Previous Year Question Paper 2012

General Instuctions *:*

1. All questions are **compulsory**.

2. The question paper consists of **34** questions divided into **four sections** A, B, C, and D.

3. Section A contains of **10** questions of 1 mark each, which are multiple choice type question, **Section B** contains of **8** questions of 2 marks each, **Section C** contains of **10** questions of 3 marks each and **Section D** contains of **6** questions of 4 marks each.

4. Question numbers **1 to 8** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.

5. There is no overall choice. However, internal choice has been provided in **one** question of **2 marks**, **three** questions of **3 marks** each and **two** questions of 4 **marks** each. You have to attempt only one of the alternatives in all such questions.

6. Use of calculator is **not** permitted.

SECTION – A

1. The length of shadow of a tower on the plane ground is 3 times the height of the tower. The angle of elevation of sun is:

(A) 45°

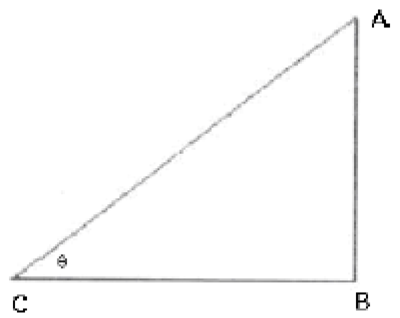
(B) 30°

(C) 60°

(D) 90°

Solution:

Correct answer: B



Let AB be the tower and BC be its shadow. Let θ be the angle of elevation of the sun.

According to the given information,

BC = $\sqrt{3}$ AB ... (1)

In ΔABC,

$\tan\theta = \frac{AB}{BC} = \frac{AB}{\sqrt{3}AB} = \frac{1}{\sqrt{3}}$ [Using (1)]

We know that tan 30 = $\frac{1}{\sqrt{3}}$

$\therefore \theta = 30°$

Hence, the angle of elevation of the sun is 30°.

2. If the area of a circle is equal to sum of the areas of two circles of diameters 10 cm and 24 cm, then the diameter of the larger circle (in cm) is:

(A) 34

(B) 26

(C) 17

(D) 14

Solution:

Correct answer: B

Diameters of two circles are given as 10 cm and 24 cm.

Radius of one circle = r_1 = 5 cm

Radius of one circle = r_2 = 12 cm

According to the given information,

Area of the larger circle = $\pi(r_1)^2 + \pi(r_2)^2$

$= \pi(5)^2 + \pi(12)^2$

$= \pi(25+144)$

$= 169\pi$

$= \pi(13)^2$

$\therefore$ Radius of larger circle = 13 cm

Hence, the diameter of larger circle = 26 cm

3. If the radius of the base of a right circular cylinder is halved, keeping the height the same, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is:

(A) 1 : 2

(B) 2 : 1

(C) 1 : 4

(D) 4 : 1

Solution:

Correct answer: C

Let the original radius and the height of the cylinder be r and h respectively.

Volume of the original cylinder = $\pi r^2 h$

Radius of the new cylinder = $\frac{r}{2}$

Height of the new cylinder = h

Volume of the new cylinder = $\pi\left(\frac{r}{2}\right)^2 h = \frac{\pi r^2 h}{4}$

Required ratio $\frac{\text{Volume of the new cylinder}}{\text{Volume of the original cylinder}} = \frac{\frac{\pi r^2 h}{4}}{\pi r^2 h} = \frac{1}{4} = 1:4$

4. Two dice are thrown together. The probability of getting the same number on both dice is:

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $\frac{1}{12}$

Solution:

Correct answer: C

When two dice are thrown together, the total number of outcomes is 36.

Favourable outcomes = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$\therefore$ Required probability = $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{6}{36} = \frac{1}{6}$

5. The coordinates of the point P dividing the line segment joining the points A(1,3) and B(4,6) in the ratio 2 : 1 are:

(A) (2,4)

(B) 3,5)

(C) (4,2)

(D) 5,3)

Solution:

Correct answer: B

It is given that the point P divides AB in the ratio 2: 1.

Using section formula, the coordinates of the point P are

$$\left(\frac{1\times1+2\times4}{2+1}, \frac{1\times3+2\times6}{2+1}\right) = \left(\frac{1+8}{3}, \frac{3+12}{3}\right) = (3,5)$$

Hence the coordinates of the point P are (3, 5).

6. If the coordinates of the one end of a diameter of a circle are (2,3) and the coordinates of its centre are (-2,5), then the coordinates of the other end of the diameter are:

(A) (-6,7)
(B) (6,-7)
(C) (6,7)
(D) (-6,-7)

Solution:
Correct answer: A
Let the coordinates of the other end of the diameter be (x, y).
We know that the centre is the mid-point of the diameter. So, O(-2, 5) is the mid-point of the diameter AB.
The coordinates of the point A and B are (2, 3) and (x, y) respectively.
Using mid-point formula, we have,

$$-2=\frac{2+x}{2}\Rightarrow -4=2+x\Rightarrow x=-6$$
$$5=\frac{3+y}{2}\Rightarrow 10=3+y\Rightarrow y=7$$

Hence, the coordinates of the other end of the diameter are (-6, 7).

