Previous Year Question Paper 2009

General Instuctions *:*
(i) All questions are compulsory.
(ii) The question paper consists of 30 questions divided into four sections — A, B, C and D. Section A comprises of ten questions of 1 mark each, Section B comprises of five questions of 2 marks each, Section C comprises of ten questions of 3 marks each and Section D comprises of five questions of 6 marks each
(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
(iv) There is no overall choke. However, an internal choice has been provided in one question of 2 marks each, three questions of 3 marks each and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) In question on construction, the drawings should be neat and exactly as per the given measurement.
(vi) Use of calculators is not permitted.

SECTION – A

Questions number 1 to 10 carry 1 mark each.

1. Find the (HCF $\times$ LCM] for the numbers 100 and 190.

Solution:

Given two numbers 100 and 190

Product of 100 and 190

$$\therefore HCF \times LCM = 100 \times 190$$
$$= 19000$$

2. If 1 is a zero of the polynomial p(x) = ax^2 - 3 (a - 1)x - 1, then find the value of a.

Solution:

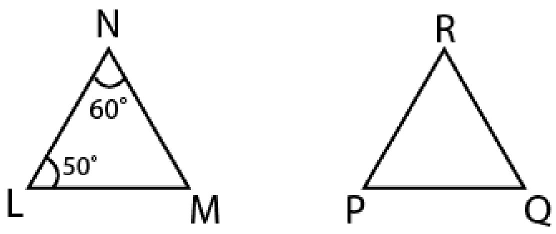
If x = 1 is the zero of the polynomial $p(x) = ax^2 - 3(a-1)x - 1$

Then $p(1) = 0$

$$\therefore a(1)^2 - 3(a-1) - 1 = 0$$
$$-2a + 2 = 0$$
$$a = 1$$

3. In Δ LMN, $\angle$L = 50° and $\angle$N = 60°. If Δ LMN ~ Δ PQR, then find $\angle$Q.

Solution:



Given $\Delta LMN \sim \Delta PQR$

In similar triangles, corresponding angles are equal.

$\therefore \angle L = \angle P$

$\angle M = \angle Q$ x

$\angle N = \angle R$

In ΔLMN

$\angle L + \angle M + \angle N = 180^\circ$

$\angle M = 180^\circ - 50^\circ - 60^\circ$

$\angle M = 70^\circ$

$\therefore \angle Q = 70^\circ$

4. If $\sec^2$ $\sec^2\theta(1+\sin\theta)(1-\sin\theta) = k$, then find the value of k.

Solution:

$\sec^2\theta(1+\sin\theta)(1-\sin\theta) = k$

$\Rightarrow \sec^2\theta(1-\sin^2\theta) = \kappa$

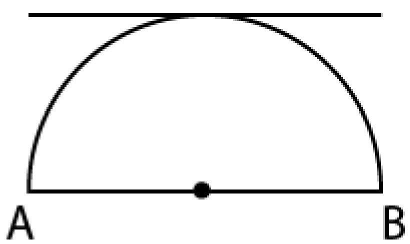
$\Rightarrow \sec^2\theta.\cos^2\theta = \kappa$

$\Rightarrow \dfrac{\cos^2\theta}{\cos^2\theta} = k$

$\Rightarrow k = 1$

5. If the diameter of a semicircular protractor is 14 cm, then find its perimeter.

Solution:



Given diameter of semicircular protractor (AB) = 14cm

Perimeter of a semicircle $= \pi\left(\dfrac{d}{2}\right) + d$

$\therefore$ Perimeter of protractor $= \pi\left(\dfrac{14}{2}\right) + 14$

$= \dfrac{22}{7} \times \dfrac{14}{2} + 14$

$= 36cm$

6. Find the number of solutions of the following pair of linear equations :

x + 2y — 8 = 0
2x + 4y = 16

Solution:

$x+2y-8=0$

$2x+4y-16=0$

For any pair linear equations

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

If $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, then

There exists infinite solutions

Here $\frac{a_1}{a_2}=\frac{1}{2},\frac{b_1}{b_2}=\frac{2}{4},\frac{c_1}{c_2}=\frac{-8}{-16}$

$\therefore \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{1}{2}$

Lines are coincident and will have infinite solutions.

7. Find the discriminant of the quadratic equation
$\therefore$
$3\sqrt{3}x^2+10x+\sqrt{3}=0.$

Solution:

$3\sqrt{3}x^2+10x+\sqrt{3}=0$

Discriminant for $ax^2+bx+c=0$ will be b^2-4ac

$\therefore$ For the given quadratic equation

$=(10)^2-4(3\sqrt{3})(\sqrt{3})$

$=100-36$

$=64$

8. If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a.

Solution:

Given $\frac{4}{5},a,2$ are in A.P.

