Previous Year Question Paper 2014

General Instuctions *:*
1. All questions are **compulsory**.
2. The question paper consists of **34** questions divided into **four sections** A, B, C, and D.
3. Section A contains of **8** questions of 1 mark each, which are multiple choice type question, **Section B** contains of **6** questions of 2 marks each, **Section C** contains of **10** questions of 3 marks each and **Section D** contains of **10** questions of 4 marks each.
4. Use of calculator is **not** permitted.

SECTION – A

Q-1 The first three terms of an AP respectively are $3y – 1$, $3y +5$ and $5y +1$. Then y equals

(A) -3

(B) 4

(C) 5

(D) 2

Solution:
The first three terms of an AP are $3y-1$, $3y+5$ and $5y+1$, respectively.

We need to find the value of y.

We know that if a, b and c are in AP, then:

$b − a = c − b \Rightarrow 2b = a + c$

$\therefore 2 (3y+5) = 3y – 1 +5y +1$

$\Rightarrow 6y +10 = 8y$

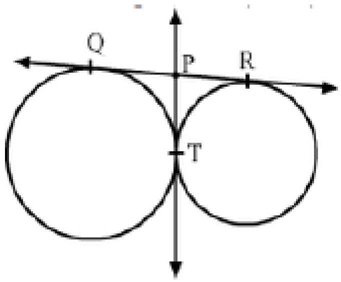
$\Rightarrow 10 = 8y -6y$

$\Rightarrow 2y =10$

$\Rightarrow y = 5$

Hence the correct option is C.

Q-2 In Fig. 1, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If PT = 3.8 cm, then the length of QR (in cm) is:



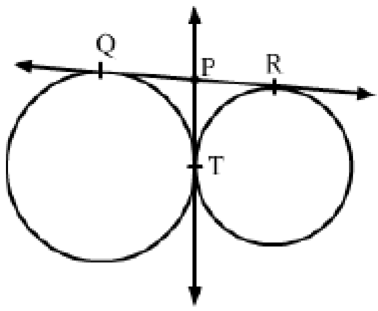
(A) 3.8

(B) 7.6

(C) 5.7

(D) 1.9

Solution:



It is known that the length of the tangents drawn from an external point to a circle is equal.

∴ QP = PT = 3.8 cm ... (1)

PR = PT = 3.8 cm ... (2)

From equations (1) and (2), we get:

QP = PR = 3.8 cm

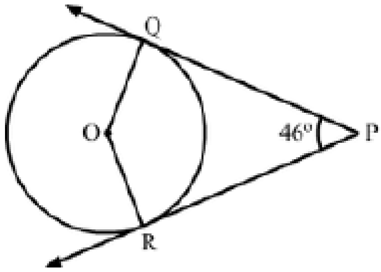
Now, QR = QP + PR

= 3.8 cm + 3.8 cm

= 7.6 cm

Hence, the correct option is B.

3. In Fig. 2, PQ and PR are two tangents to a circle with centre O. If ∠QPR = 46° , then ∠QOR equals:



(A) 67°

(B) 134°

(C) 44°

(D) 46°

Solution:
Given: ∠QPR = 46°

PQ and PR are tangents.

Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

So, we have OQ ⊥ PQ and OR ⊥ RP.

⇒ ∠OQP = ∠ORP = 90°

So, in quadrilateral PQOR, we have

∠OQP +∠QPR + ∠PRO + ∠ROQ = 360°

⇒ 90° + 46° + 90° + ∠ROQ = 360°

⇒ ∠ROQ = 360° − 226° = 134°

Hence, the correct option is B.

4. A Ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall, then the length (in meters) is:

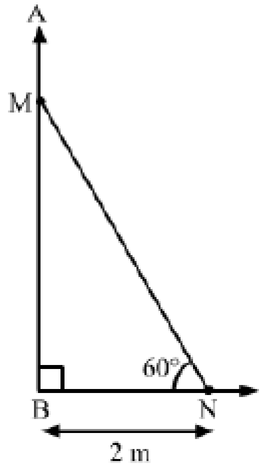
(A) $\frac{4}{\sqrt{3}}$

(B) $4\sqrt{3}$

(C) $2\sqrt{2}$

(D) 4

Solution:



In the figure, MN is the length of the ladder, which is placed against the wall AB and makes an angle of 60° with the ground.

The foot of the ladder is at N, which is 2 m away from the wall.

∴ BN = 2 m

In right-angled triangle MNB:

$$\cos 60^\circ = \frac{BN}{MN} = \frac{2m}{MN}$$
$$\Rightarrow \frac{1}{2} = \frac{2m}{MN}$$
$$\Rightarrow MN = 4m$$

Therefore, the length of the ladder is 4 m.

Hence, the correct option is D

Q5. If two different dice are rolled together, the probability of getting an even number on both dice, is:

(A) 1/36

(B) 1/2

(C) 1/6

(D) 1/4

Solution:
Possible outcomes on rolling the two dice are given below:

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Total number of outcomes = 36

Favourable outcomes are given below:

{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)}

Total number of favourable outcomes = 9

∴ Probability of getting an even number on both dice =

$$\frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{9}{36} = \frac{1}{4}$$

Hence, the correct option is D.