7. The sum of first 20 odd natural number is :
(A) 100
(B) 210
(C) 400
(D) 420
Solution:
Correct answer: C
The first 20 odd numbers are 1, 3, 5, … … 39
This is an AP with first term 1 and the common difference 2.

Sum of 20 terms = S_{20}

$$S_{20}=\frac{20}{2}[2(1)+(20-1)(2)]=10[2+38]=400$$

Thus, the sum of first 20 odd natural numbers is 400.

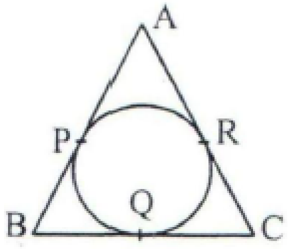
8. If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then ab equals:
(A) 3
(B) $-\frac{7}{2}$
(C) 6
(D) -3
Solution:

Correct answer: A
It is given that 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$.
Therefore, y = 1 will satisfy both the equations.

$\therefore a(1)^2 + a(1) + 3 = 0$
$\Rightarrow a + a + 3 = 0$
$\Rightarrow 2a + 3 = 0$
$\Rightarrow a = \frac{-3}{2}$
Also, $(1)^2 + (1) + b = 0$
$\Rightarrow 1 + 1 + b = 0$
$\Rightarrow b = -2$

$\therefore ab = \frac{-3}{2} \times -2 = 3$

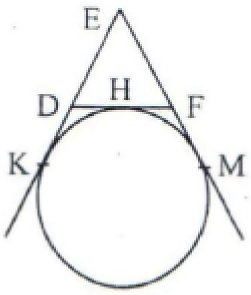
9. In Fig., the sides AB, BC and CA of a triangle ABC, touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11 cm, then the length of BC (in cm) is:



(A) 11
(B) 10
(C) 14
(D) 15

Solution:
Correct answer: B
It is known that the lengths of tangents drawn from a point outside a circle are equal in length.
Therefore, we have:
AP = AR ... (1) (Tangents drawn from point A)
BP = BQ ... (2) (Tangents drawn from point B)
CQ = CR ... (3) (Tangents drawn from point C)
Using the above equations,
AR = 4 cm (AP = 4 cm, given)
BQ = 3 cm (BP = 3 cm, given)
AC = 11 cm $\Rightarrow$ RC = 11 cm – 4 cm = 7 cm
$\Rightarrow$ CQ = 7 cm
Hence, BC = BQ + CQ = 3 cm + 7 cm = 10 cm

10. In Fig., a circle touches the side DF of Δ EDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then the perimeter of Δ EDF (in cm) is:



(A) 18
(B) 13.5
(C) 12
(D) 9

Solution:

Correct answer: A

It is known that the tangents from an external point to the circle are equal.

$\therefore$ EK = EM, DK = DH and FM = FH ... (1)

Perimeter of ΔEDF = ED + DF + FE

= (EK – DK) + (DH + HF) + (EM – FM)

= (EK – DH) + (DH + HF) + (EM – FH) [Using (1)]

= EK + EM

= 2 EK = 2 (9 cm) = 18 cm

Hence, the perimeter of EDF is 18 cm.

SECTION – B

11. If a point A(0,2) is equidistant from the points B(3,p) and C(p,5) then find the value of p.

Solution:

Solution:

It is given that the point A (0, 2) is equidistant from the points B(3, p) and C(p, 5).

So, AB = AC $\Rightarrow$ $AB^2 = AC^2$

Using distance formula, we have:

$\Rightarrow (0-3)^2 + (2-p)^2 = (0-p)^2 + (2-5)^2$

$\Rightarrow 9 + 4 + p^2 - 4p = p^2 + 9$

$\Rightarrow 4 - 4p = 0$

$\Rightarrow 4p = 4$

$\Rightarrow p = 1$

Hence, the value of p = 1.

12. A number is selected at random from first 50 natural numbers. Find the probability that it is a multiple of 3 and 4.

Solution:

Solution:

The total number of outcomes is 50.