$\therefore a-\frac{4}{5}=2-a$

$\Rightarrow 2a=\frac{4}{5}+2$

$2a=\frac{14}{5}$

$\therefore a=\frac{7}{5}$

9. In Figure 1, ABC is circumscribing a circle. Find the length of BC.

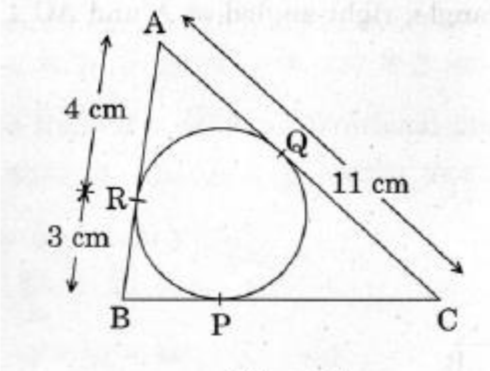
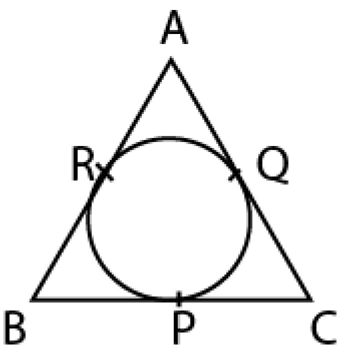


Figure 1

Solution:



Given BR = 3cm, AR = 4cm & AC = 11 cm

$BP = BR$

$AR = AQ$

$CP = CQ$

{Lengths of tangents to circle from external point will be equal}

$\therefore AQ = 4\ cm$ and $BP = 3cm$

As AC = 11 cm

QC + AQ = 11cm

$\Rightarrow QC = 7cm$

$\therefore PC = 7cm$

We know $BC = BP + PC$

$\therefore BC = 3 + 7$

$BC = 10cm$

10. Two coins are tossed simultaneously. Find the probability of getting exactly one head.

Solution:

Two coins are tossed simultaneously

Total possible outcomes $= \{HH, HT, TH, TT\}$

No. of total outcomes = 4

Favourable outcomes for getting exactly
One head = $\{Ht, TH\}$

Probability = $\frac{2}{4} = \frac{1}{2}$

SECTION B

Questions number 11 to 15 carry 2 marks each.

11. Find all the zeroes of the polynomial $x^3 + 3x^2 — 2x — 6$, if two of its zeroes are $-\sqrt{2}$ and $\sqrt{2}$.

Solution:

$x^3 + 3x^2 - 2x - 6 = 0$

Given two zeros are $-\sqrt{2}, \sqrt{2}$

Sum of all zeros = -3

Let the third zero be x

$\therefore x + \sqrt{2} + \left(-\sqrt{2}\right) = -3$

$x = -3$

$\therefore$ All zeros will be $-3, -\sqrt{2}, \sqrt{2}$

12. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21st term ?

Solution:

Given an A.P. 3, 15, 27, 39, ………

Lets say n^{th} term is 120 more than 12st term

$\therefore T_n = 120 + T_{21}$

$a + (n-1)d = 120 + (a + 20d)$

$(n-1)12 = 120 + 20 \times 12$

$n - 1 = 30$

$n = 31$

31st term is 120 more than 12st term.

13. In Figure 2, ABD is a right triangle, right-angled at A and AC $\perp$ BD. Prove that $AB^2 = BC \cdot BD$.

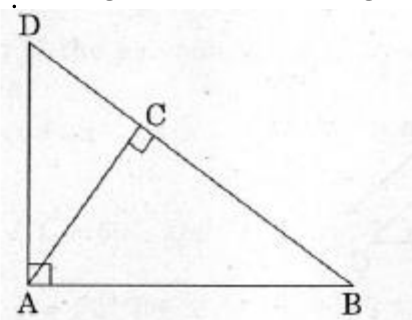
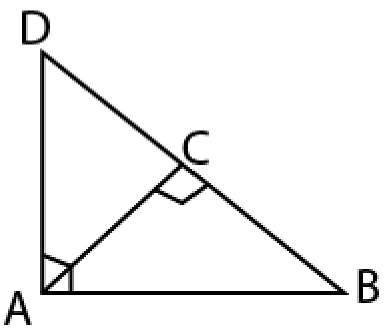


Figure 2

Solution:



In ΔABC

$AB^2 + AD^2 = BD^2$ ....(1)

In ΔABC

$AC^2 + BC^2 = AB^2$ ......(2)

In ΔACD

$AC^2 + CD^2 = AD^2$ ....(3)

Subtracting (3) from (2)

$AB^2 - AD^2 = BC^2 - CD^2$ ....(4)

Adding (1) and (4)

$2AB^2 = BD^2 + BC^2 - CD^2$

$2AB^2 = \left(BC + CD\right)^2 + BC^2 - CD^2$

$2AB^2 = BC^2 + CD^2 + 2BC.CD + BC^2 - CD^2$

$AB^2 = BC(BC + CD)$

$AB^2 = BC.BD$

14. If $\cot\theta = \frac{15}{8}$, then evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$.

OR

Find the value of tan 60°, geometrically.