Q6. A number is selected at random from the numbers 1 to 30. The probability that it is a prime number

(A) 2/3

(B) 1/6

(C) 1/3

(D) 11/30

Solution:
Total number of possible outcomes = 30

Prime numbers between 1 to 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Total number of favourable outcomes = 10

∴ Probability of selecting a prime number from 1 to 30

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{10}{30} = \frac{1}{3}$$

Hence, the correct option is C.

Q7 If the points A(x, 2), B (-3,-4) and C (7, -5) are collinear, then the value of x is:

(A) -63

(B) 63

(C) 60

(D) -60

Solution:
It is given that the three points A(*x*, 2), B(−3, −4) and C(7, −5) are collinear.

∴ Area of ΔABC = 0

$\Rightarrow \frac{1}{2}\ [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\ ] = 0$

Here, $x_1 = x$, $y_1 = 2$, $x_2 = -3$, $y_2 = -4$, and $x_3 = 7$, $y_3 = -5$

$\Rightarrow x[-4 - (-5)] - 3(-5 - 2) + 7[2 - (-4)\ ] = 0$

$\Rightarrow x(-4 + 5) - 3(-5 - 2) + 7(2 + 4) = 0$

$\Rightarrow x - 3 \times (-7) + 7 \times 6 = 0$

$\Rightarrow x + 21 + 42 = 0 \Rightarrow x + 63 = 0$

$\Rightarrow x = -63$

Thus, the value of *x* is − 63.

Hence, the correct option is A.

Q8 The number of solid of solid spheres, each of diameter 6cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm is:

(A) 3

(B) 5

(C) 4

(D) 6

Solution:
Let *r* and *h* be the radius and the height of the cylinder, respectively.

Given: Diameter of the cylinder = 4 cm

∴ Radius of the cylinder, *r* = 2 cm

Height of the cylinder, *h* = 45 cm

Volume of the solid cylinder = $\pi r^2 h = \pi \times (2)^2 \times 45\ cm^3 = 180\pi\ cm^3$

Suppose the radius of each sphere be *R* cm.

Diameter of the sphere = 6 cm

∴ Radius of the sphere, *R* = 3 cm

Let n be the number of solids formed by melting the solid metallic cylinder.

∴ n × Volume of the solid spheres = Volume of the solid cylinder

$$\Rightarrow n\times\frac{4}{3}\pi R^3=180\pi$$

$$\Rightarrow n\times\frac{4}{3}\pi R^3=180\pi \text{ k}$$

$$\Rightarrow \text{n}=\frac{180\times3}{4\times27}=5$$

Thus, the number of solid spheres that can be formed is 5.

Hence, the correct option is B.

SECTION-B

Q9 Solve the quadratic equation $2x^2+ax-a^2=0$ for x.

Solution:
We have: $2x^2+ax-a^2=0$

Comparing the given equation with the standard quadratic equation ($ax^2+bx+c=0$), we get a =2, b = a and c =-a^2

Using the quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, we get:

$$x=\frac{-a\pm\sqrt{a^2-4\times2\times(-a)^2}}{2\times2}$$

$$=\frac{-a\pm\sqrt{9a^2}}{4}$$

$$=\frac{-a\pm3a}{4}$$

$$\Rightarrow x=\frac{-a+3a}{4}=\frac{a}{2} \text{ or } x=\frac{a-3a}{4}=-a$$

So, the solutions of the given quadratic equation are $x=\frac{a}{2}$ *or* $x=-a$.

Q10. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, Find its common difference.

Solution:
Let a be the first term and d be the common difference.

Given: a = 5

$T_n = 45$
$S_n = 400$

We know:

$T_n = a + (n-1)d$
$\Rightarrow 45 = 5 + (n-1)d$
$\Rightarrow 40 = (n-1)d$ ....(1)

And $S_n = \frac{n}{2} a + T_n$

$\Rightarrow 400 = \frac{n}{2}(5 + 45)$

$\Rightarrow \frac{n}{2} = \frac{400}{50}$
$\Rightarrow n = 2 \times 8 = 16$

On substituting n = 16 in (1), we get:

40 =(16-1)d

⇒ 40 = (15)d

⇒d = $\frac{40}{15} = \frac{8}{3}$

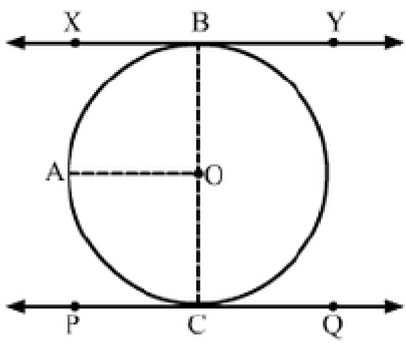
Thus, the common difference is 83.

Q11. Prove that the line segment joining the point of contact of two parallel tangles of a circle passes through its centre.

Solution:
Let XBY and PCQ be two parallel tangents to a circle with centre O.

Construction: Join OB and OC.

