Previous Year Question Paper 2011

- Please check that this question paper contains **16** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **34** questions.
- **Please write down the serial number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer script during this period.

SUMMATIVE ASSESSMENT-II

MATHEMATICS

Time Allowed: **3** hours Maximum Marks: **80**

General Instructions

1. **All** questions are compulsory.
2. The question paper consists of **34** questions divided into four sections A, B, C and D.
3. Section A contains **30** questions of **1** mark each, which are multiple choice type questions, Section B contains **8** questions of **2** marks each, Section C contains **10** questions of **3** marks each, Section D contains **6** questions of **4** marks each.
4. There is no overall choice in the paper. However, internal choice is provided in one question of **2** marks. **3** questions of **3** marks each and two questions of **4** marks each.

5. Use of calculators is not permitted.

SECTION-A

Question Numbers 1 to 30 carry 1 mark each. For each of the questions 1 to 30, four alternative choices have been provided, of which only one is correct. Select the correct choice.

1. The roots of the equation $x^2+x-p\left(p+1\right)=0$, where p is a constant, are:

(A) p,p+1

(B) -p,p+1

(C) p,-(p+1)

(D) -p,-(p+1) **1 Mark**

Ans: The given equation is $x^2+x-p(p+1)=0$. Now, solving this equation using Quadratic formula, i.e. $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Putting the values in the formula, we get :

$$\Rightarrow x=\frac{1\pm\sqrt{1-4(1)\left(-p\left(p+1\right)\right)}}{2}$$

$$\Rightarrow x=\frac{1\pm\sqrt{1+4p^2+p}}{2}$$

$$\Rightarrow x=\frac{-1\pm\sqrt{\left(2p+1\right)^2}}{2}=-\frac{1}{2}\pm\frac{\left(2p+1\right)}{2}$$

Thus, the two roots are:

$$\alpha=-\frac{1}{2}+p+\frac{1}{2}=p$$

$$\beta=-\frac{1}{2}-p-\frac{1}{2}=-p-1$$

Therefore, option is the correct answer.

2. In an AP, if, d=-2, n=5 and an=0, then the value of a is : **1 Mark**

a) 10

(B) 5

(C) -8

(D) 8

Ans: It is given that

d=-2

n=5

an=0

The formula for an is given as an=a+(n-1)d . Putting the values, we get

0=a+(5-1)-2

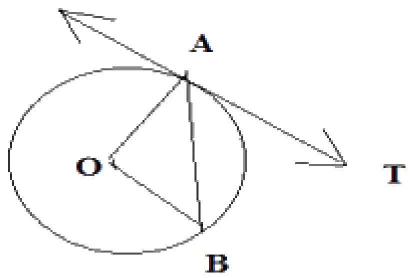
0=a+4(-2)

a=8

Hence, option (D) 8 is the correct answer.

3. In Fig. 1, O is the centre of a circle, AB is a chord and AT is the tangent at A. If

< AOB =100° then < BAT is equal to: 1 Mark



Ans: It is given in this question that AO and BO are radius therefore, triangle AOB is an isosceles triangle:

< AOB + < OBA + < OAB = 180°

Putting values, we get

100° + < OAB + < OAB = 180°

$\Rightarrow$ 2 < AOB=80°

$\Rightarrow < AOB = 40^{o}$

We know that tangent and radius are perpendicular to each other,

$\therefore < OAT = 40^{o}$

Simplifying,

$\therefore < OAB + < BAT = 40^{o}$

$< BAT = 90^{o} - 40^{o} = 50^{o}$

Hence, option (C) 50° is the correct answer.

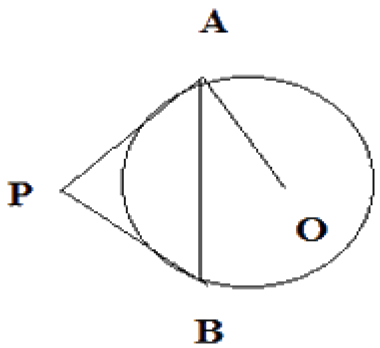
4. In Fig. 2, PA and PB are tangents to the circle with centre O. If <APB =60^{o}, then < OAB is : 1 Mark

a. 30^{o}

b. 60^{o}

c. 90^{o}

d. 15^{o}



Ans: It is given that PA and PB are two tangents. O is the centre of the circle and OA and OB are joined. Given: $< APB = 60^{\circ}$. We need to find $< OAB$ PA and PB are tangents to the circle, so, $PA = PB$, $< PAB = < PBA$. But $< APB = 60^{\circ}$. So, $< PAB + < PBA = 180^{\circ} - 60^{\circ} = 120^{\circ}$.

$\Rightarrow 2 < PAB = 120^{\circ}$

$\Rightarrow < PAB = 60^{o}$

Now, OA is radius and PA is tangent, hence, $OA \perp PA$,

$\Rightarrow < OPA = 90^{o}$

$\Rightarrow \angle OAB + \angle PAB = 90^\circ$

Putting the values, we get

$\Rightarrow \angle OAB + 60^\circ = 90^\circ$

$\Rightarrow \angle OAB = 30^\circ$

Hence, option (A) 30° is the correct answer.

5. The radii of two circles are 4 cm and 3 cm respectively. The diameter of the circle having area equal to the sum of the areas of the two circles (in cm) is: 1 Mark

a. 5

b. 7

c. 10

d. 14

Ans: The radius of one circle is $4\,\text{cm}$. Therefore, the area of the circle is :

$A = \pi \times (4)^2 = 16\pi$. The radius of the second circle is $3\,\text{cm}$. Therefore, the area of the circle is: $A = \pi \times (3)^2 = 9\pi$. Thus, the sum of the areas of two circles is:

$A' = 9\pi + 16\pi$

$\Rightarrow \pi r'^2 = 25\pi$

$\Rightarrow r' = 5\text{cm}$

Hence, the diameter of the circle is $10\,\text{cm}$. So, option (C) 10 is the correct answer.

6. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged, then the water level rises (in cm) by : 1 Mark

a) 3

(B) 4

(C) 5

(D) 6

Ans: It is given that: The diameter of the sphere is 18 cm. So, the radius is 9 cm. The diameter of the cylindrical vessel is 36cm. So, the radius is 18cm. Let us consider the height of water rise is H cm. Now, the sphere is completely submerged, Volume of the vessel is equal to the volume of the sphere. Putting the values, we get.

$$\pi R^2 H = \frac{4}{3}\pi r^3$$

$$\Rightarrow H = \frac{\frac{4}{3}r^3}{R^2} = \frac{4\times(9)^3}{3\times(18)^2} = 3\text{cm}$$

Hence, option (A)3 is the correct answer.

7. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 45°. The height of the tower (in metres) is:

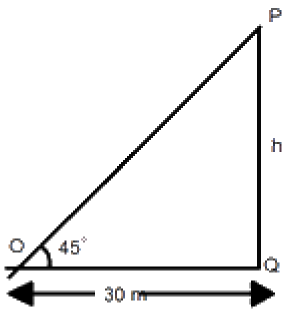
a) 15

(B) 30

(C) $30\sqrt{3}$

(D) $10\sqrt{3}$ **1 Mark**

Ans: Let us draw a figure for the given tower and the point in the question :



Let us consider the height of the tower to be h cm, now, in ΔOPQ

, $\tan 45^\circ = \frac{PQ}{OQ}$

Now the distance of the tower from the point is 30m, so,

$$\tan 45 = \frac{h}{30}$$

$$\Rightarrow 1 = \frac{h}{30}$$

$$\Rightarrow h = 30m$$

Hence, option (B)30 m is the correct answer.

8. The point P which divides the line segment joining the points A(2, - 5) and B(5, 2) in the ratio 2:3 lies in the quadrant: 1 Mark

a) I

(B) II

(C) III

(D) IV

Ans: Given the points A(2, - 5) and B(5, 2) are divided in the ratio 2:3. Thus, the point P is given by:

$$P = \left(\frac{2\times5+3\times2}{2+3}, \frac{2\times2-3\times5}{2+3} \right) = \left(3, -\frac{11}{5} \right)$$

Thus, the point clearly lies in the fourth quadrant. Hence option (D) IV is the correct answer.

9. The midpoint of segment AB is the point P(0, 4). If the coordinates of Bare (-2, 3) then the coordinates of A are :

(A) (2,5)

(B) (-2,-5)

(C) (2,9)

(D) (-2,11) **1 Mark**

Ans: It is given that P(0, 4) is the midpoint of segment AB .The point B is (-2, 3). Let the point A be (x, y), then,

$\frac{x+(-2)}{2}$. Solving this we get; x=2. For the point y.

$$\frac{y+3}{2}=4$$

$\Rightarrow y=8-3=5$. Hence, the point A is given by (2,5). Therefore, option (a) (2, 5) is the correct answer.

10. Which of the following cannot be the probability of an event? **1 Mark**

(A) 1.5

(B) $\frac{3}{5}$

(C) 25%

(D) 0.3

Ans: Out of these four options, 1.5 cannot be a probability of an event because it is greater than 1 and probability of any event cannot be greater than 1.

SECTION-B

Question Numbers 11 to 18 carry 2 marks each.

11. Find the value of p so that the quadratic equation px(x - 3) + 9 = 0 has two equal roots. **2 Marks**

Ans: Given the quadratic equation is $px(x-3)+9=0$.Simplifying this equation further, we get, $px^2-3px+9=0$.

Using the quadratic formula, we get $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Putting the values, we get: $x=\frac{3p\pm\sqrt{9p^2-4p(9)}}{2p}$. It is given that the equation has two equal roots, this implies, $9p^2-4p(9)=0$. Solving this to find p, we get

$9p^2-4p(9)=0$

$9p(p-4)=0$

$\Rightarrow p=0,4$

Therefore, the value of p is (0,4).

12. Find whether -150 is a term of the AP 17, 12, 7, 2,..........? 2 Marks

Ans: The given AP is 17,12,7,2............ Thus, we can see here that:

a=17

d=5

If -150 is a term, then, a_n =-150.Expanding the term to find the value of n,

$a_n = a+(n-1)d$

$\Rightarrow$ -150=17+(n-1)5

$\Rightarrow$ -150-17=(n-1)5

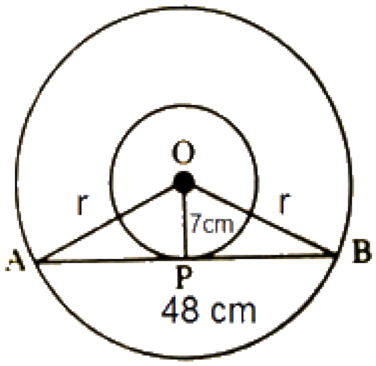
$\Rightarrow \frac{-167}{5}=n-1$

$\Rightarrow n=-\frac{162}{5}$

But this can't be an actual n^{th} term, thus, -150 does not belong to this AP.

13. Two concentric circles are of radii 7 cm and r cm respectively, where r >7. A chord of the larger circle, of length 48cm touches the smaller circle, find the value of r. 2 Marks

Ans: First of all, let us draw a figure of the two concentric circles,



We need to find the value of r. In the figure, we can clearly see that point P is the midpoint of AB. So, in ΔOPB, using Pythagoras theorem, we get:

$$OB^2 = OP^2 + PB^2$$

$$\Rightarrow r^2 = (7)^2 + (24)^2$$

$$\Rightarrow r^2 = 49 + 576$$

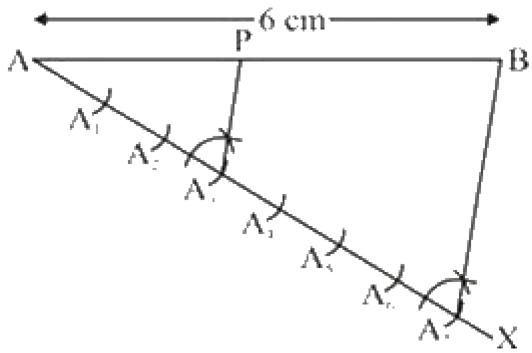
$$\Rightarrow r^2 = 625$$

$$\Rightarrow r^2 = 25\text{cm}$$

Hence, the value of r is 25cm.

