a^2 .....(2)$$

$$\frac{A(\Delta ABC)}{A(\Delta ADE)} = \frac{\sqrt{3}/4 \times a^2}{\sqrt{3}/4 \times 2a^2}$$

$$A(\Delta ABC) = \frac{1}{2} A(\Delta ADE)$$

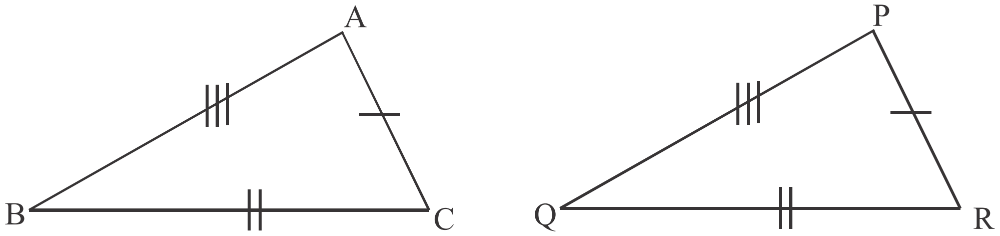
Area of equivalent triangle describes on

$$= \frac{1}{2}(area\ of\ equilateral\ \Delta\ described\ an\ one\ of\ its\ diagonal)$$

OR

If the area of two similar triangles are equal, prove that they are congruent.

Ans.



Let ΔABC is ΔPQR

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

Given that $(\Delta ABC) = A(\Delta PQR)$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = 1$$

$$1 = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

$\therefore AB = PQ$

$BC = QR$

$AC = PR$

Hence corresponding sides are equal.

$\therefore \Delta ABC \cong \Delta PQR$ (SSS rule)

hence proved.

18. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

Ans. Let the usual speed of the plane be x km/hr

Time taken to cover 1500 km with usual speed $= \frac{1500}{x} hrs$

Time taken to cover 1500 km with speed of $(x+100) km/hr = \frac{1500}{x+100}$ hrs.

$$\therefore \frac{1500}{x} = \frac{1500}{x+100} + \frac{1}{2}$$

$$\frac{1500}{x} - \frac{1500}{x+100} = \frac{1}{2}$$

$$1500\left(\frac{x+100-x}{x(x+100)}\right) = \frac{1}{2}$$

$$150000 \times 2 = x(x+100)$$

$x^2 + 100x - 300000 = 0$

$x^2 + 100x - 300000 = 0$

$x = -600$ or $x = 500$

But speed can't be negative

Hence usual speed 500 *km/hr*.

19. The table below shown the salaries of 280 persons:

Salary (In thousand □)	No. of Person
$5-10$	49
$10-15$	133
$15-20$	63
$20-25$	15
$25-30$	6
$30-35$	7
$35-40$	4
$40-45$	2
$45-50$	1

Calculate the median salary of the data.

Ans.

Class	Frequency	Cumulative Frequency
5 - 10	49	49
10 - 15	133	182
15 - 20	63	245
20 - 25	15	260
25 - 30	6	266
30 – 35	7	273
35 - 40	4	277
40 – 45	2	279
45 – 50	1	280

Let $N =$ total frequency

$\therefore$ we have $N = 280$

$$\therefore \frac{N}{2} = \frac{280}{2} = 140$$

The cumulative frequency just greater than $\frac{N}{2}$ is 182 and the corresponding class is $10-15$

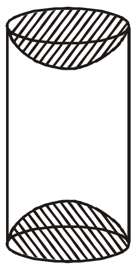
Thus, $10-15$ is the median class such that

$l=10, f=133, F=49$ and $h=5$

$$\text{Median} = l+\frac{\frac{N}{2}-F}{f}\times h = 10+\left(\frac{140-49}{133}\right)\times 5$$

$=13.42$

20. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 2. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.