Draw OA||XY



Now, XB ||AO

$\Rightarrow \angle XBO + \angle AOB = 180°$ (sum of adjacent interior angles is 180°)

Now, $\angle XBO = 90°$ (A tangent to a circle is perpendicular to the radius through the point of contact)

$\Rightarrow 90° + \angle AOB = 180°$

$\Rightarrow \angle AOB = 180° - 90° = 180°$

Similarly , $\angle AOC = 90^{o}$

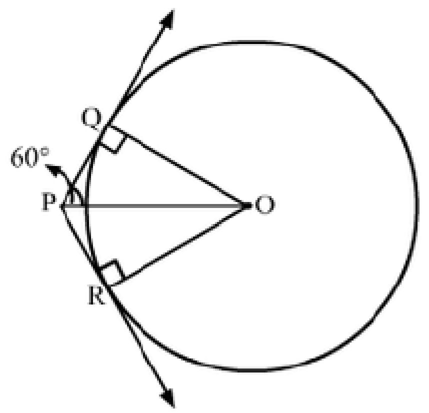
$\angle AOB + \angle AOC = 90^{o} + 90^{o} = 180°$

Hence, BOC is a straight line passing through O.

Thus, the line segment joining the points of contact of two parallel tangents of a circle passes through its centre.

Q12 If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that $\angle QPR = 120°$, prove that 2PQ = PO.

Solution:
Let us draw the circle with extent point P and two tangents PQ and PR.



We know that the radius is perpendicular to the tangent at the point of contact.

$\therefore \angle OQP = 90°$

We also know that the tangents drawn to a circle from an external point are equally inclined to the joining the centre to that point.

$\therefore \angle QPO = 60°$

Now, in ΔQPO:

$\text{Cos } 60° = \frac{PQ}{PO}$

$\Rightarrow \frac{1}{2} = \frac{PQ}{PO}$

$\Rightarrow 2PQ = PO$

Q13 Rahim tosses two different coins simultaneously. Find the probability of getting at least one tail.

Solution:
Rahim tosses two coins simultaneously. The sample space of the experiment is {HH, HT, TH, and TT}.

Total number of outcomes = 4

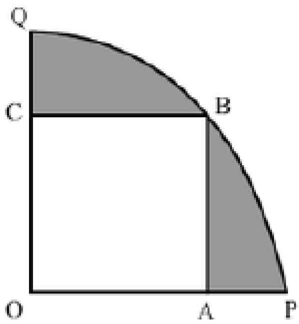
Outcomes in favour of getting at least one tail on tossing the two coins = {HT, TH, TT}

Number of outcomes in favour of getting at least one tail = 3

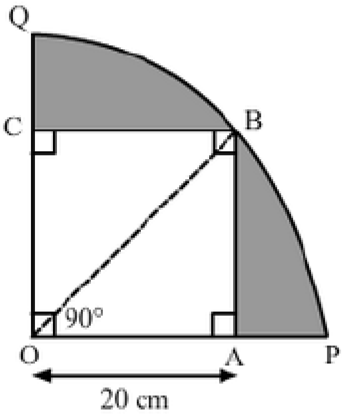
∴ Probability of getting at least one tail on tossing the two coins

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{4}$

Q14 In fig. 3, a square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20 cm, find the area of the shaded region (Use π= 3.14)



Let us join OB.



In ΔOAB: $OB^2 = OA^2 + AB^2 = (20)^2 + (20)^2 = 2 \times (20)^2$

$\Rightarrow OB = 20\sqrt{2}$

Radius of the circle, $r = 20\sqrt{2}cm$

Area of quadrant OPBQ $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times (20\sqrt{2})^2 cm^2$

$= \frac{1}{4} \times 3.14 \times 800\ cm^2$

$= 628\ cm^2$

Area of square OABC = $(\text{Side})^2 = (20)^2\ cm^2 = 400\ cm^2$

∴ Area of the shaded region = Area of quadrant OPBQ – Area of square OABC

= (628 - 400) cm

$= 228\ cm^2$

SECTION-C

Q15 Solve the equation $\frac{4}{x} - 3 = \frac{5}{2x+3}$; $x \neq 0, -3/2$, for x.

Solution:

Given equation:

$$\frac{4}{x} - 3 = \frac{5}{2x+3};\ x \neq 0, -\frac{3}{2}$$

$$\frac{4}{x} - 3 = \frac{5}{2x+3}$$

$$\Rightarrow \frac{4-3x}{x} = \frac{5}{2x+3}$$

$$\Rightarrow (4-3x)(2x+3) = 5x$$

$\Rightarrow -6x^2 + 8x - 9x + 12 = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$

$\Rightarrow (x+2) = 0\ (x-1) = 0$

$\Rightarrow$ x = -2 or x = 1

Thus, the solution of the given equation is -2 or 1.

Q16. If the seventh term of an AP is 1/9 and its ninth term is 1/7, find its 63rd term.

Solution:
Let a be the first term and d be the common difference of the given A.P.

Given:

$$a_7 = \frac{1}{9}$$
$$a_9 = \frac{1}{7}$$

$$a7 = a + (7-1)d = \frac{1}{9}$$

$$\Rightarrow a + 6d = \frac{1}{9} \quad \text{...............(1)}$$
$$a_9 = a + (9-1)d = \frac{1}{7}$$
$$\Rightarrow a + 8d = \frac{1}{7} \quad \text{................(2)}$$

Subtracting equation (1) from (2), we get:

$$2d = \frac{2}{63}$$
$$\Rightarrow d = \frac{1}{63}$$

Putting $d = \frac{1}{63}$ in equation (1), we get:

$$a + \left(6 \times \frac{1}{63}\right) = \frac{1}{9}$$
$$\Rightarrow a = \frac{1}{63}$$
$$\therefore a_{63} = a + (63-1)d = \frac{1}{63} + 62\left(\frac{1}{63}\right) = \frac{63}{63} = 1$$

Thus, the 63rd term of the given A.P. is 1.