14. Draw a line segment of length 6 cm. Using compasses and ruler, find a point P on it which divides it in the ratio 3:4. 2 Marks

Ans: In order to solve this question, we are first of all going to draw a line segment AB which is equal to length 6cm.

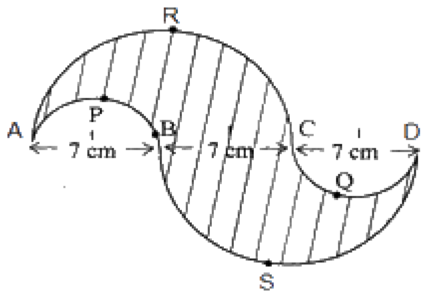


Next, you need to put the compass on points A and B and a perpendicular bisector of AB is drawn. The point at which the perpendicular bisector meets

AB is named O. The compass is put on points O and B and a perpendicular bisector of OB is drawn. Then, you should note that the Perpendicular bisector meets OB at a point which is named P which as given in the question divides the segment into 3:4.Thus,

$$\text{AP}=\frac{3}{4}\text{ of AB}$$

15. Fig. 3, APB and CQD are semi-circles of diameter 7 cm each, while ARC and BSD are semi-circles of diameter 14 cm each. Find the perimeter of the shaded region. [Use $\pi=\frac{22}{7}$]. **2 Marks**



Ans: Let us consider the semicircle ARC and BSD. They have diameters 14cm each, thus, the perimeter of ARC and BSD is equal to:

$$p_1=p_2=2\pi r=2\pi(7)=14\pi$$

The semicircles APB and CQD have diameters 7cm each.

Perimeter of the shaded region = Length of APB + Length of ARC + Length of CQD + Length of DSB . Now,

perimeter of APB $\frac{1}{2}\times 2\pi\left[\frac{7}{2}\text{cm}\right]=\frac{22}{7}\times\frac{7}{2}=11\text{cm}$.

Perimeter of ARC $=\frac{1}{2}\times 2\pi(7\text{cm})=\frac{22}{7}\times 7\text{cm}=22\text{cm}$

Perimeter of CQD $=\frac{1}{2}\times 2\pi\left[\frac{7}{2}\text{cm}\right]=\frac{22}{7}\times\frac{7}{2}\text{cm}=11\text{cm}$

Perimeter of DSB $=\frac{1}{2}\times 2\pi(7\text{cm})=\frac{22}{7}\times 7\text{cm}=22\text{cm}$

Thus, the perimeter of the shaded region = 11 cm + 22 cm + 11 cm + 22 cm = 66 cm.

Therefore, the perimeter of the shaded region is 66cm.

Or

Find the area of a quadrant of a circle, where the circumference of the circle is 44 cm. [Use $\pi=\frac{22}{7}$] **2 Marks**

Ans: It is given that the circumference of the circle is equal to 44 cm. If we let the radius of the circle to be r cm. $2\pi r=44$.

Solving this to find the radius.

$$r=\frac{44}{2\pi}=\frac{44}{2\times\frac{22}{7}}=7\text{ cm}$$

Thus, the area of the quadrant is

$A=\frac{1}{4}\times\pi r^2=\frac{1}{4}\times\frac{22}{7}\times7=\frac{77}{2}$. Thus, the area of the quadrant of the circle is equal to $\frac{77}{2}\text{ cm}^2$.

16. Two cubes, each 4 cm, are joined end to end. Find the surface area of the resulting cuboid. **2 Marks**

Ans: It is given that the cubes of side 4 cm each are joined end to end. The joining of two cubes will result in a cuboid of length, breadth and height equal to 8cm, 4cm, 4cm respectively. Hence, the surface area of the resulting cuboid is:

$S= 2(lb+bh+hl)$

$S=(8\times4+4\times4+4\times8)$

$S=160\text{ cm}^2$

17. Find that value(s) of x for which the distance between the points P(x, 4) and Q(9, 10) is 10 units. **2 Marks**

Ans: It is given that the distance between the points P(x, 4) and Q(9, 10) is 10 units. Finding the distance using the distance formula for two points, we get

$$PQ=\sqrt{(9-x)^2+(10-4)^2}$$

$$\Rightarrow PQ=\sqrt{(9-x)^2+36}$$

Putting the value of the distance, we get:

$$\Rightarrow 10=\sqrt{(9-x)^2+36}$$

$$\Rightarrow 100=(9-x)^2+36$$

$$\Rightarrow 64=(9-x)^2$$

$$\Rightarrow 9-x=\pm 8$$

Taking the two different cases, we get :

$$\Rightarrow x=9-8=1$$

And

$$\Rightarrow 9-x=-8$$

$$\Rightarrow x=9+9=17$$

Thus, $x=1,17$.

18. A coin is tossed two times. Find the probability of getting at least one head. 2 Marks

Ans: When a coin is tossed twice, then the total number of outcomes is 4. The probability of getting no heads is: The possible outcome is (T,T) The probability is $\frac{1}{4}$. Thus, the probability of getting at least one head is:

$$1-\frac{1}{4}=\frac{3}{4}.$$

SECTION-C

Question numbers 19 to 28 carry 3 marks each.