Favourable outcomes = {12, 24, 36, 48}

$\therefore$ Required probability $= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{4}{50} = \frac{2}{25}$

13.The volume of a hemisphere is $2425\frac{1}{2}$ cm^3. Find its curved surface area.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution:

Solution:

Given volume of a hemisphere $= 2425\frac{1}{2} cm^2 = \frac{4851}{2} cm^3$

Now, let r be the radius of the hemisphere

Volume of a hemisphere $= \frac{2}{3}\pi r^3$

$\therefore \frac{2}{3}\pi r^3 = \frac{4851}{2}$

$\Rightarrow \frac{2}{3} \times \frac{22}{7} \times r^3 = \frac{4851}{2}$

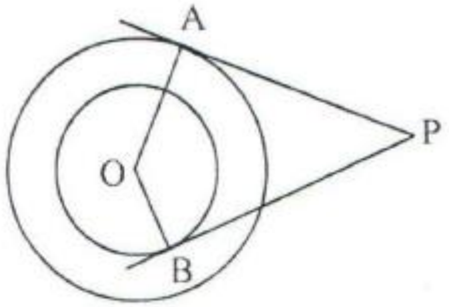
$\Rightarrow r^3 = \frac{4851}{2} \times \frac{3}{2} \times \frac{7}{22} = \left(\frac{21}{2}\right)^3$

$\therefore r = \frac{21}{2} cm$

So, Curved surface area of the hemisphere $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 693\ sq.\ cm$

14.Tangents PA and PB are drawn from an external point P to two concentric circle with centre O and radii 8 cm and 5 cm respectively, as shown in Fig., If AP = 15 cm, then find the length of BP.

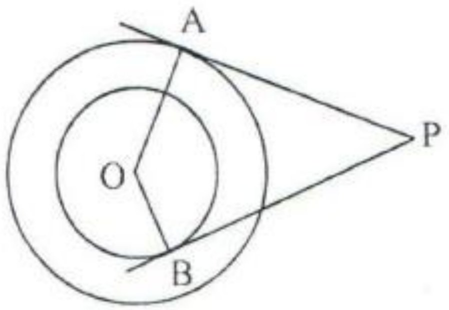


Solution:

Given: Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii OA = 8 cm, OB = 5 cm respectively. Also, AP = 15 cm

To find: Length of BP

Construction: We join the points O and P.



Solution: OA ⊥ AP ; OB ⊥ BP

[Using the property that radius is perpendicular to the tangent at the point of contact of a circle]

In right angled triangle OAP,

$OP^2 = OA^2 + AP^2$ [Using Pythagoras Theorem]

$= (8)^2 + (15)^2 = 64 + 225 = 289$

∴ OP = 17 cm

In right angled triangle OBP,

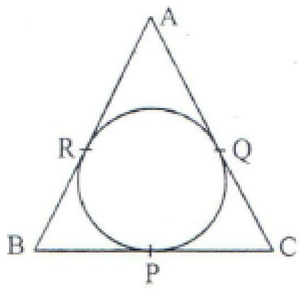
$OP^2 = OB^2 + BP^2$

$\Rightarrow BP^2 = OP^2 - OB^2$

$= (17)^2 - (5)^2 = 289 - 25 = 264$

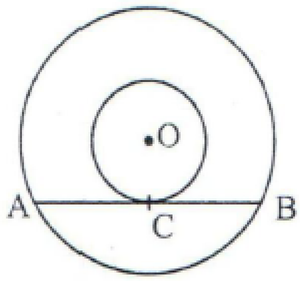
$\therefore BP = \sqrt{264} = 2\sqrt{66}\ cm.$

15.In fig., an isosceles triangle ABC, with AB = AC, circumscribes a circle. Prove that the point of contact P bisects the base BC.



OR

In fig., the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that AC = CB.



Solution:

Given: ABC is an isosceles triangle, where AB = AC, circumscribing a circle.

To prove: The point of contact P bisects the base BC.

i.e. BP = PC

Proof: It can be observed that

BP and BR ; CP and CQ; AR and AQ are pairs of tangents drawn to the circle

from the external points B , C and A respectively.

So, applying the result that the tangents drawn from an external point to a circle, we get

BP = BR --- (i)

CP = CQ --- (ii)

AR = AQ --- (iii)

Given that AB = AC

⇒ AR + BR = AQ + CQ

⇒ BR = CQ [from (iii)]

⇒ BP = CP [from (i) and (ii)]

∴ P bisects BC.

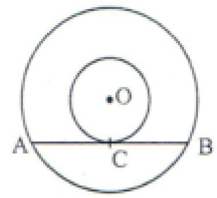
Hence proved.

OR

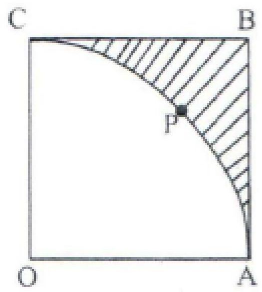
Given: The chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C.

To prove: AC = CB

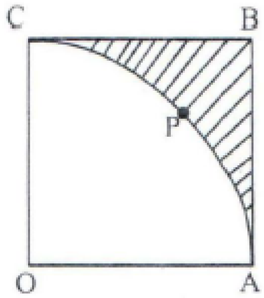
Construction: Let us join OC.