Q17. Draw a right triangle ABC is which AB = 6 cm, BC = 8 cm and ∠B = 90°. Draw BD perpendicular from B on AC and draw a circle passing through the points B, C and D. Construct tangents from A to this circle.

Solution:

Follow the given steps to construct the figure.

Step 1

Draw a line BC of 8 cm length.

Step 2

Draw BX perpendicular to BC.

Step 3

Mark an arc at the distance of 6 cm on BX. Mark it as A.

Step 4

Join A and C. Thus, ΔABC is the required triangle.

Step 5

With B as the centre, draw an arc on AC.

Step 6

Draw the bisector of this arc and join it with B. Thus, BD is perpendicular to AC.

Step 7

Now, draw the perpendicular bisector of BD and CD. Take the point of intersection as O.

Step 8

With O as the centre and OB as the radius, draw a circle passing through points B, C and D.

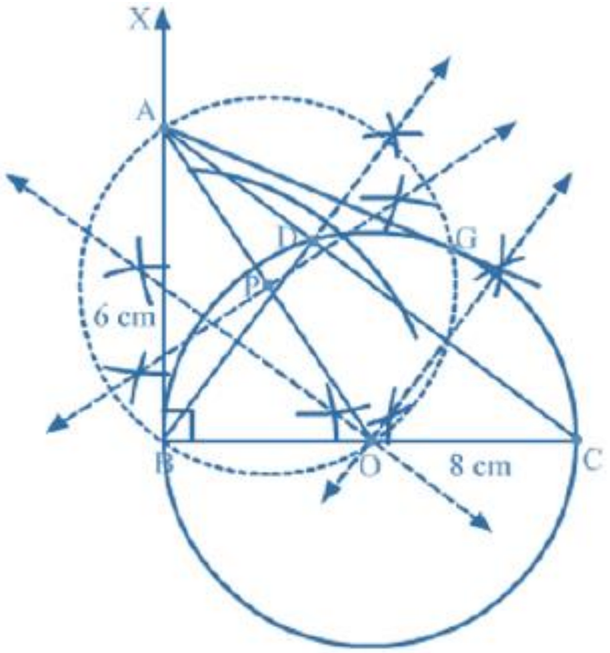
Step 9

Join A and O and bisect it Let P be the midpoint of AO.

Step 10

Taking P as the centre and PO as its radius, draw a circle which will intersect the circle at point B and G. Join A and G.

Here, AB and AG are the required tangents to the circle from A.



Q18. If the point A (0,2) is equidistant from the points B(3, p) and C(p, 5), find P. Also find the the length of AB.

Solution:
The given points are A (0, 2), B (3, p) and C (p, 5).

It is given that A is equidistant from B and C.

∴ AB = AC

⇒ $AB^2 = AC^2$

⇒ $(3\text{-}0)^2 + (p\text{-}2)^2 = (p\text{-}0)^2 + (5\text{-}2)^2$

⇒ $9 + p^2 + 4 - 4p = p^2 + 9$

⇒ $4 - 4p = 0$

⇒ $4p = 4$

⇒ $p = 1$

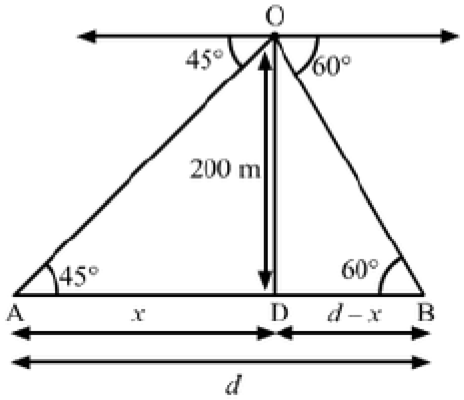
Thus, the value of p is 1

Length of AB $=\sqrt{(3-0)^2+(1-2)^2}=\sqrt{3^2+(-1)^2}=\sqrt{9+1}=\sqrt{10}$ units.

Q19. Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of two ships as observed from the top of the light house are 60° and 45°. If the height of the light house is 200 m, find the distance between the two ships. [Use $\sqrt{3}=1.73$]

Solution:

Let d be the distance between the two ships. Suppose the distance of one of the ships from the light house is X meters, then the distance of the other ship from the light house is (d-x) meter.



In right –angled ΔADO, we have.

$$\tan 45° = \frac{OD}{AD} = \frac{200}{X}$$
$$\Rightarrow 1 = \frac{200}{X}$$
$$\Rightarrow X = 200 \quad \text{............(1)}$$

In right-angled Δ BDO, we have

$$\tan 60° = \frac{OD}{BD} = \frac{200}{d = x}$$
$$\Rightarrow \sqrt{3} = \frac{200}{d - x}$$
$$\Rightarrow d - x = \frac{200}{\sqrt{3}}$$

Putting x=200. We have:

$$d - 200 = \frac{200}{\sqrt{3}}$$
$$d = \frac{200}{\sqrt{3}} + 200$$
$$\Rightarrow d = 200\left(\frac{\sqrt{3+1}}{\sqrt{3}}\right)$$
$$\Rightarrow d = 200 \times 1.58$$
$$\Rightarrow d = 316 \quad \text{(approx.)}$$

Thus, the distance between two ships is approximately 316 m.