19. Find the roots of the following quadratic equation: $2\sqrt{3}x^2-5x+\sqrt{3}=0$.

3 Marks

Ans: We are given the equation, $2\sqrt{3}x^2-5x+\sqrt{3}=0$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{5\pm\sqrt{(-5)^2-4\times2\sqrt{3}\times\sqrt{3}}}{2\times2\sqrt{2}}$$

$$x=\frac{5\pm\sqrt{25-24}}{4\sqrt{2}}$$

$$x=\frac{5\pm1}{4\sqrt{3}}$$

Further simplifying, we get

$$x=\frac{6}{4\sqrt{3}};\frac{4}{4\sqrt{3}}$$

$$x=\frac{\sqrt{3}}{2};\frac{1}{\sqrt{3}}$$

Thus, the roots of the equation are: $x=\frac{\sqrt{3}}{2};\frac{1}{\sqrt{3}}$

20. Find the value of the middle term of the following AP: - 6, -2, 2, ..., 58.
3 Marks

Ans: We are given the AP, - 6, -2, 2, ..., 58

For finding the middle term , we need to find the position of the middle term. Thus, we will first find the total number of terms in this AP.

That is,

$a=-6$

$d=4$

$l=58$

Thus, finding the value of n: $l=a+(n-1)d$

Putting the values in this, we get:

$$58=-6+(n-1)4$$

$$\Rightarrow 58+6=(n-1)4$$

$$\Rightarrow \frac{64}{4}=n-1$$

$$\Rightarrow 16+1=n$$

$$\Rightarrow n=17$$

Hence, the middle term will be,

$$n_m=\frac{n+1}{2}.$$

$$\Rightarrow n_m=\frac{17+1}{2}=9$$ So, we need to find the 9th term,

$$a_9=a+(n-1)d=-6+(9-1)4$$

$$\Rightarrow a_9=-6+8\times4=-6+32$$

$$\Rightarrow a_9=26$$

Thus, the middle term of the AP - 6, -2, 2, ..., 58 is 26.

Or

Determine the AP whose fourth term is 18 and the difference of the ninth term from the fifteenth term is 30. 3 Marks

Ans: In this question, we are given the fourth term and the difference between ninth and fifteenth term, i.e. $a_4=18$. Expanding the terms in terms of the formula to find the AP, $a_{15}-a_9=30$

$a+(4-1)d=18$. Finding the other equation as well,

$$\Rightarrow a+3d=18$$

$$a+14d-a-8d=30$$

$$\Rightarrow 6d=30$$

$$\Rightarrow d=5$$

So, the common difference is 5. Putting the above equation:

$a+(4-1)d=18$

$\Rightarrow a+3(5)=18$

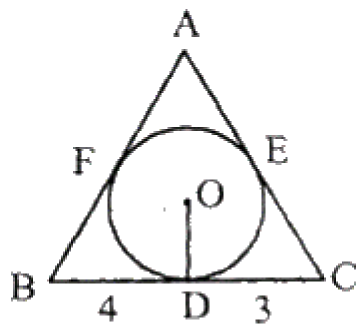
$\Rightarrow a=18-15=3$

Thus, the AP which is a,a+d,a+2d,18,…3+14d,…… becomes:

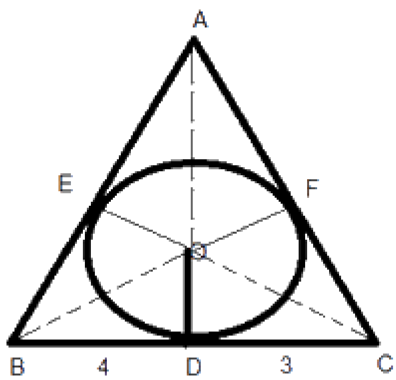
$3,3+5,3+10,18,.....3+70,....$

$3,8,13,18,.....,73,....$

21. In Fig. 4, a triangle ABC is drawn to circumscribe a circle' of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 4 cm and 3 cm respectively. If area of ABC= 21 cm2, then find the lengths of sides AB and AC. 3 Marks



Ans: In this question, we are given that the radius of the circle is 2cm and the area of triangle ABC is 21 cm^2, we need to find the lengths of the triangle. Construction: we need to join OA, OB, OC, OE ⊥ AB at E and OF ⊥ AC at F. The resulting figure will be:



Now we have, the following data:

AE = AF

$BD = BE = 4$ cm

$CD = CF = 3$ cm

Let the lengths of AE and Af be x. This implies, area of the triangle equals:

$$A = \text{area}(\Delta BOC) + \text{area}(\Delta AOB) + \text{area}(\Delta AOC)$$

$$\Rightarrow 21 = \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AB \times OE + \frac{1}{2} \times AC \times OF$$

$$\Rightarrow 42 = 7 \times 4 + (4 + x) \times 2 + (3 + x) \times 2$$

Solving this further, we get:

$$\Rightarrow 21 = 7 + (4 + x) + (3 + x)$$

$$\Rightarrow 21 = 14 + 2x$$

$$\Rightarrow x = 3.5$$

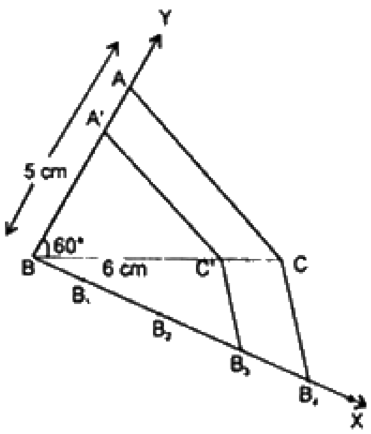
Thus, the lengths of the sides AB and AC are:

$$AB = 4 + 3.5 = 7.5$$

$$AC = 3 + 3.5 = 6.5.$$

22. Draw a triangle ABC in which AB = 5 cm, BC = 6 cm and $\angle ABC = 60^{o}$. Then construct a triangle whose sides are times the corresponding sides of ABC. 3 Marks

Ans: Construction of the triangle:



1. A line segment AB of length equal to 5 cm is drawn.

2. An angle ABY equal to 60° is drawn from point B.

3. Now in order to measure angle at B:

3.1 With B as centre and with any radius, another arc are drawn cutting the line which is present at D.

3.2 Taking the point D as centre and taking the same radius, an arc cutting the first arc is drawn at point E.

3.3 A ray BY that passes through the point E is forming the angle equal to 60° with the line AB.

4. Taking point B as centre and radius equal to 6 cm, an arc intersecting the line BY is drawn at C.

5. A and C are joined, the required triangle is ΔABC.

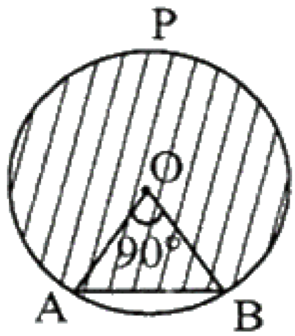
6. A ray from point A, named AX is drawn downwards which makes an acute angle.