Solution:

$\cot\theta = \frac{15}{8}$

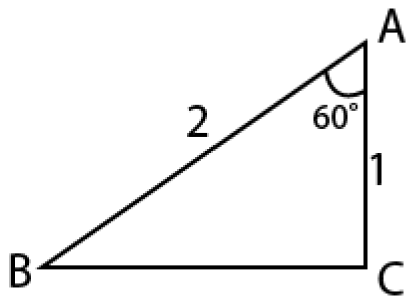
$\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos)(2-2\cos\theta)} = \frac{2(1-\sin^2\theta)}{2(1-\cos^2\theta)}$

$= \frac{\cos^2\theta}{\sin^2\theta}$

$= \cot^2\theta$

$= \frac{225}{64}$

OR



Consider a right triangle with $\angle A = 60°, AC = 1\,cm$ and AB = 2cm

$\tan 60° = \frac{BC}{AC}$

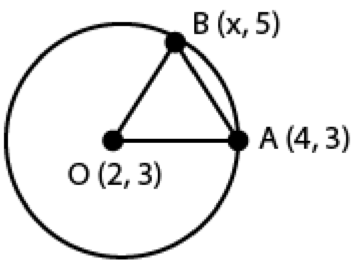
By Pythagoras theorem

$BC^2 = 4 - 1 \Rightarrow BC = \sqrt{3}$

$\therefore \tan 60° = \sqrt{3}$

15. If the points A (4, 3) and B (x, 5) are on the circle with the centre O (2, 3), find the value of x.

Solution:



OA = OB (radii)

$OA = \sqrt{(2-4)^2 + (0)^2} = 2$

$OB = \sqrt{(2-x)^2 + (3-5)^2} = \sqrt{(2-x)^2 + 4}$

$\Rightarrow 2 = \sqrt{(2-x)^2 + 4}$

$4 = (2-x)^2 + 4$

$\Rightarrow \boxed{x = 2}$

SECTION C

Questions number 16 to 25 carry 3 marks each.

16. Prove that $3 + \sqrt{2}$ is an irrational number.

Solution:

Lets assume $3 + \sqrt{2}$ is a rational number

$\therefore 3 + \sqrt{2} = \frac{p}{q}$ {p, q are integers and q$\neq$0}

$$\Rightarrow \sqrt{2} = \frac{p}{q} - 3$$

$$\Rightarrow \sqrt{2} = \frac{p-3q}{q}$$

$\frac{P-2q}{q}$ is rational number but we know $\sqrt{2}$ is an irrational.

Irrational $\neq$ rational

$\therefore 3+\sqrt{2}$ is not a rational number.

17. Solve for x and y

$$\frac{ax}{b} - \frac{by}{a} = a+b$$

$$ax - by = 2ab$$

OR

The sum of two numbers is 8. Determine the numbers if the sum of their . reciprocals is $\frac{8}{15}$.

Solution:

$$\frac{ax}{b} - \frac{by}{a} = a+b \quad ....(1)$$

$$ax - by = 2ab \quad ....(2)$$

Multiply (2) with $\frac{1}{b}$ and subtract (1) from (2)

$$\frac{a}{b}x - y = 2a$$

$$-\frac{a}{b} - \frac{by}{a} = a+b$$

$$+ \qquad -$$

$$y\left(\frac{b-a}{a}\right) = a-b$$

$$y = -a$$

Substituting y = -a in (1)

$$\frac{a}{b}x - \frac{b}{a}(-a) = a+b$$

$$\frac{a}{b}x = a$$

$$x = b$$

$\therefore x = b$ and $y = -a$

OR

Lets assume 2 numbers are x, y

Given $x + y = 8 \Rightarrow x = 8 - y \quad .....(1)$

$$\frac{1}{x}+\frac{1}{y}=\frac{8}{15}$$

$$\frac{x+y}{xy}=\frac{8}{15}\Rightarrow\frac{8}{xy}=\frac{8}{15}$$

$$\Rightarrow xy=15$$

From (1) $xy=y(8-y)=15$

$$\therefore y^2-8y+15=0$$

$$y=3,5\Rightarrow x=5,3$$

The numbers are 3 and 5

18. The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth term of the A.P.

Solution:

Lets say first term of given A.P. = a

Common difference = d

Sum of first six terms = 42

$$\therefore\frac{6}{2}\left(2a+5d\right)=42$$

$$2a+5d=14 \quad .....(1)$$

Also given $T_{10}:T_{30}=1:3$

$$\Rightarrow\frac{a+9d}{a+29d}=\frac{1}{3}$$

$$3a+27d=a+29d$$

$$\Rightarrow 2a=2d$$

$$\Rightarrow\boxed{a=d} \quad ......(2)$$

Substituting (2) in (1)

$$\Rightarrow 2a+5a=14$$

$$a=2 \text{ and } d=2$$

$$T_{13}=a+12d$$

$$=2+24$$

$$T_{13}=26$$

19. Evaluate :

$$\frac{2}{3}\text{cosec}^2 58^\circ-\frac{2}{3}\cot 58^\circ\tan 32^\circ-\frac{5}{3}\tan 13^\circ\tan 37^\circ\tan 45^\circ\tan 53^\circ\tan 77^\circ$$