Q20 If the points A(-2. 1), B (a, b) and C (4, -1) are collinear and a-b = 1, find the values of a and b.

Solution:
The given points are A (-2, 1), B (a, b) and C (4-1).

Since the given points are collinear, the area of the triangle ABC is 0.

$$\Rightarrow \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x3(y_1-y_2)\right]=0$$

Here, x_1=-2,y_1 =1,x_2=a, y_2 =b and x_3=4, y_3=-1

$$\therefore \frac{1}{2}\left[-2(b+1)+a(-1-1)+4(1-b)\right]=0$$
$$\Rightarrow -2b-2-2a+4-4b=0$$
$$\Rightarrow 2a+6b=2$$
$$\Rightarrow a+3b=1 \quad ....(1)$$

Given :

a-b= 1 ....(2)

Subtracting equation (1) from (2) we get:

4b =0

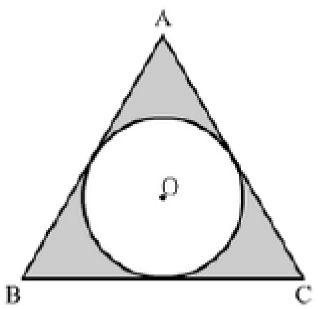
⇒b=0

Subtracting b= 0 in (2), we get:

a= 1

Thus, the values of a and b are 1 and 0, respectively.

Q21 In Fig 4, a circle is inscribed in an equilateral triangle ABC of side 12 cm. Find the radius of inscribed circle and the area of the shaded region. [Use π =3.14 and $\sqrt{3}=1.73$]



Solution:
It is given that ABC is an equilateral triangle of side 12 cm.

Construction:

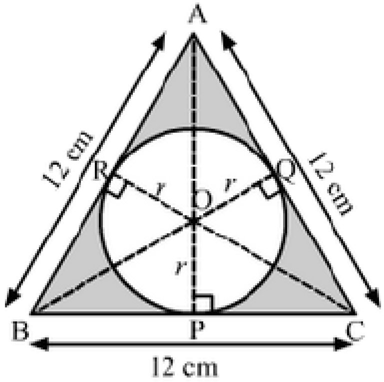
Join OA, OB and OC.

Draw.

OP ⊥ BC

OQ ⊥ AC

OR ⊥ AB



Let the radius of the circle be r cm.

Area of ΔAOB +Area of ΔBOC + Area of ΔAOC = Area of ΔABC

$$\Rightarrow \frac{1}{2}\times AB\times OR+\frac{1}{2}\times BC\times OP+\frac{1}{2}\times AC\times OQ=\frac{\sqrt{3}}{4}\times(\text{Side})^2$$
$$\Rightarrow \frac{1}{2}\times 12\times r+\frac{1}{2}\times 12\times r+\frac{1}{2}\times 12\times r=\frac{\sqrt{3}}{4}\times(12)^2$$
$$\Rightarrow 3\times\frac{1}{2}\times 12\times r=\frac{\sqrt{3}}{4}\times 12\times 12$$
$$\Rightarrow r=2\sqrt{3}=2\times 1.73=3.46$$

Therefore the radius of the inscribed circle is 3.46 cm.

Now, area of the shaded region = Area of ΔABC – Area of the inscribed circle

$$=\left[\frac{\sqrt{3}}{4}\times(12)^2-\pi(2\sqrt{3})^2\right]cm^2$$
$$=\left[36\sqrt{3}-12\pi\right]cm^2$$

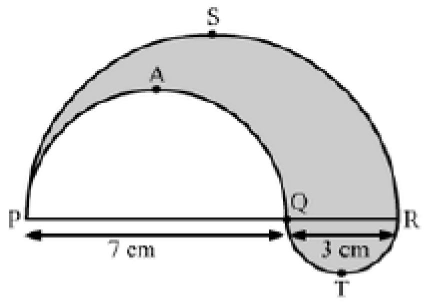
= [36 ×1.73 – 12 × 3.14] cm^2

= [62.28 – 37.68] cm^2

= 24.6 cm^2

Therefore, the area of the shaded region is 24.6 cm^2.

Q22. In Fig.5. PSR, RTQ and PAQ are three semicircles of diameters 10cm, 3cm and 7 cm respectively. Find the perimeter of the shaded region. [Use π = 3.14]



Solution:

Radius of Semicircle PSR = $\frac{1}{2}$ × 10 cm =5 cm

Radius of Semicircle RTQ = $\frac{1}{2}$ × 3 = 1.5 cm

Radius of semicircle PAQ = $\frac{1}{2}$ ×7 cm = 3.5 cm

Perimeter of the shaded region = Circumference of semicircle PSR + Circumference of semicircle RTQ + Circumference of semicircle PAQ

$$= \left[\frac{1}{2}\times 2\pi(5)+\frac{1}{2}\times 2\pi(1.5)+\frac{1}{2}\times 2\pi(3.5)\right]cm$$
$$= \pi(5+1.5+3.5)cm$$
$$= 3.14\times 10cm$$
$$= 31.4\ cm$$

Q23 A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep? If the water flows through the pipe at the rate of 4 km per hour, in how much time will the tank be filled completely?

Solution:

For the given tank.