7. Some points are marked named on the segment AX such that the lengths:

$AA_1=A_1A_2=A_2A_3=A_3A_4=A_4A_5=A_5A_6=A_6A_7$

8. The points A_7 and B are joined and from the point A_5, the segment A_5M is drawn parallel to A_7B that intersects AB at M. 9. Taking the point M, a segment MN is drawn parallel to BC intersecting AC at the point N. Then, ΔAMN is the required triangle, the sides of this triangle are equal to $\frac{5}{7}$ of the corresponding sides of ΔABC.

23. Find the area of the major segment APB, in Fig 5, of a circle of radius 35 cm and $\angle AOB=90^{o}$. [Use $\pi=\frac{22}{7}$] 3 Marks



Ans: For finding the area of the major segment, firstly we will write the equation for area of minor segment. Area of minor segment = Area of sector AOBC- Area of AOB

Putting the values in this equation, we get :

$$A=\frac{90^{o}}{360^{o}}\times\pi(OA)^{2}-\frac{1}{2}\times OA\times OB$$

$$\Rightarrow A=\frac{1}{4}\times\frac{22}{7}\times(35)^{2}-\frac{1}{2}\times35\times35$$

$$\Rightarrow A=962.5-612.5$$

$$\Rightarrow A=350cm^{2}$$

Thus, the area of major segment is equal to Area of major segment= Area of circle – Area of minor segment. Therefore,

$$A'=\pi(OA)^{2}-350$$

$$\Rightarrow A'=\frac{22}{7}\times(35)^{2}-350$$

$$\Rightarrow A'=3850-350=3500cm^{2}$$

Hence, the area of the major segment is $3500cm^{2}$.

24. The radii of the circular ends of a bucket of height 15 cm are 14 cm and r cm (r < 14 cm). If the volume of the bucket is 5390 cm3, then find the value of r. [Use $\pi=\frac{22}{7}$]. **3 Marks**

Ans: In the question, we are given that: Height of bucket, $h=15cm$. Radius of outer end,

$R=14cm$. Radius of inner end, $=r$. Volume of the bucket is, $V=5390cm^3$. Putting the formula for the volume above, we get:

$$\frac{1}{3}\pi h\left[R^2+r^2+Rr\right]=5390cm^3$$

$$\Rightarrow \frac{1}{3}\times\frac{22}{7}\times15\left[(14)^2+r^2+14r\right]=5390$$

$$\Rightarrow \frac{110}{7}\left[196+r^2+14r\right]=5390$$

$$\Rightarrow r^2+14r+196=343$$

$$\Rightarrow r^2+14r-147=0$$

Now solving this equation to find the value of r.

$$\Rightarrow r^2+21r-7r-147=0$$

$$\Rightarrow r(r+21)-7(r+21)=0$$

$$\Rightarrow r=7,-21$$

The value of a radius cannot be negative, so, the value of r is 7cm.

25. Two dice are rolled once. Find the probability of getting such numbers on two dice, whose product is a perfect square. 3 Marks

Ans: When two dice are rolled, the total number of possible outcomes is equal to 36

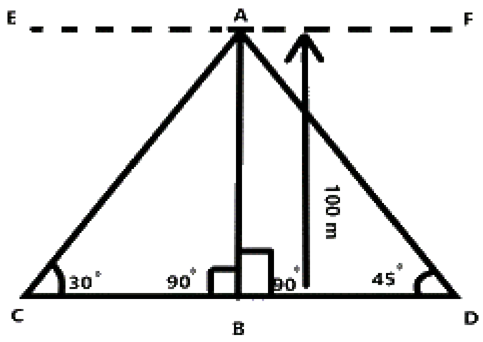
Now, the number pairs on the two dices whose product form perfect square have the possibilities, $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,4),(4,1)$. Thus, the number of possible outcomes for this event is 8. Hence, the probability of getting such numbers on two dice, whose product is a perfect square is $\frac{8}{36}=\frac{2}{9}$.

Or

A game consists of tossing a coin 3 times and noting its outcome each time. Hanif wins if he gets three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game. 3 Marks

Ans: It is given that the coin is tossed thrice which means that the total number of outcomes is $2^3 = 8$. The probability that Hanif loses is equal to one minus the probability that he wins. So, we will find the probability of him winning the game first Now, the outcomes for getting three heads or three tails will be: (H,H,H), (T,T,T). Thus, the probability of Hanif winning is $\frac{2}{8}=\frac{1}{4}$. Thus, the probability that Hanif will lose the game is $1-\frac{1}{4}=\frac{3}{4}$.

26. From the top of a tower 100 m high, a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the cars. [Use $\sqrt{3}$=1.73] 3 Marks



Ans: First of all, we are going to construct a diagram for the given situation.

Now it is given that Height of tower, $H=100m$. Angle of depression of car 1, $\angle EAC=30^o$. Angle of depression of car 2, $\angle FAD=45^o$. Now, in the right angled triangle ABC, $\tan30^o=\frac{AB}{BC}$. As $AB=100m$ and $\tan30^o=\frac{1}{\sqrt{3}}$. Thus, putting these values in above equation, $\frac{1}{\sqrt{3}}=\frac{100}{BC}$. In right angled triangle ABD, $\tan45^o=\frac{AB}{BD}$

$\Rightarrow BC=100\sqrt{3}$

Now, $AB=100m$ and

$\tan45^o=1$. Substituting the values in above equation.