Proof: In the smaller circle, AB is a tangent to the circle at the point of contact C.
∴ OC ⊥ AB ------ (i)
(Using the property that the radius of a circle is perpendicular to the tangent at the point of contact)
For the larger circle, AB is a chord and from (i) we have OC ⊥ AB
∴ OC bisects AB
(Using the property that the perpendicular drawn from the centre to a chord of a circle bisects the chord)
∴ AC = CB



16. In fig., OABC is a square of side 7 cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Solution:
Given, OABC is a square of side 7 cm
i.e. OA = AB = BC = OC = 7cm
∴ Area of square OABC = $(\text{side})^2 = 7^2 = 49$ sq.cm
Given, OAPC is a quadrant of a circle with centre O.

∴ Radius of the sector = OA = OC = 7 cm.
Sector angle = 90°

$\therefore$ Area of quadrant OAPC $\frac{90^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times (7)^2 = \frac{77}{2} sq.\,cm = 38.5\, sq.\,cm$

$\therefore$ Area of shaded portion = Area of Square - OABC Area of quadrant OAPC

$= (49 - 38.5) sq.\,cm = 10.5\, sq.cm$

17.Find the sum of all three digit natural numbers, which are multiples of 7.

Solution:

First three- digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore the series is 105, 112, 119,…

The maximum possible three digit number is 999.

When we divide by 7, the remainder will be 5.

Clearly, 999 – 5 = 994 is the maximum possible three – digit number divisible by 7.

The series is as follows:

105, 112, 119, …., 994

Here a = 105, d = 7

Let 994 be the nth term of this A.P.

$a_n = a + (n-1)d$

$\Rightarrow 994 = 105 + (n-1)7$

$\Rightarrow (n-1)7 = 889$

$\Rightarrow (n-1) = 127$

$\Rightarrow n = 128$

So, there are 128 terms in the A.P.

$\therefore$ Sum $= \frac{n}{2}$ {first term + last term}

$= \frac{128}{2}\{a_1 + a_{128}\}$

$= 64\{105 + 994\} = (64)(1099) = 70336$

18.Find the values (s) of k so that the quadratic equation $3x^2 - 2kx + 12 = 0$ has equal roots.

Solution:

Given quadratic equation is $3x^2 - 2kx + 12 = 0$

Here a = 3, b = -2k and c = 12

The quadratic equation will have equal roots if Δ = 0

$\therefore b^2 - 4ac = 0$

Putting the values of a,b and c we get

$(2k)^2 - 4(3)(12) = 0$

$\Rightarrow 4k^2 - 144 = 0$

$\Rightarrow 4k^2 = 144$

$\Rightarrow k^2 = \frac{144}{4} = 36$

Considering square root on both sides,

$k = \sqrt{36} = \pm 6$

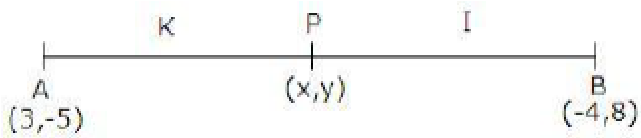
Therefore, the required values of k are 6 and -6.

SECTION – C

19.A point P divides the line segment joining the points A(3,-5) and B(-4,8) such that $\frac{AP}{PB} = \frac{K}{1}$. If P lies on the line x + y = 0, then find the value of K.

Solution:

Solution:



Let the co-ordinates of point P be (x, y)

Then using the section formula co-ordinates of P are.

$x = \frac{-4K+3}{K+1} \qquad y = \frac{8K-5}{K+1}$

Since P lies on x+y=0

$\therefore \frac{-4K+3}{K+1} + \frac{8K-5}{K+1} = 0$

$\Rightarrow 4K - 2 = 0$

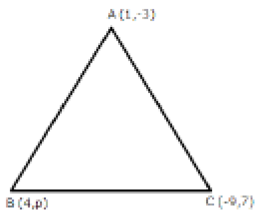
$\Rightarrow K = \frac{2}{4}$

$\Rightarrow K = \frac{1}{2}$

Hence the value of $K = \frac{1}{2}$.

20.If the vertices of a triangle are (1,-3), (4,p) and (-9,7) and its area is 15 sq. units, find the value (s) of p.

Solution:



The area of a Δ, whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$\Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Substituting the given coordinates

Area of $\Delta = \frac{1}{2}|1(p-7) + 4(7+3) + (-9)(-3-p)|$

$$\Rightarrow \frac{1}{2}|(p-7) + 40 + 27 + 9p| = 15$$

$$\Rightarrow 10p + 60 = \pm 30$$

$$\Rightarrow 10p = -30 \quad or \quad 10p = -90$$

$$\Rightarrow p = -3. \quad or \quad p = -9$$

Ans hence the value of p=-3 or -9

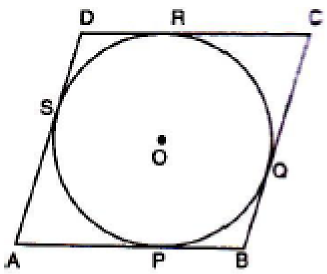
21.Prove that the parallelogram circumscribing a circle is a rhombus.