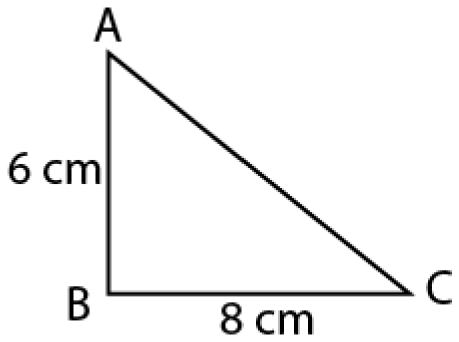
Solution:

$$\frac{2}{3}\cos ec^2 58^\circ-\frac{2}{3}\cot 58^\circ\tan 32^\circ-\frac{5}{3}\tan 13^\circ\tan 37^\circ\tan 45^\circ\tan 58^\circ$$

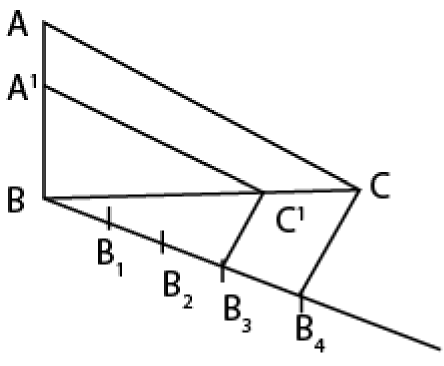
$\cot 58^\circ = \tan(90^\circ - 58^\circ) = \tan 32^\circ$
$\tan 77^\circ = \cot(90^\circ - 77^\circ) = \cot 13^\circ$
$\tan 53^\circ = \cot(90^\circ - 53^\circ) = \cot 37^\circ$
$\tan 45^\circ = 1$
Substituting in the given expression
$\therefore \Rightarrow \frac{2}{3}\operatorname{cosec}^2 58^\circ - \frac{2}{3}\cot^2 58^\circ - \frac{5}{3}$
$\Rightarrow \frac{2}{3}\left[\operatorname{cosec}^2 58^\circ - \cot^2 58^\circ\right] - \frac{5}{3}$
$\Rightarrow \frac{2}{3}(1) - \frac{5}{3}$
$\Rightarrow -1$

20. Draw a right triangle in which sides (other than hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the first triangle.

Solution:



Given ΔABC which is a right angled triangle $\angle B = 90^\circ$
Steps:
1. Draw line segment BC = 8cm, draw a ray BX making 90 with BC
2. Draw an arc with radius 6cm from B so that it cuts BX at A
3. Now join AC to form ΔABC



4. Draw a ray by making an acute angle with BC, opposite to vertex A
5. Locate 4 points B_1, B_2, B_3, B_4, on by such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$
6. Join B_4C and now draw a line from B_3 parallel to B_4C so that it cuts BC at C’

7. From C’ draw a line parallel to AC and cuts AB at A’
8. $\Delta A'BC'$ is the required triangle

Justification:

ΔABC and $\Delta A'BC'$

$\angle ABC = \angle A'BC'$ (Common)

$\angle ACB = \angle A'C'B$ (Corresponding angles)

By AA criterion $\Delta ABC \sim \Delta A'BC'$

$\therefore \frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'}$

In ΔBB_4C and $\Delta BB_3C'$

$B_4C || B_3C'$ [By construction]

$\therefore \Delta BB_4C \sim \Delta BB_3C'$

$\Rightarrow \frac{BB_4}{BB_3} = \frac{BC}{BC'} = \frac{B_4C}{B_3C'}$

We know that $\frac{BB_4}{BB_3} = \frac{4}{3}$

$\therefore \frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{3}{4}$

21. In Figure 3, AD $\perp$ BC and BD = $\frac{1}{3}$ CD. Prove that

2 CA^2 = 2 AB^2 + BC^2

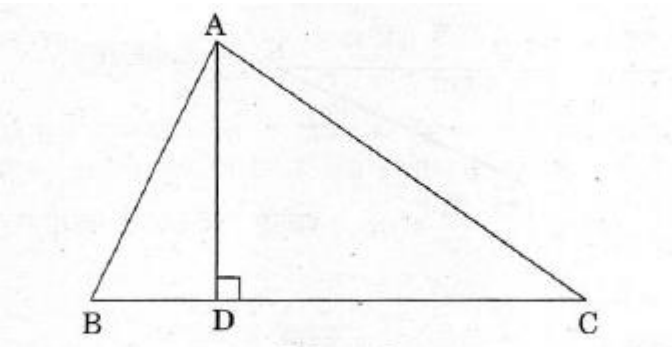


Figure 3

OR

In Figure 4, M is mid-point of side CD of a parallelogram ABCD. The line BM is drawn intersecting AC at L and AD produced at E. Prove that EL = 2 BL.