Ans. Let *r* be the radius of the base of the cylinder and *h* be its height. Then, total surface area of the article

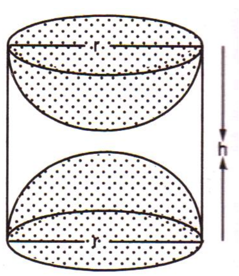
= Curved surface area of the cylinder + 2 (surface area of a hemisphere)

$=2\pi rh+2\left(2\pi r^2\right)$

$=2\pi r\left(h+2r\right)$

$=2\times\frac{22}{7}\times 3.5\left(10+2\times 3.5\right)cm^2$

$=22\times 17\,cm^2$ $=374\,cm^2$



OR

A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

Ans. Given

Base diameter = 24 m

Base radius = 12 m

Height = 3.5 m

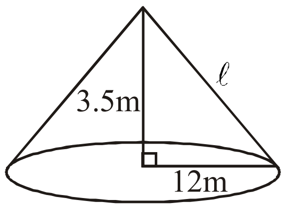
$\text{Volume} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5$

$= 22 \times 4 \times 12 \times 0.5$

$= 264 \times 2$

$= 528$ cubic meter



$\therefore \; \ell^2 = 12^2 + 3.5^2 = 144 + 12.25$

$\ell^2 = 156.25$

$\ell = \sqrt{156.25} = 12.5\, m$

Curved surface area $= \pi r \ell$

$$= \frac{22}{7} \times 12 \times 12.5 = \frac{150 \times 22}{7} = 471.428 \text{ sq.meter}$$

21. Find the area of the shaded region in Fig. 3, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square of side 12 cm, [Use $\pi = 3.14$]

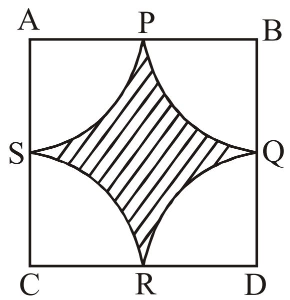


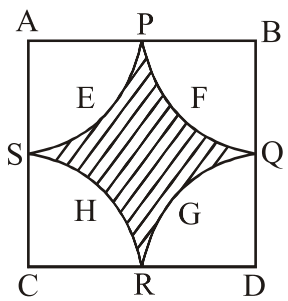
Fig.–3

Ans. Given that ABCD is a square & P,Q,R & S are the mid points of AB,BC,CD & DA respectively

$\& \; AB = 12\, cm$

$\Rightarrow AP = 6cm \qquad \{P \text{ bisects } AB\}$

area of the shaded region = Area of square ABCD – (Area of sector APEC + Area of sector PFQB + ... Area of sector RGQC + Area of sector RHSD)



$= 12^2 - \left(\frac{\pi(6^2)}{4} + \frac{\pi 6^2}{4} + \frac{\pi 6^2}{4} + \frac{\pi 6^2}{4} \right)$

$= 12^2 - \pi \times 36$

$=144-113.04$

$=30.96\,cm^2$

22. If $4\tan\theta=3$, evaluate $\left(\dfrac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}\right)$

Ans. Given that,

$\tan\theta=\dfrac{3}{4}$ $\qquad\qquad \therefore\ \tan^2\theta=\dfrac{9}{16}$

we know that,

$\sec^2\theta=1+\tan^2\theta$

$\therefore\ \sec^2\theta=1+\dfrac{9}{16}=\dfrac{25}{16}$

$\therefore\ \cos^2\theta=\dfrac{16}{25}$

$\therefore\ \cos\theta=\dfrac{4}{5}$

we know that,

$\sin^2\theta=1-\cos^2\theta$

$\therefore\ \sin^2\theta=1-\dfrac{16}{25}=\dfrac{9}{25}$

$\Rightarrow \sin\theta=\dfrac{3}{5}$

Now,

$$\left(\frac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}\right)=\left(\frac{4\times\left(\frac{3}{5}\right)-\frac{4}{5}+1}{4\times\left(\frac{3}{5}\right)+\left(\frac{4}{5}\right)-1}\right)$$

$$=\frac{12-4+5}{12+4-5}$$

$$=\frac{13}{11}$$

OR

If $\tan 2A = \cot(A - 18°)$, where $2A$ is an angle, find the value of A.

Ans. Given that,

$$\tan 2A = \cot\left(A - 18^0\right)$$

Now,

we know that,

$$\tan\theta = \cot\left(90^0 - \theta\right)$$

$$\therefore \cot\left(90^0 - 2A\right) = \cot\left(A - 18^0\right)$$

$$\therefore 90^0 - 2A = A - 18^0$$

$$\therefore 3A = 108^0$$

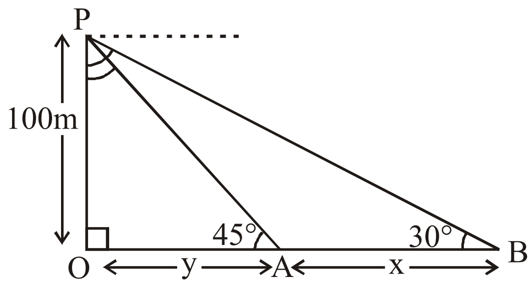
$$\therefore A = \frac{108^0}{3} = 36^0$$

$$\therefore A = 36^0$$

SECTION D

23. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behing the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$]