OR

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:

Let ABCD be a parallelogram such that its sides touching a circle with centre O. We know that the tangents to a circle from an exterior point are equal in length.



$\therefore$ AP = AS [From A] ...(i)

BP = BQ [From B] ...(ii)

CR = CQ [From C] ...(iii)

and, DR = DS [From D] ...(iv)

Adding (i), (ii), (iii) and (iv), we get

AP+BP+CR+DR=AS+BQ+CQ+DS

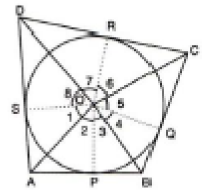
$\Rightarrow$ (AP+BP)+(CR+DR)=(AS+DS)+(BQ+CQ)

$\Rightarrow$ AB+CD = AD+BC

$\Rightarrow$ 2 AB = 2 BC [$\because$ ABCD is a parallelogram $\therefore$ AB=CD and BC = AD]
$\Rightarrow$ AB=BC
Thus, AB=BC=CD=AD
Hence, ABCD is a rhombus.

OR

A circle with centre O touches the sides AB, BC, CD, and DA of a quadrilateral ABCD at the points P, Q, R and S respectively.
TO PROVE : $\angle$AOB + $\angle$COD = 180°
and, $\angle$AOD + $\angle$BOC = 180°

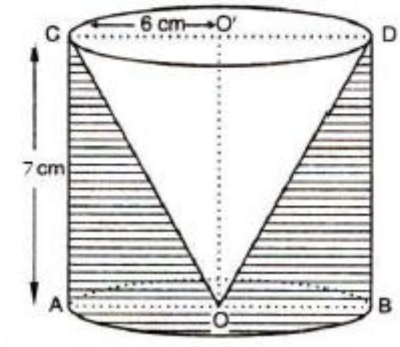


CONSTRUCTION
Join OP, OQ, OR and OS.
PROOF Since the two tangents drawn from a external point to a circle subtend equal angles at the centre.
$\therefore$ $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$ and $\angle 7 = \angle 8$
Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$
$\left[\begin{array}{l}\text{Sum of all the angles} \\ \text{subtended at a point is } 360°\end{array}\right]$
$\Rightarrow$ $2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360$ and $2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360°$
$\Rightarrow$ $(\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180$ and $(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$
$\left[\begin{array}{l}\because \angle 2+\angle 3=\angle AOB, \angle 6+\angle 7=\angle COD \\ \angle 1+\angle 8=\angle AOD \text{ and } \angle 4+\angle 5=\angle BOC\end{array}\right]$
$\Rightarrow \angle AOB+\angle COD=180°$
and $\angle AOD+\angle BOC=180°$
Hence Proved

22. From a solid cylinder of height 7 cm and base diameter 12 cm, a conical cavity of same height and same base diameter is hollowed out. Find the total surface area of the remaining solid. $\left[\text{Use } \pi=\frac{22}{7}\right]$

OR

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, then find the radius and slant height of the heap.
Solution:



Given: radius of cyl=radius of cone=r=6cm

Height of the cylinder=height of the cone=h=7cm

Slant height of the cone= l

$\sqrt{7^2+6^2}$

$=\sqrt{85}cm$

Total surface area of the remaining solid

= curved surface area of the cylinder + area of the base of the cylinder + curved surface area of the cone

$(2\pi rh+\pi r^2+\pi rl)$

$=2\times\frac{22}{7}\times 6\times 7+\frac{22}{7}\times 6^2+\frac{22}{7}\times 6\times\sqrt{85}$

$=264+\frac{792}{7}+\frac{132}{7}\sqrt{85}$

$=377.1+\frac{132}{7}\sqrt{85}cm^2$

OR

Volume of the conical heap=volume of the sand emptied from the bucket.

Volume of the conical heap=

$\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi r^2\times 24cm^2$ (height of the coneis24)----------(1)