Diameter = 10 m

Radius, R = 5m

Depth, H =2m

Internal radius of the pipe = r = $\frac{20}{2}$ cm =10 cm = $\frac{1}{10}$ m

Rate of flow of water = v = 4 km/h = 4000 m/h

Let t be the time taken to fill the tank.

So, the water flown through the pipe in t hours will equal to the volume of the tank

$$\therefore \pi r^2 \times v \times t = \Pi R^2 H$$

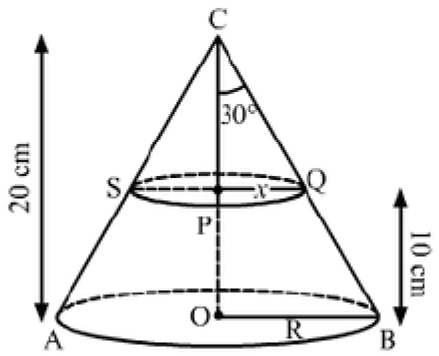
$$\Rightarrow \left(\frac{1}{10}\right)^2 \times 4000 \times t = (5)^2 \times 2$$

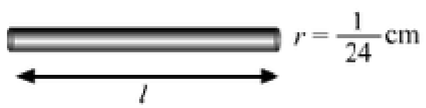
$$\Rightarrow t = \frac{25 \times 2 \times 100}{4000} = 1\frac{1}{4}$$

Hence, the time taken is $1\frac{1}{4}$ hours.

Q24. A solid metallic right circular cone 20 cm high and whose vertical angle is 60°, is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{12}$ cm, find the length of the wire.

Solution:





Let ACB be the cone whose vertical angle ∠ACB = 60°. Let R and x be the radii of the lower and upper end of the frustum.

Here, height of the cone, OC = 20 cm = H

Height CP = h = 10 cm

Let us consider P as the mid-Point of OC.

After cutting the cone into two parts through P.

OP = $\frac{20}{2}$ = 10 cm

Also, ∠ACO and ∠OCB = $\frac{1}{2}$ × 60° = 30°

After cutting cone CQS from cone CBA, the remaining solid obtained is a frustum.

Now, in triangle CPQ:

$$\tan 30^\circ = \frac{x}{10}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{10}$$
$$\Rightarrow x = \frac{10}{\sqrt{3}} cm$$

In triangle COB:

$$\text{Tan}30^\circ = \frac{R}{CO}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{R}{20}$$
$$\Rightarrow R = \frac{20}{\sqrt{3}} cm$$

Volume of the frustum, $V = \frac{1}{3}\pi(R^2H - x^2h)$

$$\Rightarrow V = \frac{1}{3}\pi\left(\left(\frac{20}{\sqrt{3}}\right)^2 \cdot 20 - \left(\frac{10}{\sqrt{3}}\right)^2 \cdot 10\right)$$
$$= \frac{1}{3}\pi\left(\frac{8000}{3} - \frac{1000}{3}\right)$$
$$= \frac{1}{3}\pi\left(\frac{7000}{3}\right)$$
$$= \frac{1}{9}\pi \times 7000$$
$$= \frac{7000}{9}\pi$$

The volumes of the frustum and the wire formed are equal.

$$\pi \times \left(\frac{1}{24}\right)^2 \times l = \frac{7000}{9} \times [\text{Volume of wire} = \pi r^2 h]$$

$$\Rightarrow l = \frac{7000}{9} \times 24 \times 24$$
$$\Rightarrow l = 448000cm = 4480m$$

Hence, the length of the wire is 4480 m.

SECTION –D

Q 25. The difference of two natural number is 5 and the difference of their reciprocals is 110. Find the numbers.

Solution:
Let the two natural numbers be X and Y such that x > y.

Given:

Difference between the natural numbers = 5

∴ X - Y =5 …..(1)

Difference of their reciprocals $=\frac{1}{10}$ (given)

$$\frac{1}{y}-\frac{1}{x}=\frac{1}{10}$$

$$\Rightarrow \frac{x-y}{xy}=\frac{1}{10}$$

$$\Rightarrow \frac{5}{xy}=\frac{1}{10}$$

$$\Rightarrow xy=50 \quad \text{.............(ii)}$$

Putting the value of x from equation (i) in equation (ii), we get

(y+5) y =50

⇒ y^2 +5y - 50 = 0

⇒ y^2 + 10y -5y - 50 = 0

⇒ y (y+10) – 5 (y+10) = 0

⇒ (y -5) (y + 10) = 0

⇒ y = 5 or -10

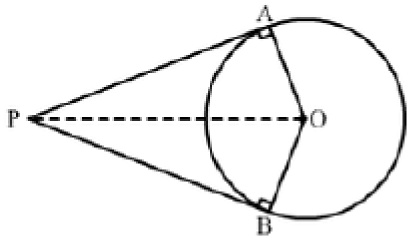
As y is a natural number, therefore y = 5

Other natural number = y + 5 = 5 + 5 = 10

Thus, the two natural numbers are 5 and 10.

Q26. Prove that the length of the tangents drawn from an external point to a circle are equal.

Solution:
Let AP and BP be the two tangents to the circle with centre O.