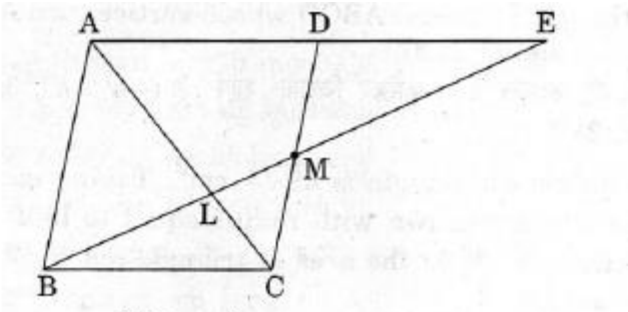
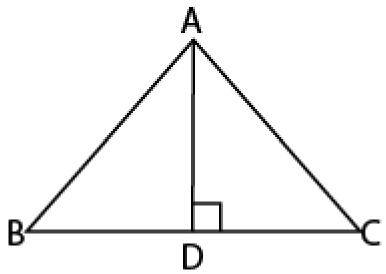


Figure 4

Solution:



$BD = \frac{1}{3}CD;$

$BD + CD = BC$

$CD = \frac{3}{4}BC$

$BD = \frac{1}{4}BC$

In right ΔACD,

$AC^2 = AD^2 + CD^2$ …………(1)   (Pythagoras Theorem)

In right ΔABD,

$AB^2 = AD^2 - BD^2$ ....(2)   (Pythagoras Theorem)

From (1) and (2), we get

$AC^2 = AB^2 - BD^2 + CD^2$

$\Rightarrow AC^2 = AB - \left(\frac{BC}{4}\right)^2 + \left(\frac{3BC}{4}\right)^2$

$\Rightarrow AC^2 = AB^2 - \frac{BC^2}{16} + \frac{9BC^2}{16}$

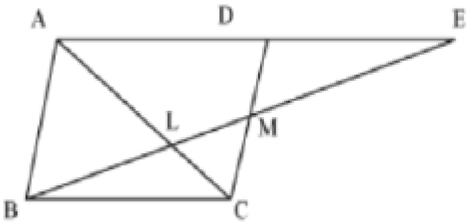
$\Rightarrow AC^2 = AB^2 + \frac{9BC^2 - BC^2}{16}$

$\Rightarrow AC^2 = AB^2 + \frac{8BC^2}{16}$

$\Rightarrow AC^2 = AB^2 + \frac{BC^2}{2}$

$\Rightarrow AC^2 = \frac{2AB^2 + BC^2}{2}$

OR



In ΔDME and ΔCMD

$\angle EDM = \angle MCB$ [Alternate angles]

DM = CM [mis midpoint of CD]

$\angle DME = \angle BMC$ [VOA]

By ASA congruency $\Delta DME \cong \Delta CMB$

By CPCT

BM = ME

DE = BC

Now in

ΔALE and ΔBLC

$\angle ALE = \angle BLC$ [VOA]

$\angle LAE = \angle LCB$ [Alternate angles]

By AA similarly

$\Delta ALE \sim \Delta BLC$

$\Rightarrow \frac{AE}{BC} = \frac{AL}{BL} = \frac{LE}{LC}$

$\Rightarrow \frac{EL}{BL} = \frac{AE}{BC}$

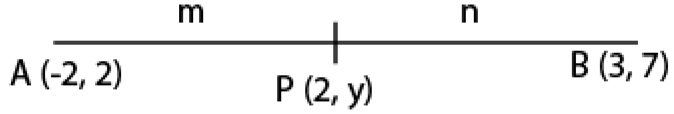
$\Rightarrow \frac{EL}{BL} = \frac{AD + DE}{BC}$

$\Rightarrow \frac{EL}{BL} = \frac{BC + BC}{BC}$

$\Rightarrow \boxed{EL = 2BL}$

22. Find the ratio in which the point (2, y) divides the line segment joining the points A (-2, 2) and B (3, 7). Also find the value of y.

Solution:



Lets say ratio = m : n

$\therefore (2, y) = \left(\frac{3m-2n}{m+n}, \frac{2n+7m}{m+n} \right)$

$2 = \frac{3m-2n}{m+n}$

$2m + 2n = 3m - 2n$

$m : n = 4 : 1$

$y = \frac{2+7\times 4}{5}$

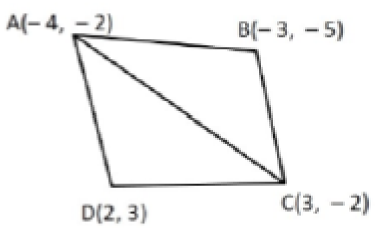
$y = \frac{30}{5}$

$y = 6$

$\therefore p(2,6)$

23. Find the area of the quadrilateral ABCD whose vertices are A (— 4, — 2), B (— 3, — 6), C (3, — 2) and D (2, 3).

Solution:



Joining AC

Area of Quadrilateral ABCD

$= ar(\Delta ABC) + ar(\Delta ADC)$

Area of triangle ABC = $\frac{1}{2}\left[-4(-5-(-2)) + (-3)(-2-(-2)) + 3(-2-(-5))\right]$