Volume of the sand in the bucket $=\pi r^2 h$

$=\pi(18)^2 32cm^2 -------(2)$

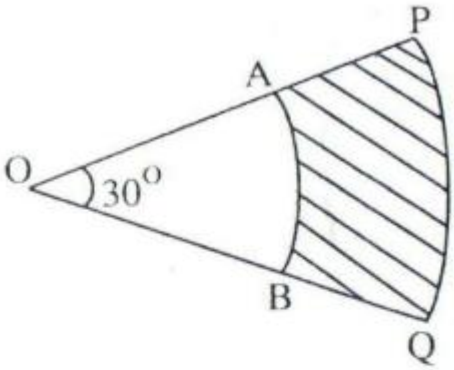
Equating 1 and 2

$\frac{1}{3}\pi r^2\times 24=\pi(18)^2 32$

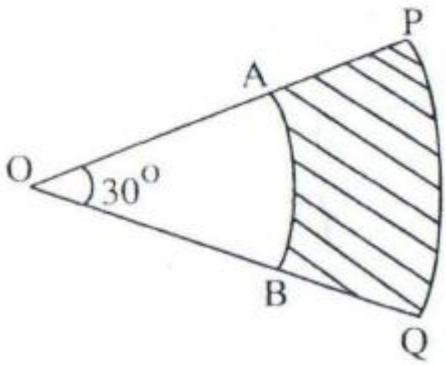
$\Rightarrow r^2=\frac{(18)^2\times 32\times 3}{24}$

$\Rightarrow r=36cm$

23. In fig., PQ and AB are respectively the arcs of two concentric circles of radii 7 cm and 3.5 cm and centre O. If ∠ POQ = 30°, then the area of the shaded region. $\left[\text{Use } \pi=\frac{22}{7}\right]$



Solution:



Area of the shaded region=
Area of sector POQ-Area of sector AOB

$$\left(\frac{\theta}{360}\pi R^2 - \frac{\theta}{360}\pi r^2\right)$$

Area of Shaded region $= \frac{30}{360} \times \frac{22}{7} \times (7^2 - 3.5^2)$

$$= \frac{77}{8} cm^2$$

24. Solve for x: $4x^2 - 4ax + (a^2 - b^2) = 0$

Or

Solve for x: $3x^2 - 6x + 2 = 0$

Solution:

$4x^2 - 4ax + (a^2 - b^2) = 0$

$\Rightarrow (4x^2 - 4ax + a^2) - b^2 = 0$

$\Rightarrow \left[(2x^2) - 2.2x.a + a^2\right] - b^2 = 0$

$\Rightarrow \left[\left(2x - a\right)^2\right] - b^2 = 0$

$\Rightarrow \left[(2x - a)^2 - b\right]\left[(2x - a) + b\right] = 0$

$\Rightarrow \left[(2x - a) - b\right] = 0$ or $\left[(2x - a) + b\right] = 0$

$\Rightarrow x = \dfrac{a + b}{2}; x = \dfrac{a - b}{2}$

OR

$3x^2 - 2\sqrt{6}x + 2 = 0$

$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\Rightarrow \sqrt{3} \times \left[\sqrt{3}x - \sqrt{2}\right] - \sqrt{2}\left[\sqrt{3}x - \sqrt{2}\right] = 0$

$\Rightarrow \left(\sqrt{3}x - \sqrt{2}\right)\left(\sqrt{3}x - \sqrt{2}\right) = 0$

$\Rightarrow \left(\sqrt{3}x - \sqrt{2}\right)^2 = 0$

$\therefore \sqrt{3}x - \sqrt{2} = 0$

$\Rightarrow \sqrt{3}x = \sqrt{2}$

$\Rightarrow x = \dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{\sqrt{2} \times \sqrt{3}}{(\sqrt{3})^2} = \dfrac{\sqrt{6}}{3}$

25. A kite is flying at a height of 45 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60 . Find the length of the string assuming that there is slack in the string.

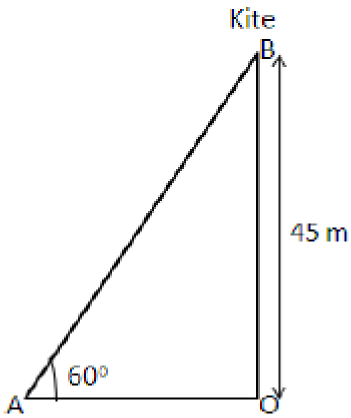
Solution:

Given: Position of kite is B.

Height of kite above ground= 45 m

Angle of inclination = 60°

Required length of string = AB



In right angled triangle AOB,

$$\sin A = \frac{OB}{AB}$$

$$\Rightarrow \sin 60^\circ = \frac{45}{AB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{45}{AB}$$

$$\Rightarrow AB = \frac{45\times 2}{\sqrt{3}} = \frac{90}{\sqrt{3}} = 30\sqrt{3}m$$

Hence, the length of the string is $30\sqrt{3}m$

26. Draw a triangle ABC with side BC = 6 cm, $\angle$C = 30° and $\angle$A = 105°. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of ΔABC.

Solution:

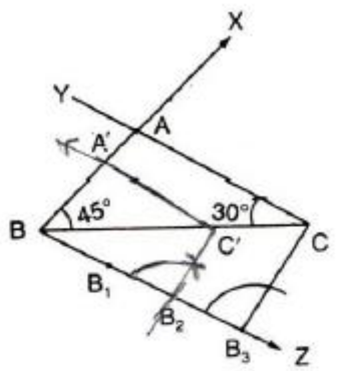
It is given that $\angle$A = 105°, $\angle$C = 30 .