Ans. Let ships are at distance x from each other



In ΔAPO

$$\tan 45° = \frac{100}{y} = 1 \qquad \therefore y = 100\,m \qquad \text{...(i)}$$

In ΔPOB

$$\tan 30° = \frac{OP}{OB} = \frac{100}{x+y} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3} = \frac{x+y}{100}$$

$$x + y = 100\sqrt{3} \qquad \text{...(ii)}$$

$$x = 100\sqrt{3} - y = 100\sqrt{3} - 100 = 100\left(\sqrt{3} - 1\right)$$

$\therefore\ x = 100(1.732 - 1)$

$= 100 \times 0.732$

$= 73.2\,m$

$\therefore$ Ships are 73.2 meters apart.

24. The diameters of the lower and upper ends of a bucket in the form of a frustum f a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find:

(i) The area of the metal sheet used to make the bucket.

(ii) Why we should avoid the bucket made by ordinary plastic? [Use $\pi = 3.14$]

Ans. Let $r_1 = 5\,cm$ and $r_2 = 15\,cm$ are radii of lower and upper circular faces.

Metal sheet required = Area of curved surface + Area of Base

$$= \pi\,(r_1 + r_2)\,\ell + \pi r_1^2 \qquad \text{...(i)}$$

From diagram

$AB = CD = 5$ cm

$DE = 15 - 5 = 10$ cm

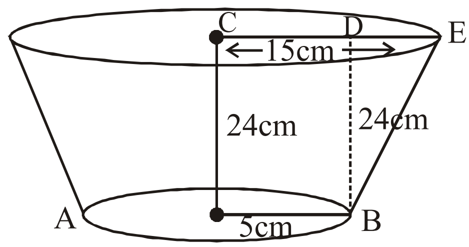
and $BD = 24$ cm

$\therefore\ BE^2 = BD^2 + DE^2$

$= 576 + 100$

$BE^2 = 676$

$BE = 26\,cm = \ell$



Metal required $= \pi(5+15)26 + \pi(5)^2$

$= \pi \times 20 \times 26 + \pi \times 25$

$= 5\pi(4 \times 26 + 5)$

$= 5\pi(109)$

$= 5 \times \frac{22}{7} \times 109$

$= 1712.85\,cm^2$

There is a chance of breakdown due to stress an ordinary plastic.

25. Prove that $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A.$

Ans. To prove

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$$

$$L.H.S = \frac{\sin A\left(1 - 2\sin^2 A\right)}{\cos A\left(2\cos^2 A - 1\right)}$$

We know that, $\sin^2 A + \cos^2 A = 1$

$$= \frac{\sin A}{\cos A}\left(\frac{\left(\sin^2 A + \cos^2 A - 2\sin^2 A\right)}{\left(2\cos^2 A - \sin^2 A - \cos^2 A\right)}\right)$$

$$= \tan A\left(\frac{\cos^2 A - \sin^2 A}{\cos^2 A - \sin^2 A}\right)$$

$= \tan A$

$= R.H.S.$ hence proved.

26. The mean of the following distribution is 18. Find the frequency f of the class 19-21.

Class	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

Ans.

Class	Mid values x_i	Frequence f_i	$d_i = x_i - 18$	$u_i = \frac{x_i - 18}{2}$	$f_i u_i$
11 – 13	12	3	-6	-3	-9
13 - 15	14	6	-4	-2	-12
15 - 17	16	9	-2	-1	-9
17 - 19	18	13	0	0	0
19 - 21	20	f	2	1	f
21 - 23	22	5	4	2	10
23 - 25	24	4	6	3	12
		$\sum f_i = 40 + f$			

$\sum f_i u_i = f - 8$

we have

$h = 2; A = 18, N = 40 + f, \sum f_i u_i = f - 8 \quad \overline{X} = 18$

$$\therefore\ Mean = A + h\left\{\frac{1}{N}\sum f_i u_i\right\}$$

$$18 = 18 + 2\left\{\frac{1}{40+f}(f-8)\right\}$$

$$\frac{2(f-8)}{40+f} = 0$$

$f - 8 = 0$

$f = 8$

OR

The following distribution gives the daily income of 50 workers of a factory :