$= \frac{1}{2}\left[-4(-5+-2) + (-3)(-2+2) + 3(-2+5)\right]$

$= \frac{1}{2}[-4(-3) - 3(0) + 3(3)]$

$= \frac{1}{2}[12 - 0 + 9]$

$= \frac{1}{2}[21]\,\text{square units}$

Area of triangle ADC = $\frac{1}{2}\left[-4(3-(-2)) + 2(-2-(-2)) + 3(-2-3)\right]$

$= \frac{1}{2}[-4(3+2)-3(-2+2)+3(-2-3)]$

$= \frac{1}{2}[-4(5)-3(0)+3(-5)]$

$= \frac{1}{2}[-20-0-15]$

$= \frac{1}{2}|-35| = \frac{35}{2}$ sq. units

$\therefore$ Area of quadrilateral (ABCD) = $\frac{21}{2}+\frac{35}{2}$

$= 28$ sq. units

24. The area of an equilateral triangle is $49\sqrt{3}$ cm^2. Taking each angular point as centre, circles are drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles. [Take $\sqrt{3}$ = 1.73]

OR

Figure 5 shows a decorative block which is made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm and the hemisphere, fixed on the top, has a diameter of 4.2 cm. Find the total 2 surface area of the block. [Take $\pi = \frac{22}{7}$]

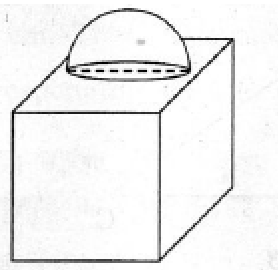


Figure 5

Solution:

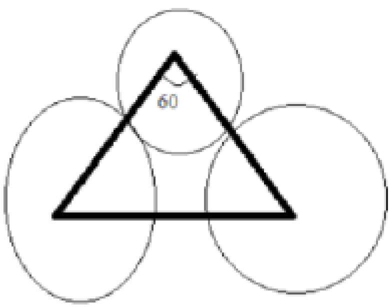
Let be the side of equilateral triangle.

$\frac{\sqrt{3}a^2}{4} = 49\sqrt{3};$

$a^2 = 49 * 4;$

a= 7 * 2 = 14 cm;

Radius of circle = 14/2 = 7 cm



Area of the first circle occupied by triangle = area of sector

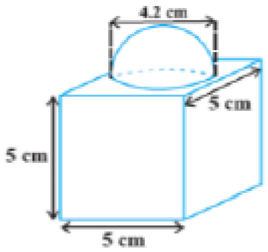
$$= \frac{1}{2} r^2 \theta = \frac{1}{2} * 7^2 * \pi * \frac{60}{180} = \frac{77}{3} cm^2$$

Area of all the 3 sectors = 77/3 * 3 = 77 cm^2

Area of triangle not Included In the circle = area of triangle- area of all the 3 sectors

$= 49\sqrt{3} — 77 — 7.87 cm^2$

OR



The total surface area of the cube = 6 (edge)^2 = 6 5 5 cm^2 = 150 cm^2

Note that the part of the cube where the hemisphere is attached is not Included in the surface area.

So, the surface area of the block = TSA of cube — base area of hemisphere + CSA of hemisphere

$$= 150 - \pi r^2 + 2\pi r^2 = (150 + \pi r^2) cm^2$$

$$= 150\, cm^2 + \left(\frac{22}{7} \times \frac{4.2}{2} \times \frac{4.2}{2} \right) cm^2$$

$$= (150 + 13.86)\, cm^2 = 163.86\, cm^2$$

× × ×

25. Two dice are thrown simultaneously. What is the probability that

(i) 5 will not come up on either of them ?

(ii) 5 will come up on at least one ?

(iii) 5 will come up at both dice ?

Solution:

Total outcomes = $6 \times 6 = 36$

(i). Total outcomes when 5 comes up on either dice are (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6) (6, 5) (4, 5) (3, 5) (2, 5) (1, 5)

P (5 will come up on either time) $\frac{11}{36}$

P (5 will not come up) = $1 - \frac{11}{36}$

$= \frac{25}{36}$

(ii) P (5 will come at least once) = $\frac{11}{36}$

(iii) P (5 will come up on both dice = $\frac{1}{36}$)

SECTION D

Questions number 26 to 30 carry 6 marks each.

26. Solve the following equation for x :

$9x^2 — 9 (a + b) x + (2a^2 + 5ab + 2b^2) = 0$

OR

If (-5) is a root of the quadratic equation $2x^2 + px — 15 = 0$ and the quadratic equation $p (x^2 + x) + k = 0$ has equal roots, then find the values of p and k.