To Prove : AP = BP

Proof:

In ΔAOP and ΔBOP

OA =OB (radii of the same circle)

∠OAP = ∠OBP = 90° (since tangent at any point of a circle is perpendicular to the radius through the point of contact)

OP = OP (common)

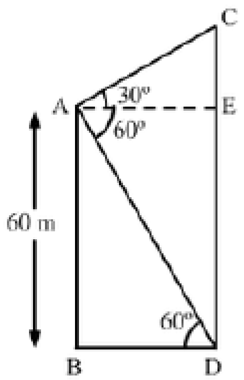
∴ ΔAOP ≅ Δ OBP (by R.H.S. congruence criterion)

∴ AP = BP (corresponding parts of congruent triangles)

Hence the length of the tangents drawn from an external point to a circle are equal.

Q27. The angles of elevation and depression of the top and the bottom of a tower from the top Of a building, 60 m high, are 30° and 60° respectively. Find the difference between the heights of the building and the tower and the distance between them.

Solution:
Let AB be the building and CD be the tower.



In right ΔABD.

$$\frac{AB}{BD} = \tan 60°$$
$$\Rightarrow \frac{60}{BD} = \sqrt{3}$$
$$\Rightarrow BD = \frac{60}{\sqrt{3}}$$
$$\Rightarrow BD = 20\sqrt{3}$$

In right ΔACE:

$$\frac{CE}{AE} = \tan 30°$$
$$\Rightarrow \frac{CE}{BD} = \frac{1}{\sqrt{3}} \ (\because AE = BD)$$
$$\Rightarrow CE = \frac{20\sqrt{3}}{\sqrt{3}} = 20$$

Height of the tower = CE + ED = CE + AB = 20 m + 60 m = 80 m

Difference between the heights of the tower and the building = 80 m – 60m = 20 m

Distance between the tower and the building = BD = 20 $\sqrt{3}$ m.

Q28. A bag contains cards numbers from 1 to 49. A card is drawn from the bag at random, after mixing the cards thoroughly. Find the probability that the number on the drawn card is:

(1) An odd number

(2) A multiple of 5

(3) A perfect Square

(4) An even prime number.

Solution:
Total number of cards = 49

(1)

Total number of outcomes = 49

The odd numbers form 1 to 49 are 1, 3,5,7,9,11,13,15,17, 19,21, 23, 2527,29,31,33,35, 37, 39, 41, 43, 45, 47 and 49.

Total number of favourable outcomes = 25

$\therefore$ Required probability $= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{25}{49}$

(ii) Total number of outcomes = 49

The number 5,10,15,20,25,30,35,40 and 45 multiples of 5.

The number of favourable outcomes = 9

$\therefore$ Required probability $= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{9}{49}$

(iii) Total number of outcomes = 49

The number 1, 4, 9, 16, 25, 36 and 49 are perfect squares.

Total number of favourable outcomes = 7

$\therefore$ Required probability $= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{7}{49} = \dfrac{1}{7}$

(iv)

Total number of outcomes = 49

We know that there is only one even prime number which is 2

Total number of favourable outcomes = 1

$\therefore$ Required probability $= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{1}{49}$

Q29. Find the ratio in which the point P (X, 2) divides the line segment joining the points A (12, 5)and B (4, -3). Also find the value of X

Solution:
Let the Point P (x, 2) divide the line segment joining the points A (12, 5) and B (4, -3) in the ratio k: 1

Then, the coordinates of P are $\left(\dfrac{4k+12}{k+1}, \dfrac{-3k+5}{k+1}\right)$

Now, the coordinates of P are (x,2)

$\therefore \dfrac{4k+12}{k+1} = x$ and $\dfrac{-3k+5}{k+1} = 2$

$\dfrac{-3k+5}{k+1} = 2$

$\Rightarrow -3k+5 = 2k+2$

$\Rightarrow 5k = 3$

$\Rightarrow k = \dfrac{3}{5}$

Substituting $k = \dfrac{3}{5}$ in $\dfrac{4k+12}{k+1} = x$, we get

$$x=\frac{4\times\frac{3}{5}+12}{\frac{3}{5}+1}$$

$$\Rightarrow x=\frac{12+60}{3+5}$$

$$\Rightarrow x=\frac{72}{8}$$

$$\Rightarrow x=9$$

Thus, the value of x is 9

Also, the point P divides the line segment joining the points A (12, 5) and (4, -3) in the ratio $\frac{3}{5}:1$, i.e. 3:5.

Q30. Find the values of k for which the quadratic equation (k + 4) x^2 + (k + 1)x + 1 = 0 has equal roots. Also find these roots.

Solution:

Given quadratic equation:

$(k + 4)x^2 + (k + 1)x + 1=0$

Since the given quadratic equation has equal roots, Its discriminant should be zero.

∴ D = 0

$\Rightarrow(k+1)^2- 4 \times (k+4) \times 1 = 0$

$\Rightarrow k^2+2k+1- 4k - 16=0$

$\Rightarrow k^2- 2k -15 =0$

$\Rightarrow k^2 - 5k+ 3k -15 = 0$

$\Rightarrow(k-5) (k+3) =0$

$\Rightarrow k-5 =0$ or $k+3 =0$

$\Rightarrow k = 5$ or -3

Thus, the values of k are 5 and -3

For k = 5 $(k+4)x^2 +(k+1)x +1= 0$

$\Rightarrow 9x^2 +6x +1 = 0$

$\Rightarrow(3x)^2 +2(3x) +1 =0$

$\Rightarrow(3x +1)^2 =0$

$\Rightarrow x = -\frac{1}{3}, -\frac{1}{3}$

For k = -3 $(k+4)x^2 + (k+1)x + 1 = 0$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x - 1)^2 = 0$

$\Rightarrow$ x = 1, 1

Thus, the equal root of the given quadratic equation is either 1 or $-\frac{1}{3}$

Q31. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P.