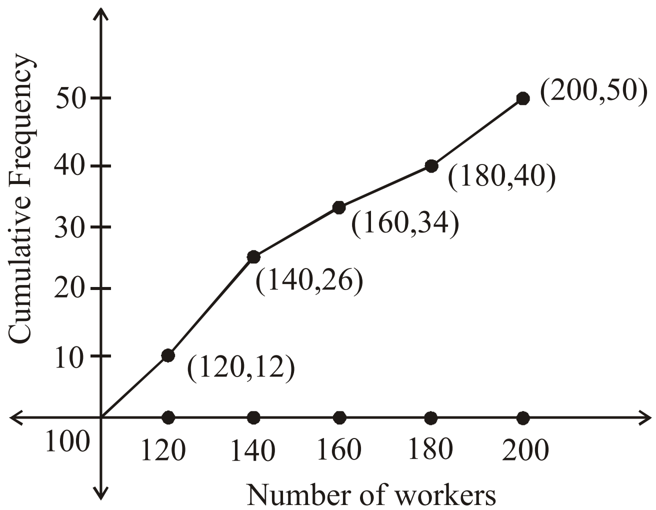
Daily Income(In)	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

Ans.

Daily income	Frequency	Income less than	Cumulative frequency
100-120	12	120	12
120-140	14	140	26
140-160	8	160	34
160-180	6	180	40
180-200	10	200	50

Other than the given class intervals, we assum a class interval 80-100 with zero frequency.



27. A motor boat whose speed is 18 km/hr in still water 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans. Let the speed of stream be $x\ km/hr$

Now, for upstream: speed $=(18-x)\,km/hr$

$\therefore$ time taken $=\left(\frac{24}{18-x}\right)hr$

Now, for downstream: speed $=(18+x)\,km/hr$

$\therefore$ time taken $=\left(\frac{24}{18+x}\right)hr$

Given that,

$$\frac{24}{18-x}=\frac{24}{18+x}+1$$

$$-1=\frac{24}{18+x}-\frac{24}{18-x}$$

$$-1=\frac{24\left[(18-x)-(18+x)\right]}{(18)^2-x^2}$$

$$-1=\frac{24\left[-2x\right]}{324-x^2}$$

$$-324+x^2=-48x$$

$$x^2+48x-324=0$$

$$x^2+54x-6x-324=0$$

$$(x+54)(x-6)=0$$

$$x=-54 \quad \text{or} \quad x=6$$

$x=-54\,km/hr$ (not possible)

Therefore, speed of the stream = 6 km/hr.

OR

A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. It it takes 3 hours to complete total journey, what is the original average speed ?

Ans. Let x be the original average speed of the train for 63 km.

Then, (x + 6) will be the new average speed for remaining 72 km.

Total time taken to complete the journey is 3 hrs.

$$\therefore \frac{63}{x}+\frac{72}{(x+6)}=3$$

$$\left(\because \text{ Time} = \frac{\text{Distance}}{\text{Speed}}\right)$$

$$\therefore \frac{63x+378+72x}{x(x+6)}=3$$

$$\Rightarrow 135x+378=3x^2+18x$$

$$\Rightarrow x^2-39x-126=0$$

$$\Rightarrow (x-42)(x+3)=0$$

$$\Rightarrow \boxed{x=42} \text{ OR } \boxed{x=-3}$$

Since speed can not be negative.
Therefore x = 42 km/hr.

28. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers.

Ans. Let the numbers be (a, –3d), (a – d), (a + d) and (a + 3d)