$1=\frac{100}{BD}$.

$\Rightarrow$ BD=100m

Now, $\Rightarrow$ CD=CB+BD=$100\sqrt{3}+100=100\left(\sqrt{3}+1\right)$.

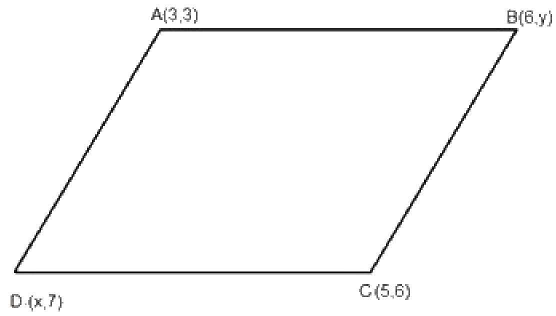
Putting $\sqrt{3}=1.732$. Thus, CD=$100\left(1.73+1\right)=273.2\text{m}$.

Therefore, the distance between the cars is 273.2m.

27. If (3, 3), (6, y), (x, 7) and (5, 6) are the vertices of a parallelogram taken in order, find the values of x and y. 3 Marks

Ans: It is given that the vertices of the parallelogram are (3, 3), (6, y), (x, 7) and (5, 6).

The figure is:



Now finding the midpoints of the diagonals AC and BD Coordinates of mid-point of diagonal BD are:

$$\left(\frac{5+6}{2},\frac{6+y}{2}\right).$$

$$\left(\frac{11}{2},\frac{6+y}{2}\right)$$

Coordinates of mid-point of diagonal AC are:

$$\left(\frac{3+x}{2},\frac{7+3}{2}\right).$$

$$\left(\frac{3+x}{2},\frac{10}{2}\right)$$

Now comparing the x and y coordinates of the midpoint as the mid points of both the diagonals coincide:

$\frac{3+x}{2}=\frac{11}{2}$. And the y-coordinate :

$\Rightarrow x=11-3=8$

$\frac{6+y}{2}=\frac{10}{2}$

$\Rightarrow y=10-6=4$

Thus, the values of x and y are 8 and 4 respectively.

28. If two vertices of an equilateral triangle are (3, 0) and (6, 0), find the third vertex. 3 Marks

Ans: Let the third vertex of the triangle be (x ,y). Thus, the vertices of the triangle are:

$A=(3,0)$

$B=(6,0)$

$C=(x,y)$

Finding the lengths of the sides of the triangle:

$AB=\sqrt{(6-3)^2}=3$

$BC=\sqrt{(x-6)^2+y^2}$

$CA=\sqrt{(x-3)^2+y^2}$

Equating CA and BC:

$(x-6)^2+y^2=(x-3)^2+y^2$

$(x-6)^2=(x-3)^2$

$\Rightarrow x-6=-(x-3)$

$\Rightarrow 2x=9$

$\Rightarrow x=\frac{9}{2}$

Using equation AB=BC and putting value of x:

$$(x-6)^2+y^2=9$$

$$\Rightarrow\left(\frac{9}{2}-6\right)^2+y^2=9$$

$$\Rightarrow\left(-\frac{3}{2}\right)^2-9=-y^2$$

$$\Rightarrow y^2=-\frac{9}{4}+9$$

$$\Rightarrow y^2=\frac{27}{4}$$

$$\Rightarrow y=\pm\frac{3\sqrt{3}}{2}$$

Thus, the values of x and y are $\frac{9}{2}$ and $\pm\frac{3\sqrt{3}}{2}$ respectively.

Or

Find the value of k, if the points P(5, 4), Q(7, k) and R(9, -2) are collinear.
3 Marks

Ans: The given points are collinear, hence taking two of them in pairs each, and finding the slope for them and equating the two slopes will give us the value of k. Thus, slope of PQ

$$\frac{k-4}{7-5}=\frac{k-4}{2}$$

Slope of QR:

$$\frac{-2-k}{9-7}=\frac{-k-2}{2}$$

Comparing both the slopes , we get:

$$\frac{k-4}{2}=\frac{-k-2}{2}$$

$$\Rightarrow 2k=-2+4=2$$

$\Rightarrow k=1$

SECTION: D

Question Numbers 29 to 34 carry 4 marks each.

29. A motor boat whose speed is 20 km/h in still water, takes 1 hour more to go 48 km upstream than to return downstream to the same spot. Find the speed of the stream. 4 Marks

Ans: Before proceeding with the question, we must know the formulas that will be required to solve this question.

We have a formula, time = distance/speed ....(1). In the question, it is given that the motorboat takes 1 hour more to go 48 km upstream than to return downstream to the same spot. Also, it is given that the speed of the motor boat in still water is equal to 20 km/hr. We are required to find the speed of the stream.

Let us assume the speed of the stream = x km/hr. It is given that the speed of the motor boat in still water is equal to 20 km/hr. While going upstream, the direction of the stream will be against the direction of the motorboat. So, the speed of the motorboat while going upstream will be equal to (20-x) km/hr. It is given that the distance to be travelled is equal to 48 km. So, using formula (1), the time taken by the motorboat to go upstream is equal to

$\frac{48}{20-x}$..........(2).