Solution:
Let a and d be the first term and the common difference of an A. P. respectively.

n^{th} term of an A. P, $a_n = a + (n - 1)d$

Sum of n terms of an A. P , $S_n = \frac{n}{2}[2a + (n-1)d]$

We have:

Sum of the first 10 terms = $\frac{10}{2}[2a + 9d]$

$\Rightarrow 210 = 5[2a+9d]$

$\Rightarrow 42 = 2a+9d$ .... (1)

15^{th} term from the last = $(50-15+1)^{th}$ = 36^{th} term from the beginning

Now, $a_{36} = a + 35d$

$\therefore$ Sum of the last 15 terms = $\frac{15}{2}(2a_{36} + (15-1)d)$

$= \frac{15}{2}[2(a+35d) + 14d]$

$= 15[a + 35d + 7d]$

$\Rightarrow 2565 = 15[a+42d]$

$\Rightarrow 171 = a+42d$ ....(2)

From (1) and (2), we get,

d = 4

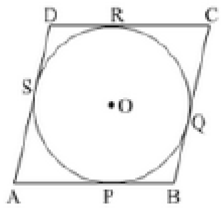
a = 3

So, the A. P. formed is 3, 7, 11, 15 …. and 199.

Q32 . Prove that a parallelogram circumscribing a circle is a rhombus.

Solution:
Given ABCD be a parallelogram circumscribing a circle with centre O.

To Prove: ABCD is a rhombus.



We know that the tangents drawn to a circle from an exterior point are equal is length.

∴ AP = AS, BP = BQ, CR =CQ AND DR = DS.

AP +BP +CR +DR = AS +BQ +CQ +DS

(AP +BP) + (CR+DR) = (AS+DS) +BQ +CQ)

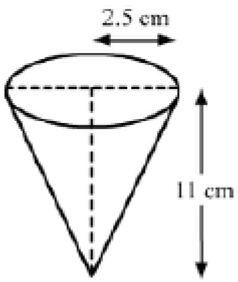
∴AB +CD =AD +BC OR 2AB = 2BC (since AB =DC and AD = BC)

∴ AB = BC = DC =AD

Therefore, ABCD is a rhombus.

Q33. Sushant has a vessel, of the form of an inverted cone, open at the top, of height 11 cm and Radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put In the vessel due to which $\frac{2}{5}$th of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the Flower beds. What value has been shown by Sushant?

Solution:



Height (h) of the conical vessel = 11 cm

Radius (r_1) of the conical Vessel = 2.5 cm

Radius (r_2) of the metallic spherical balls = $\frac{0.5}{2} = 0.25$ cm

Let n be the number of spherical balls = that were dropped in the the vessel.

Volume of the water spilled = Volume of the spherical balls dropped

$\frac{2}{5}\times$ Volume of cone = n × Volume of one spherical ball

$$\Rightarrow \frac{2}{5}\times\frac{1}{3}\pi r\frac{2}{1}h = n\times\frac{4}{3}\pi r\frac{3}{2}$$

$$\Rightarrow r\frac{2}{1}h = n\times 10r\frac{3}{2}$$

$$\Rightarrow (2.5)^2\times 11 = n\times 10\times(0.25)^3$$

⇒ 68.75 = 0.15625n

⇒ n = 440

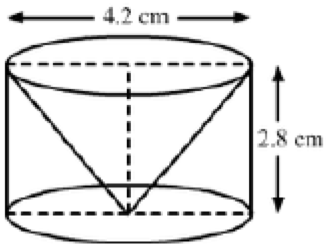
Hence, the number of spherical balls that were dropped in the vessel is 440.

Sushant made the arrangement so that the water that flows out, irrigates the flower beds.

This shows the judicious usage of water.

Q34. From a solid cylinder of height 2.8 cm and diameter 4.2 cm. a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Take π=22/7]

Solution:
The following figure shows the required cylinder and the conical cavity.



Given Height (b) of the conical Part = Height (h) of the cylindrical part = 2.8 cm

Diameter of the cylindrical part = Diameter of the conical part = 4.2 cm

∴ Radius $\rightarrow$ of the cylindrical part = Radius $\rightarrow$ of the conical part = 2.1 cm

Slant height (l) of the conical part $= \sqrt{r^2 + h^2}$

$$= \sqrt{(2.1)^2 + (2.8)^2}\,cm$$
$$= \sqrt{4.41 + 7.81}\,cm$$
$$= \sqrt{12.25} cm$$
$$= 3.5\,cm$$

Total surface area of the remaining solid = Curved surface area of the cylindrical part +Curved surface area of the conical part + Area of the cylindrical base

$$= 2\pi rh + \pi rl + \pi r^2$$
$$= \left(2 \times \frac{22}{7} \times 2.1 \times 2.8 + \frac{22}{7} \times 2.1 \times 3.5 + \frac{22}{7} \times 2.1 \times 2.1\right) cm^2$$

= (36.96 + 23.1 + 13.86) cm^2

=73.92 cm^2

Thus, the total surface area of the remaining solid is 73.92 cm^2