Solution:

$$9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$$

Discriminant

$$D = 81(a+b)^2 - 36\left(2a^2 + 5ab + 2b^2\right)$$

$$D = 9\left[9a^2 + 9b^2 + 18ab - 8a^2 - 8b^2 - 20ab\right]$$

$$D = 9\left[a^2 + b^2 - 2ab\right]$$

$$\therefore \boxed{D = 9(a-b)^2}$$

$$\therefore x = \frac{+9(a+b) \pm \sqrt{9(a-b)^2}}{2 \times 9}$$

$$x = \frac{9(a+b) \pm 3(a-b)}{18}$$

$$x = \frac{3a+3b+a-b}{6}, \frac{3a+3b-a+b}{6}$$

$$\therefore x = \frac{2a+b}{3}; \frac{a+2b}{3}$$

OR

-5 is root of $2x^2 + 9x – 15 = 0$

$$\therefore 2(-5)^2 + p(-5) - 15 = 0$$

$$10 - p - 3 = 0$$

$$\therefore p = 7$$

$p(x^2 + x) + k = 0$ has equal roots

$7x^2 + 7x + k = 0$ [as we know p =7]

$\therefore$ Discriminant = 0

$$D = 49 - 28k$$

$$28k = 49$$

$$k = \frac{7}{4}$$

27. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

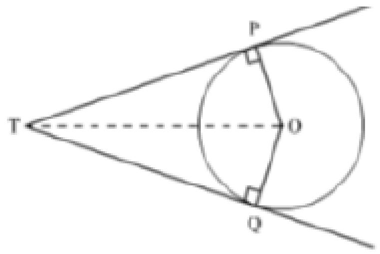
Using the above theorem prove that:

If quadrilateral ABCD is circumscribing a circle, then

AB + CD = AD + BC.

Solution:

PT and TQ are two tangent drawn from an external pant T to the circle C (O, r)



To prove: 1. PT = TQ

2. OTP = OTQ

Construction Join OT

Proof: We know that, a tangent to circle is perpendicular to the radius through the point of contact

OPT = OQT = 90

In OPT and OQT

OT = OT (Common)

OP = OQ (Radius of the circle)

OPT = OQT (90)

∠OPT ∠

≅

OQT (RHS congruence criterion)

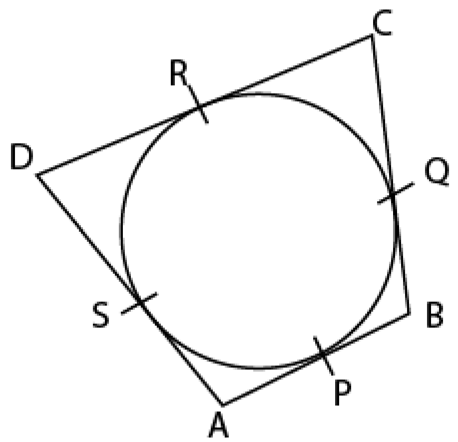
∴∠ ⇒ PT = TQ ∠and OTP = OTQ (CPCT)

PT Δ= TQ Δ

The lengths of the tangents drawn from an external point to a circle are equal.

OTP = OTQ

∠Centre ∠lies on the bisector of the angle between the two tangents.



Let AB touches the circle at P.BC touches the circle at Q.DC touches the circle at R.AD. touches the circle at S

THEN, PB = QB (Length of the tangents drawn from the external point are always equal)

QC =RC'

AP : AS

DS = DP

NOW, AB + CD

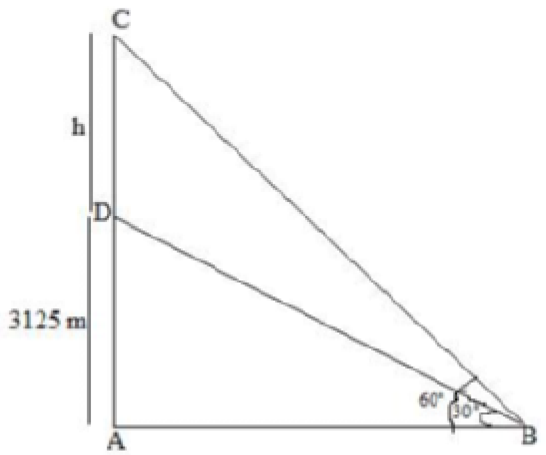
= AP + PB+DR+RC

= AS+QB+DS+CQ

= AS + DS + OB + CQ

= AD + BC
HENCE PROVED

28. An aeroplane when flying at a height of 3125 m from the ground passes vertically below another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 30° and 60° respectively. Find the distance between the two planes at that instant.
Solution:



Let the distance between the two planes = h m
Given that: AD = 3125 m and
$\angle ABC = 60^\circ$
$\angle ABD = 30^\circ$
In
ΔABD
$\tan 30^\circ = \frac{AD}{AB}$
$\frac{1}{\sqrt{3}} = \frac{3125}{AB}$
$\Rightarrow AB = 3125\sqrt{3}$ ....(1)
ΔABC
$\tan 60^\circ = \frac{AC}{AB}$
$\sqrt{3} = \frac{AD + DC}{AB}$
$\sqrt{3} = \frac{3125 + h}{AB}$
$\Rightarrow AB = \frac{3125 + h}{\sqrt{3}}$ ....(2)
Equating equation (1) and (2), we have

$$\frac{3125+h}{\sqrt{3}}=3125\sqrt{3}$$
$$h=3125\times 3-3125$$
$$h=6250$$
Hence, distance between the two planes is 6250 m.