Using angle sum property of triangle, we get, $\angle$B = 45

The steps of construction are as follows:

1. Draw a line segment BC = 6 cm.
2. At B, draw a ray making an angle of 45° with BC.
3. At C, draw a ray making an angle of 30° with BC. Let the two rays meet at point A.
4. Below BC, make an acute angle $\angle$CBX.
5. Along BX mark off three points B_1, B_2 , B_3 such that $BB_1 = B_1B_2 = B_2B_3$.
6. Join B_3C.
7. From B_2, draw B_2C' || B3C.
8. From C′, draw C′A′ || CA, meeting BA at the point A′.

Then A′BC′ is the required triangle.



27. The 16th term of an AP is 1 more than twice its 8th term. If the 12th term of the AP is 47, then find its nth term.

Solution:

Let a and d respectively be the first term and the common difference of the AP.

We know that the n^{th} term of an AP is given by $a_n = a + (n – 1)d$
According to the given information,
$A_{16} = 1 + 2\ a_8$
$\Rightarrow$ a + (16 – 1)d = 1 + 2[a + (8 – 1)d]
$\Rightarrow$ a + 15d = 1 + 2a + 14d
$\Rightarrow$ a + 15d = 1 + 2a + 14d
$\Rightarrow$ –a + d = 1 … (1)
Also, it is given that, $a_{12} = 47$
$\Rightarrow$ a + (12 – 1)d = 47
$\Rightarrow$ a + 11d = 47 … (2)
Adding (1) and (2), we have:
12d = 48
$\Rightarrow$ d = 4
From (1),
–a + 4 = 1 $\Rightarrow$ a = 3
Hence, $a_n = a + (n – 1)d = 3 + (n – 1)(4) = 3 + 4n – 4 = 4n – 1$
Hence, the n^{th} term of the AP is 4n – 1.

28.A card is drawn from a well shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) the queen of diamonds.

Solution:

Total number of outcomes=52
(i) Probability of getting a red king
Here the number of favourable outcomes=2

$$\text{probability} = \frac{\text{no.of favourable outcomes}}{\text{total number of outcome}}$$

$$= \frac{12}{52}$$

$$= \frac{3}{13}$$

(iii)Probability of queen of diamonds
number of queens of diamond=1,hence

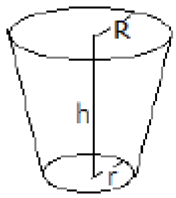
$$\text{probability} = \frac{\text{no.of favourable outcomes}}{\text{total number of outcome}}$$

$$\frac{1}{52}$$

SECTION – D

29.A bucket is in the form of a frustum of a cone and its can hold 28.49 litres of water. If the radii of its circular ends are 28 cm and 21 cm, find the height of the bucket. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution:



Here, R = 28 cm and r = 21 cm, we need to find h.

Volume of frustum = 28.49 L = 28.49×1000 cm^3 = 28490 cm^3

Now, Volume of frustum = $\frac{\pi h}{3}\left(R^2+Rr+r^2\right)$

$\Rightarrow \frac{22h}{7\times 3}\left(28^2+28\times 21+21^2\right)=28490$

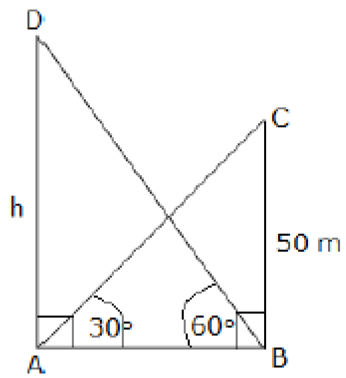
$\Rightarrow \frac{22}{21}h\times 1813=28490$

$\Rightarrow h=\frac{28490\times 21}{22\times 1813}=15cm$

Hence the height of bucket is 15 cm.

30.The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of depression from the top of the tower of the foot of the hill is 30°. If the tower is 50 m high, find the height of the hill.

Solution:



Let the height of hill is h.

In right triangle ABC,

$\frac{50}{AB}=\tan 30^\circ \Rightarrow \frac{50}{AB}=\frac{1}{\sqrt{3}}\Rightarrow AB=50\sqrt{3}$

In right triangle ABD,

$\frac{h}{AB}=\tan 60^\circ \Rightarrow \frac{h}{AB}=\sqrt{3}\Rightarrow h=\sqrt{3}AB$

$\Rightarrow h=\sqrt{3}\left(50\sqrt{3}\right)=150m$

Hence the height of hill is 150 m.