While going downstream, the direction of the stream will be along with the direction of the motorboat. So, the speed of the motorboat while going downstream will be equal to $(20+x)$km/hr. While going downstream, the direction of the stream will be along with the direction of the motorboat. So, the speed of the motorboat while going downstream will be equal to (20+x) km/hr. It is given that the distance to be travelled is equal to 48 km. So, using formula (1), the time taken by the motorboat to go downstream is equal to

$\frac{48}{20+x}$..........(3). In the question, it is given that the motor boat takes 1 hour more to go 48 km upstream than to return downstream. So, we can say that,

(Time taken while going downstream) + 1 = Time taken while going upstream

Substituting these times from equation (1) and equation (3), we get, $\frac{48}{20+x}+1=\frac{48}{20-x}$.

$$\Rightarrow \frac{48}{20-x}-\frac{48}{20+x}=1$$

$$\Rightarrow (48)\left(\frac{(20+x)-(20-x)}{(20-x)(20+x)}\right)=1$$

$$\Rightarrow (48)(2x)=(20-x)(20+x)$$

$$\Rightarrow 96x=400+20x-20x-x^2$$

$$\Rightarrow x^2+96x-400=0$$

$$\Rightarrow x^2-4x+100x-400=0$$

$$\Rightarrow x(x-4)+100(x-4)=0$$

$$\Rightarrow (x+100)(x-4)=0$$

$$\Rightarrow x=4,x=(-100)$$

Since speed cannot be negative, hence, the speed of the stream is 4 km/hr.

Or

Find the roots of the equation: $\mathbf{\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30},x^1-4,7}$. **4 Marks**

Ans: By expressing the given equation $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30},x^1-4,7$ in the form of a general quadratic equation we get:

Taking a common denominator $(x+4)(x-7)$ in the LHS,

$$\frac{(x-7)-(x+4)}{(x+4)(x-7)}=\frac{11}{30}$$

$$\frac{-11}{x^2-7x+4x-28}=\frac{11}{30}$$

Cross multiplying the terms to express as a quadratic equation, cancelling out 11 and multiplying with (-1) on both sides, $x^2-3x-28=-30$. Expressing the equation in the simplest form possible, $x^2-3x+2=0$. Comparing the above equation with the general quadratic equation we get the values of a, b, c as, $a=1, b=-3, c=2$. We know the formula to find the roots of a general quadratic equation is given as $\frac{-b\pm\sqrt{D}}{2a}$ where $D=b^2-4ac$ is known as the discriminant. Applying the above formula by substituting the values of a, b, c to find the discriminant, for the roots to be real and unequal the discriminant must be greater than 0. $\left[(-3)^2-4(1)(2)\right]>0$. Squaring -3 and multiplying -4 with 2 we get, $(9-8)>0$. Subtracting 8 from 9 the resultant is greater than 0. $1>0$. Hence the roots are real and unequal. Applying the formula to find the roots of a general quadratic equation by substituting the values of a, b, c and the value of the discriminant b^2-4ac we get, $\frac{-(-3)\pm\sqrt{1}}{2}$. By taking square root of 1 and simplifying the expression for: $\frac{-b+\sqrt{D}}{2a}$ and $\frac{-b-\sqrt{D}}{2a}$ we get, $x=2$.

$x=1$

Hence the roots of the equation $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}$ are 2 and 1.

30. If the sum of first 4 terms of an AP is 40 and that of first 14 terms is 280, find the sum of its first n terms.

Ans: Given that S4 = 40 and S14 =280'

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_n=\frac{n}{2}\left[2a+(4-1)d\right]=40$$

$$\Rightarrow 2a+3d=20.......(i)$$

$$S_{14}=\frac{14}{2}\left[2a+(14-1)d\right]=280$$

$$\Rightarrow 2a+13d=40.....(ii)$$

$$(ii)-(i)$$

$$10d=20\Rightarrow d=2$$

Substituting the value of d in (i) we get

$2a+6=10 \Rightarrow a=7$

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

$$=\frac{n}{2}\left[14+(n-1)2\right]$$

$= n(7+n-1)$

$= n(n+6)$

$=n^2+6n$

Therefore, $S_n = n^2 +6n$

Or

Find the sum of the first 30 positive integers divisible by 6

According to the question series is

6,12,18,24,….

Here,

a=6, d=6

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

n = 30

$$S_{30}=\frac{30}{2}\left[2\times6+(30-1)\times6\right]$$

$$S_{30}=15(12+29\times6)$$

$$S_{30}=15\times186$$

$$S_{30}=2790$$

31. A train travels 180 km at a uniform speed. If the speed had been 9 km/hour more, it would have take 1 hour less for the same journey. Find the speed of the train.

Ans:

Let the speed of train be x km /h

Distance = 180 km

So, time = 180 / x

When speed is 9 km/h more, time taken = 180 / x+9

According to the given information:

$$\frac{180}{1}-\frac{180}{x+9}=1$$

180 (x+9-x) / x(x+9) = 1

180×9 = x(x+9)

1620 = x2 + 9x

x2+ 9x - 1620 = 0

x2 + 45x - 36x -1620 = 0

x(x+45) - 36(x+45) = 0

(x-36)(x+45) = 0

x=36 or-45

But x being speed cannot be negative.

So, x = 36

Hence, the speed of the train is 36 km/h

Or

Find the roots of the equation $\frac{1}{2x-3}+\frac{1}{x-5}=1;x^{1}\frac{3}{2},5$

The given equation is $\frac{1}{2x-3}+\frac{1}{x-5}=1;x^{1}\frac{3}{2},5$

Taking L.C.M

$$\frac{x-5+\left(2x-3\right)}{\left(x-5\right)\left(2x-3\right)}=1$$

$$\frac{3x-8}{2x^{2}-13x+15}=1$$

Cross-multiplying

$2x^2-13x + 15 = 3x-8$

$2x^2 - 16x + 23 = 0$

Which is a quadratic equation.

Comparing with

$ax^2 + bx + c = 0$, then $a = 2$, $b = -16$, $c = 23$

So, $D = b^2 - 4ac$

$= (-16)^2 - 4.2.\ 23$

$= 256 - 184$

$= 72 > 0$

Therefore, using quadratic formula

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{6 \pm \sqrt{72}}{2.6}$$

$$= \frac{16 + 6\sqrt{2}}{4}, \frac{16 - 6\sqrt{2}}{4}$$

$$= \frac{8 + 3\sqrt{2}}{2}, \frac{8 - 3\sqrt{2}}{2}$$

So the roots are $= \frac{8 + 3\sqrt{2}}{2}, \frac{8 - 3\sqrt{2}}{2}$

32. In Figure 6, three circles each of radius 3.5 cm are drawn in such a way that each of[It them touches the other two. Find the area enclosed. between these three circles (shaded region). $\left[\textbf{use} = \pi = \frac{\mathbf{22}}{\mathbf{7}}\right]$

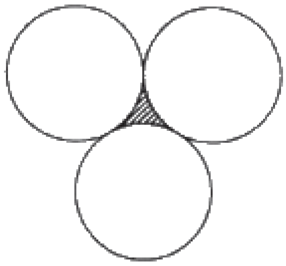


Figure 6

Given that, three circles are in such a way that each of them touches the other two.