29. A juice seller serves his customers using a glass as shown in Figure 6. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass has a hemispherical portion raised which reduces the capacity of the glass. If the height of the glass is 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14)

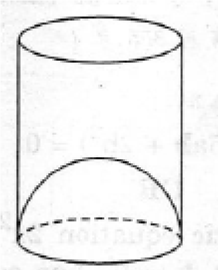
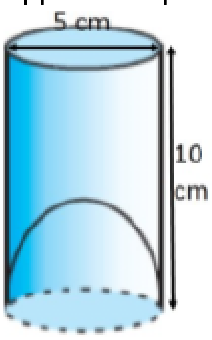


Figure 6

OR

A cylindrical vessel with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of base diameter 7 cm and height 6 cm is completely immersed in water. Find the volume of
(i) water displaced out of the cylindrical vessel.
(ii) water left in the cylindrical vessel.
[Take $\pi=\frac{22}{7}$]

Solution:
Apparent capacity of the glass = Volume of cylinder



Actual capacity of the glass = Volume of cylinder – Volume of hemisphere
Volume of the cylindrical glass = $\pi r^2 h$
$=3.14\times(2.5)^2\times 10$
$=3.14\times 2.5\times 2.5\times 10$
$=3.14\times 6.25\times 10$
$=196.25\ cm^3$

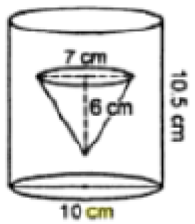
Volume of hemisphere $=\frac{2}{3}\pi r^3$

$=\frac{2}{3}\pi(2.5)^3 = 32.7\ cm^3$

Apparent capacity of the glass = Volume of cylinder = 196.25 cm^3
Actual capacity of the glass
= Total volume of cylinder — volume of hemisphere
= 196.25 — 32.7
= 163.54cm^3
Hence, apparent capacity = 196.25cm^3
Actual capacity of the glass = 163.54cm^3

OR

29.



Given, internal diameter of the cylinder =10 cm
Internal radius of the cylinder = 5cm
and height of the cylinder = 10.5 cm
Similarly, diameter of the cone = 7 cm
radius of the cone = 3.5 cm and Height of the cone = 6cm
i) Volume of water displaced out of cyindrical vessel = volume of cone

$=\frac{1}{3}\pi r^2 h$

$=\frac{1}{3}\times\frac{22}{7}\times 3.5\times 3.5\times 6 = 77\ cm^3$

ii) Volume of water left In the cylindrical vessel = volume of cylinder - volume of cone

$=\pi R^2 H - \text{Volume of cone}$

$=\frac{22}{7}\times 5\times 5\times 10.5 - 77$

= 825 —77 =748cm^3

30. During the medical check-up of 35 students of a class their weights were recorded as follows:

Weight (in kg)	*Number of students*
38 – 40	3
40 – 42	2
42 – 44	4
44 – 46	5

46 – 48	14
48 – 50	4
50 – 52	3

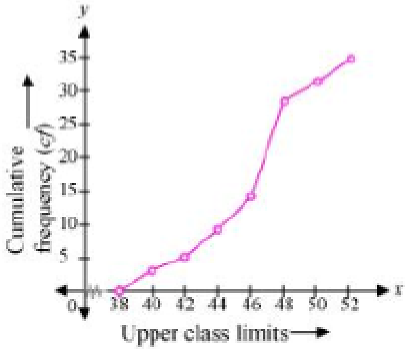
Draw a less than type and a more than type ogive from the given data. Hence obtain the median weight from the graph.

Solution:

Weight	cumulative (more than type)
More than 38	35
More than 40	32
More than 42	30
More than 44	26
More than 46	21
More than 48	7
More than 50	3
More than 52	0

Weight (in kg) Upper class limits	Number of students (cumulative frequency)
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
More than 52	35

Taking upper class limits on x-axis and their respective cumulative frequency on y-axis its give can be drawn as follows:



Here, n = 35

So,

$\frac{n}{2}$ = 17.5

Mark the point A whose ordinate is 17.5 and its x-coordinate is 46.5. Therefore, median of this data is 46.5.

It can be observed that the difference between two consecutive upper class limits is 2. The class marks with their respective frequencies are obtained as below:

Weight (In kg)	Frequency (f)	Cumulative, frequency
Less than 38	0	0
38 – 40	3 - 0 = 3	3
40 – 42	5 - 3 = 2	5
42-44	9-5=4	9
44 – 46	14 – 9 = 5	14
46- 48	28- 14 = 14	28
48-50	32 -28 = 4	32
50 – 52	35 – 32 = 3	35
Total (n)	35	

The Cumulative frequency just greater than

$$\frac{n}{2}\left(i.e., \frac{35}{2}=17.5\right)$$

is 28. Belonging to class interval 46 - 48

Medan class = 46 - 48

Lower class lima (l) of median class = 46

Frequency (f) of median class = 14

Cumulative frequency (cf) of class preceding median class = 14

Class size (h) = 2

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 46 + \left(\frac{17.5 - 14}{14}\right) \times 2$$

$$= 46 + \frac{3.5}{7}$$

$$= 46.5$$

=46.5

Therefore, median of this data is 46.5

Hence, the value of median is verified.