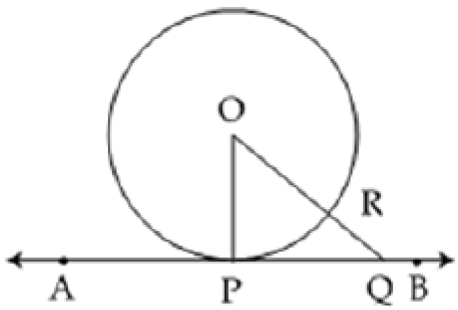
31. AB + CD = AD + BC

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

OR

A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.

Solution:



Given: AB is a tangent to a circle with centre O.

To prove: OP is perpendicular to AB.

Construction: Take a point Q on AB and join OQ.

Proof: Since Q is a point on the tangent AB, other than the point of contact P, so Q will be outside the circle.

Let OQ intersect the circle at R.

Now OQ = OR + RQ

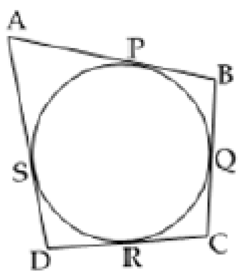
$\Rightarrow$ OQ > OR $\Rightarrow$ OQ > OP [as OR = OP]

$\Rightarrow$ OP < OQ

Thus OP is shorter than any other segment among all and the shortest length is the perpendicular from O on AB.

$\therefore$ OP $\perp$ AB. Hence proved.

OR



Let ABCD be a quadrilateral, circumscribing a circle.

Since the tangents drawn to the circle from an external point are equal,

we have

AP = AS ... (1)

PB = BQ ... (2)
RC = QC ... (3)
DR = DS ... (4)
Adding, (1), (2), (3) and (4), we get
AP + PB + RC + DR = AS + BQ + QC + DS
(AP + PB) + (DR + RC) = (AS + SD) + (BQ + QC)
AB + CD = AD + BC.
Hence, Proved.

32.A shopkeeper buys some books for ₹80. If he had bought 4 more books for the same amount, each book would have cost ₹1 less. Find the number of books he bought.

OR

The sum of two number is 9 and the sum of their reciprocals is $\frac{1}{2}$. Find the numbers.

Solution:
Total cost of books = Rs 80
Let the number of books = x

So the cost of each book = $\text{Rs}\frac{80}{x}$

Cost of each book if he buy 4 more book $=\text{Rs}\frac{80}{x+4}$

As per given in question:

$$\frac{80}{x}-\frac{80}{x+4}=1$$
$$\Rightarrow \frac{80x+320-80x}{x(x+4)}=1$$
$$\Rightarrow \frac{320}{x^2+4x}=1$$
$$\Rightarrow x^2+4x-320=0$$
$$\Rightarrow (x+20)(x-16)=0$$
$$\Rightarrow x=-20,16$$

Since number of books cannot be negative,
So the number of books he brought is 16.

OR

Let the first number be x then the second number be 9 – x as the sum of both numbers is 9.

Now the sum of their reciprocal is $\frac{1}{2}$, therefore

$\frac{1}{x}+\frac{1}{9-x}=\frac{1}{2}$

$\Rightarrow \frac{9-x+x}{x(9-x)}=\frac{1}{2}$

$\Rightarrow \frac{9}{9x-x^2}=\frac{1}{2}$

$\Rightarrow 18=9x-x^2$

$\Rightarrow x^2-9x+18=0$

$\Rightarrow (x-6)(x-3)=0$

$\Rightarrow x=6,3$

If x = 6 then other number is 3.
and if x = 3 then other number is 6.
Hence numbers are 3 and 6.

33.Sum of the first 20 terms of an AP is -240, and its first term is 7. Find its 24th term.

Solution:

Given: $S_{20}=-240$ and $a=7$

Consider, $S_{20}=-240$

$\Rightarrow \frac{20}{2}(2\times 7+19d)=-240 \quad \left[\because S_n=\frac{n}{2}[2a+(n-1)d]\right]$

$\Rightarrow 10(14+19d)=-240$

$\Rightarrow 14+19d=-24$

$\Rightarrow 19d=-38$

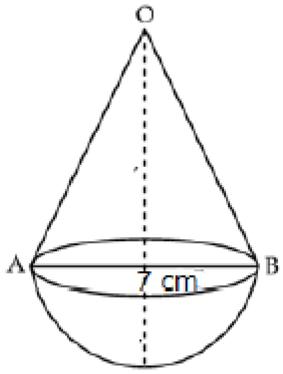
$\Rightarrow d=-2$

Now, $a_{24}=a+23d=7+23\times -2=-39$

Hence, $a_{24}=-39$

34.A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid. $\left[\text{Use } \pi=\frac{22}{7}\right]$

Solution:



Radius of hemi-sphere = 7 cm

Radius of cone = 7 cm

Height of cone = diameter = 14 cm

Volume of solid = Volume of cone + Volume of Hemi-sphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 49(14 + 14)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 49 \times 28$$

$$= \frac{22 \times 7 \times 28}{3} = \frac{4312}{3} cm^3$$