$$\therefore (a-3d)+(a-d)+(a+d)+(a+3d)=32$$

$$\Rightarrow 4a=32$$

$$a=8$$

$$\text{Also, } \frac{(a-3d)(a+3d)}{(a-d)(a+d)}=\frac{7}{15}$$

$$\Rightarrow 15a^2-135d^2=7a^2-7d^2$$

$$\Rightarrow 8a^2=128d^2$$

$$d^2=\frac{8a^2}{128}=\frac{8\times8\times8}{128}$$

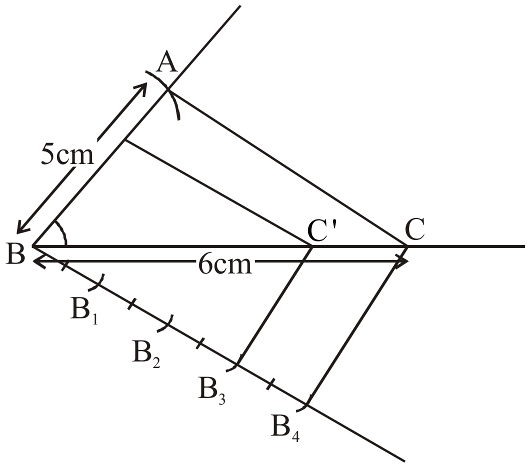
$$d^2=4$$

$$d=\pm2$$

If d =2 numbers are : 2, 6,10, 14
If $d=-2$ numbers are 14,10,16,2

29. Draw a triangle ABC with $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the ΔABC.

Ans. STEPS OF CONSTRUCTION :



(i) Draw a line segment $BC = 6$ cm, draw a ray BX making 60° with BC.

(ii) Draw an arc with radius 5 cm from B so that it cuts BX at A.

(iii) Now join AC to form ΔABC.

(iv) Draw a ray by making an acute angle with NC opposite to vertex A.

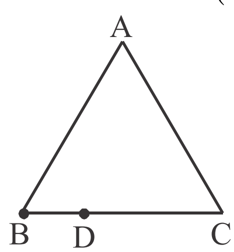
(v) Locate 4 points B_1, B_2, B_3, B_4 on by such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

(vi) Join B_4C and now draw a line from B_3 parallel to B_4C so that it cuts BC at C'.

(vii) From C' draw a line parallel to AC and cuts AB at A'.

(viii) $\Delta A'BC'$ is the required triangle.

30. In an equilateral ΔABC, is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9(AD)^2 = 7(AB)^2$.

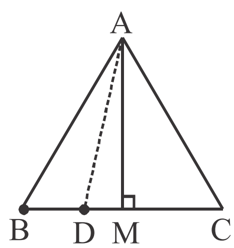


Ans. Let the each side of ΔABC be 'a' unit

$\therefore$ $BD = \frac{a}{3}$

to prove : $9(AD)^2 = 7(AB)^2$

construction : Draw $AM \perp BC$:



$DM = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$

$\therefore$ In ΔABM

$AB^2 = BM^2 + AM^2$ ....(1)

and in ΔADM

$$AD^2 = AM^2 + DM^2 \qquad ....(2)$$

In ΔABM, $\sin 60^0 = \frac{AM}{AB}$

$\Rightarrow AM = AB \sin 60^0$

$= a\frac{\sqrt{3}}{2}$

Now, taking $9(AD)^2$

$$9(AM^2 + DM^2)$$

$$9\left(\left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2\right)$$

$$9\left[\frac{3a^2}{4} + \frac{a^2}{36}\right] = 9 \times \frac{28a^2}{36}$$

$7(AB)^2 = 7a^2$

or

$\therefore 9(AD^2 =)7(AB^2)$ Hence proved.

OR

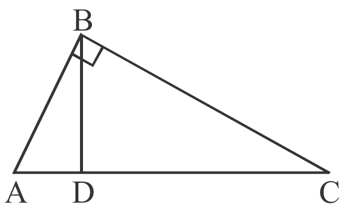
Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Ans. Given : A right - angled triangle ABC in which $\angle B = 90^0$

To Prove : $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e., $AC^2 = AB^2 + BC^2$

Construction from B draw $BD \perp AC$.



Proof : In triangle ADB and ABC, we have

$\angle ADB = \angle ABC$ [Each equal to 90^o]

and, $\angle A = \angle A$ [Common]

So, by AA - similarity criterian, we have

$\Delta ADB \sim \Delta ABC$

$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$ [$\because$ In similar triangles corresponding sides are proportional]

$\Rightarrow \quad AB^2 = AD \times AC$ ....(1)

In triangles BDC and ABC, we have

$\angle CDB = \angle ABC$ [Each equal to 90°]

and, $\angle C = \angle C$ [Common]

So, by AA-similarity criterian, we have

$\Delta BDC \sim \Delta ABC$

$\Rightarrow \dfrac{DC}{BC} = \dfrac{BC}{AC}$ [$\because$ In similar triangles corresponding sides are proportional]

$\Rightarrow BC^2 = AC \times DC$ ......(2)

Adding equation (1) and (2), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

$\Rightarrow AB^2 + BC^2 = AC(AD + DC)$

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow AB^2 + BC^2 = AC^2$

Hence, $AC^2 = AB^2 + BC^2$