Now, we join centre of all three circles to each other by a line segment. Since, radius of each circle is 3.5 cm.

So; AB = 2 × Radius of circle

= 2 × 3.5 = 7 cm.

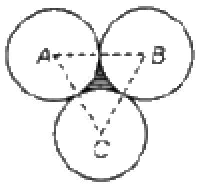
AC = BC = AB = 7cm

which shows that, Δ ABC is an equilateral triangle with side 7 cm.

We know that, each angle between two adjacent sides of an equilateral triangle is

60°

Area of sector with angle $\angle A = 60°$.



$$=\frac{\angle A}{360^\circ}\times\pi r^2=\frac{60^\circ}{360^\circ}\times\pi(3.5)^2$$

So, area of each sector = 3×Area of sector with angle A

$$=3\times\frac{60^\circ}{360^\circ}\times\pi\times(3.5)^2$$

$$=\frac{1}{2}\times\frac{22}{7}\times3.5\times3.5$$

$=11\times\frac{5}{10}\times\frac{35}{10}=\frac{11}{2}\times\frac{7}{2}$

$=\frac{77}{4}=19.25\,\text{cm}^2$

And Area of $\Delta ABC=\frac{\sqrt{3}}{4}\times(7)^2$

[area of an equilateral triangle $=\frac{\sqrt{3}}{4}$ (side)^2]

$=49\frac{\sqrt{3}}{4}\text{cm}^2$

Area of shaded region enclosed between these circles

=area of Δ ABC - Area of each sector

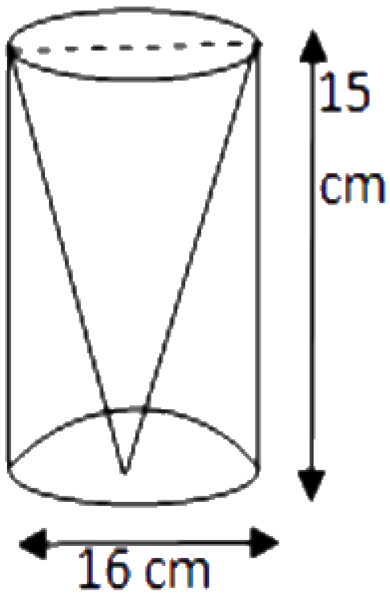
$= 49\frac{\sqrt{3}}{4}$-1925 -1225 × $\sqrt{3}$-1925

= 21.2176 - 1925=1 9676 cm^2?

Hence, the required area enclosed between these circles is 1.967 cm^2? (approx).

33. From a solid cylinder whose height is 15 cm and diameter 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Taken=3.14] 4 Marks

Ans:



In this question, we are given a solid cylinder with Height, $h=15cm$ .The diameter of the cylinder is, $d=16cm$. Thus, the radius, $R=8cm$. The slant height of the cone can be found as: $l=\sqrt{h^2+R^2}$. Putting the values above,

$l=\sqrt{(15)^2+(8)^2}=\sqrt{225+64}=\sqrt{289}$. Total surface of the solid is:

$\Rightarrow l=17$

T=Curved surface area of cone + Curved surface area of the cylinder
+Area of the bottom part

Putting the values, we get

$T=\pi Rl+2\pi Rh+\pi R^2$

$\Rightarrow T=3.14[8\times17+2\times8\times15+8\times8]$

$\Rightarrow T=3.14[440]$

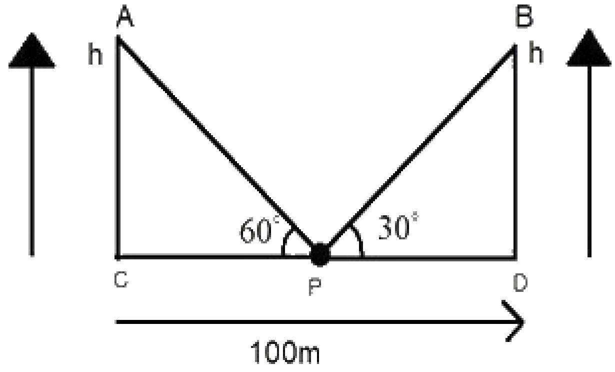
$\Rightarrow T=1381.6cm^2$

Thus, the total surface area of the remaining solid is $1381.6cm^2$.

34. Two poles of equal heights are standing opposite to each other on either side of the road, which is 100 m wide. From a point between them on the

road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles. 4 Marks

Ans: Let us firstly construct a figure reflecting the condition given in the question,



Considering the right-angled triangles, ABP and BPD, let the distance between the points C and P be x. Thus, in triangle ABP, $\tan 60^\circ = \frac{h}{x}$.

Putting here, the value of $\tan 60^\circ$

$\sqrt{3} = \frac{h}{x}$. Now, in triangle BPD,

$$\Rightarrow h = \sqrt{3}x \text{---}(1)$$

$\tan 30^\circ = \frac{BD}{PD} = \frac{h}{100-x}$. Using (1) in (2)

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100-x} \text{---}(2)$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{100-x}$$

$$\Rightarrow 3x = 100-x$$

$$\Rightarrow 4x = 100$$

$\Rightarrow$ x=25

Putting the value of x in (1); We get

$h=\sqrt{3}x=25\sqrt{3}$

The height of the poles is $25\sqrt{3}m$.